MOLYNEU
OF
DIOPT

$=$ a a ~~b~~ b

p^{d} 191

$=$ equal
$\sqsupset$ more
$-$ less
$\times$ multiplied
$\sqsupset$ lesser
$\sqsubset$ greater
$\angle$ angle
$\triangle$ triangle
$\surd$ square root
q y^{e} square of a

($\div$) this show continuall proportion
a $\div$ a to b to c to d $\div$ a to c

$\frac{a}{b}$ is letter to shew a must be divided by b

$a \times b + c$ y^{t} a must be multiplied by $b+c$

Davey
3/8/11

I think this Book fit to be Printed.

June the 4th. 1690. JOHN HOSKYNS. *V.P.R.S.*

DIOPTRICA NOVA.

A
TREATISE
OF
DIOPTRICKS,
In Two PARTS.

Wherein the

Various Effects and Appearances

OF

Spherick Glasses,

BOTH

Convex and Concave, Single and Combined,

IN

TELESCOPES and MICROSCOPES,

Together with

Their Usefulness in many Concerns of Humane Life,

ARE EXPLAINED.

By William Molyneux of *Dublin* Esq;
Fellow of the ROYAL SOCIETY.

Ex Visibilibus Invisibilia.

London: Printed for Benj. Tooke, MDCXCII.

To the Illustrious

The Royal Society.

THE *Design of the ensuing Treatise being the Promotion of a Part of Physico-Mathematical Knowledge in the* English *Nation; I know not to whom I can more properly present it, than to that noble Body of* English *Philosophers, whose Foundation by the Royal Charter of King* Charles II. *is to this very purpose. How far, and how successfully you have hitherto prosecuted the end of this excellent Institution, 'tis needless for me to declare: since the literate World is already so abundantly stored with your learned Labours, and useful Discoveries; whereof I could here recount a List of many Hundreds published by several Members of the Society. But I design not a Panegyrick, but an humble Address for your Favour and Countenance to my present Endeavours: And this I hope for, with the more Assurance, having already seen your favourable Acceptance of many Offerings of this kind, and your ready Incouragement of all such Philosophical Inquiries, as tend to the use of Life, or Advancement of Arts and Sciences.*

And on this Occasion I cannot omit expressing my Sence of that excellent Method of Experimental Philosophy, which now, by your Example and Incouragement, does so universally prevail, and is so highly advanced all over Europe, *and other Parts of the World.*

'Tis wonderful to consider, how the Schools were formerly overrun with a sensless kind of Jargon, which they call'd Philosophy;

and

and which men ſtudied with the greateſt Labour and Aſſiduity, that they might attain the name of Wiſe *and* Learned. *This certainly was the greateſt Cheat was ever impoſed on the mind of Man: But why ſay I,* impoſed? *Men drew it on themſelves, and run their own Heads into the Nooſe: and when they had intangled themſelves in a thouſand ridiculous Diſputes about empty Queſtions, they vainly thought they had attained the Perfection of Philoſophers; whilſt they had no Ideas in their Minds anſwerable to thoſe Noiſes they made with their Tongues; but took more pains to deceive both themſelves and others, than is requiſite for the Propagation of* true *Knowledge. And indeed we may well imagine, that, had the former Ages of the World been at half that Labour and Study for the Advancement of* real *Knowledg, which they ſpent in promoting verboſe Stuff; Mankind by this time might have been by many Degrees* more wiſe, *and conſequently* more happy *even in this Life; for* Wiſdom *only makes men ſo.*

But in this laſt Age the generous Undertakings of the Philoſophick Societies of Europe *(to whom your Inſtitution has ſhewn the way, and been an illuſtrious Example) have diſſipated theſe dark Miſts, and have abdicated this kind of empty Stuff; which had crept into even Natural Diſquiſitions; and like a Leproſie had quite over-run the whole Body of Philoſophy, deforming its Beauty, and ruining its Strength. Men are not ſatisfied now with* noiſy Words, *and nothing elſe; but require more ſolid Foundations of Knowledge, and believe no farther than they can find good Proofs.*

This great Change, which Philoſophy or the Proſecution of Knowledg in general has received of late years, is manifeſt in all its Parts; but in none more than in natural Enquiries. *To theſe you have given a clearly new Turn, wholly different from the Methods, by which they were formerly proſecuted in the Schools. And how advantageouſly the Change has been made, will be evident to any that conſiders the one and t'other Method.*

The

The Commentators on Aristotle, (*who was certainly himself a most diligent and profound Investigator of Nature*) *have rendred* Physicks *an heap of froathy Disputes, managing the whole Knowledge of Body and Motion, of Animals, Plants, and Minerals; of Celestial, Aerial and Terrestrial Bodies; by Hypothetical Conjectures, confirm'd by plausible Arguments of Wit and Rhetorick, ordered in a Syllogistical form; and answering Objections in like manner: But never studied to prove their Opinions by Experiments. By which Method they were as ignorant of the Properties and Affections of Natural Bodies, as if they were not at all the Subject of their Disquisitions. And yet these were the great Dictators of Physicks for many Ages in our Colleges and Schooles; and no one was accounted worthy the Name of a Philosopher, that would not on their Authority* Jurare in verba. *He that could* Dispute *and* distinguish, *about Sympathy, Antipathy, occult Qualities, Antiperistesis, and a Thousand such other fantastick Terms, was reputed a great Proficient, and deeply vers'd in Natures Secrets: tho all the while perhaps he knew not any one of the admirable Phænomena of the Magnet, or was not at all versed in the History of any one Branch in Nature. They'd rumble out indeed the Definition and Divisions of* Comets; *but knew nothing of the Laws of their Motions, or other Affections. They'd tell you the Tides depend on the* Influence *of the Moon; and when you proceed farther, and ask, what is this Influence? They'll yet give you a* Word *for it, and say, 'tis an* occult Quality: *If you inquire, what an* occult Quality *is? They'r at a Stand, and having no farther* hard Word *here to fly to, are forced to confess 'tis a Quality they know nothing of. Had they not better at first have plainly confest, they know not the Cause of the Tides? no surely; For tho this had been more becoming modest Philosphers, it would not so well captivate the Vulgar, and gain to themselves the Repute of deep Knowledge.*

Consult *Magirus, Eustathius, Zanardus, Col. Complutensis Com. &c.* the common Natural Philosophy Books read in our Colleges.

Yet this verbose Philosophy is that, which for many Generations prevail'd in the World: This it is, which is enjoyn'd to be read and studied in our Colleges and Academies, by the Statutes and Charters

thereof;

thereof: which in this Particular, to the apparent Hindrance of the Advancement of real and useful Knowledge, do yet remain unaltered in our Universities: wherein the first years of young Students may be imploy'd with much more Advantage by prosecuting other Methods. And, My thinks, it were now full time (after so happy a Reformation of our Errors in Religion, and purging our Seminaries of Learning from the Fopperies and Superstition of a false Worship) to begin a Reformation of our Human Literature, by establishing more useful Methods of Education, especially for the Employment of our more tender years.

But tho this weighty Undertaking has hitherto been deferr'd in them (the Reason whereof I leave to the Consideration of the learned and reverend Heads of our Universities) yet the strong Wits of many in this last Age have broken all these Fetters; And have happily advanced the true Method of prosecuting Knowledge upon solid Foundations.

This is manifest in every Branch of Learning. Logick *has put on a Countenance clearly different from what it appeared in formerly: How unlike is its shape in the* Ars Cogitandi, Recherches de la Verite, *&c. from what it appears in* Smiglетius, *and the Commentators on* Aristotle? *But to none do we owe for a greater Advancement in this Part of Philosophy, than to the incomparable Mr.* Locke, *Who, in his* Essay concerning Humane Understanding, *has rectified more received Mistakes, and delivered more profound Truths, established on Experience and Observation, for the Direction of Man's mind in the Prosecution of Knowledge, (which I think may be properly term'd* Logick) *than are to be met with in all the Volumes of the Antients. He has clearly overthrown all those Metaphysical Whymsies, which infected mens Brains with a Spice of Madness, whereby they* feign'd a Knowledge where they had none, by making a noise with Sounds, without clear and distinct Significations.

Natural

To the Royal Society.

Natural Philoſophy *is now proſecuted by Obſervation, Experiment, and Hiſtory thereof. And indeed if we conſider it rightly, there is really no other ſort of Natural Philoſophy, but this only. For by* Natural Philoſophy, *or* Phyſicks, *do we mean any thing elſe, but the Knowledge of the Properties and Affections of Natural Bodies? And is this to be obtain'd any otherwiſe, than by Experiment and Obſervation? Can any Man Diſpute me into the Knowledge of the* Magnet's *Attraction, Direction and Variation; or of the Phænomena of the Mercurial Baroſcope without Tryal and Experiment? Can any Arguments prove that a little Sulphur, Nitre, and Charcole ſhould produce ſuch a quick and ſtrong Blaſt as Gunpowder, before they be actually put together and tryed? Men might have diſputed to all Eternity, before their Gibberiſh could diſcover the Uſe of that ordinary deſpicable Subſtance,* Iron-Ore: *To which, for ought I ſee, (as a moſt ingenious Author has obſerv'd) we are beholding for all the Politure and Plenty; all the Learning,* State, *and Magnificence of the World, beyond the Rudeneſs, Wants, and Ignorance of the ancient ſavage* Americans: *Whoſe Natural Endowments and Proviſions equal thoſe of the moſt flouriſhing and polite Nations; But they wanted the Advantagious Uſes of this contemptible Mineral. So that he, who firſt diſcovered the Uſe of that one poor Mine; or* Tubal-cain, *that firſt taught the way of working in Iron, may be deſervedly celebrated as the Father of Arts, and Author of moſt of the Conveniences of Human Life.*

I know ſome will ſay, that by Natural Philoſophy *is meant not only the Knowledge of the Properties and Uſes of Natural Bodies; but alſo the Aſſigning the true Reaſons or Cauſes of theſe Properties. But in this Particular we are to proceed with great Caution. I know the Mind of man is of that inquiſitive, prying Nature; that upon any Appearance offer'd to the Senſes, it immediately falls to the ſearch after the Cauſe producing this Effect. But indeed in Natural Diſquiſitions, 'tis generally (I may ſay almoſt alwayes) to no purpoſe. We may make plauſible Conjectures, and ſome ſort of feaſible Gueſſes; but others perhaps may make others, and theſe alſo*

equally probable. But these deserve not the Name of Natural Philosophy; *they serve only for Chat and Diversion. For the* omnipotent Contriver *of the Universe has order'd Natures Operations to be performed by such fine Springs, secret Motions, and inexplicable Ways; that Man in this Life may well despair of attaining the intimate Knowledge thereof; and must therefore content himself with the Contemplation of plain matter of Fact, in which he cannot be deceived. But yet, that we may not wholly suppress this inquisitive Humour, but may only keep it within just Bounds: It will be granted, That whatever immediate Cause can be assigned to an Effect; and it can be proved so to be by some convincing Experiments; and these be often repeated, and diligently examin'd, and found to agree and conspire together; we may be allow'd to found thereon an* Hypothesis *or* Supposal *of this Cause, but no more. We must not positively establish it as the undoubted, adequate Cause; for this we may miss after our most diligent Inquiry. However the Experiments we use and demonstrate to sense, for the establishing our Hypothesis, shall be allowed as unquestionable Verities; and shall be embraced as so many Steps of Advancement in the Knowledge of Nature.*

But of the Uncertainty of Assigning Natural Causes I shall give but one Instance, and that perhaps as strong as any we shall meet with in Philosophy. We are apt to think, that the Cause of the Suspension of the Mercury in the Torricellian *Experiment is undoubtedly the* Gravitation *of the* Air: *And we prove it by a most convincing Experiment; for putting the Baroscope into the Pneumatick Engine, exhaust the Air, and the Mercury immediately subsides. But when we consider it a little farther, we shall find, That hereby we have obtain'd little more certain Knowledge, than plainly the matter of Fact of this latter Experiment, and not the adequate Cause of the first Experiment enquired after. For we can only conclude from hence, that the Equipoise of Liquors is the Cause of the Mercury's Suspension: But what is the Cause of this Equipoise of Liquors, or the Cause of the Gravitation of any Liquors, or any Bodies? That is, What is the Cause of Gravity in general is clearly unknown to us;*

and

and consequently the ultimate Cause of the Mercury's Suspension is not hereby discovered. 'Tis true indeed, by this Experiment we have most probably arriv'd at the Knowledge of one Link more *in the* Chain *of Natural Causes; but this is not conclusive; this puts not an end to the Enquiry: For so if one looking at a Pendulum Clock should enquire, Why the Pendulum does not cease by degrees from vibrating? And he were answered, That it is kept in motion by the next immediate Wheel that beats on the Pallats: And this were offer'd to be proved by Experiment; for stop the motion of this Wheel, and the Pendulum soon ceases: Would he not presently be satisfied, and go away secure, that he had discovered the Cause of the Continuation of the Pendulum's Motion? And yet certainly he has mistaken* one single Link *for the* whole Chain. *For if he proceeded farther; and had enquired, What moved this Wheel? He would find, the next Wheel, and so onwards to the Weight or Spring. But here he's at a Loss; for what moves* them, *is absolutely unknown.*

As to most other Reasons in Natural Philosophy, *that usually pass as satisfactory, and are received as Accounts of Nature's Proceedings: We shall generally find them little more, than farther Illustrations of the Matter enquired after in some* different Words. *Thus if it be asked, What is the Reason the Sun casts a shadow from some Bodies,* viz. *those we call opaque, and none from others,* viz. *Transparent Bodies? 'Tis answer'd, Because by the opaque Body the Rays of the Sun are stopt in their Progress, and hindred from enlightning that Part of the Ground or Floor that is behind the Body: In the Transparent Body the Rays pass freely. This would be taken by several as deep Reasoning; and yet is really no more than Tautology; as if we should say, the Sun casts a Shadow, Because he casts a Shadow. But it gives no Account of the Opacity of one Body, and Transparency of another, which truly the Question requires. If we ask, How Fire burns? 'Tis answered, by exciting a violent Motion in the Parts of the Combustible Matter: Which indeed is no more than the same thing in different Words. But how Motion is excited or communicated by one Body to another, is absolutely inexplicable. Yet these*

Kind of Verbal *Reasons do generally pass in Talk, and serve to amuse as well as the best.*

Since therefore we cannot expect to arrive at the intimate Knowledge of Natures Operations: Let us apply our selves to know as much of Her, as we may be certain of. And this only is in Matters of Experiment and Tryal; wherein by the infallible Guidance of our Sences, we cannot be deceived. For tho, Experimentum periculosum, *be an ancient Aphorism; yet we must consider of what Experiments it was pronounced,* viz. *of those in Medicine and Diseases. It agrees not to all, when diligently enquired into, and often repeated. But when we meet with any Experiment that is thus fallacious, we are not to rely on it; yet of this we may be sure, and lay it down as a discover'd Certainty, that* sometimes it hits, sometimes it misses; *and in this Truth we cannot be deceived, seeing we have so often found it.*

But I have lanch'd out thus far, before I was aware: I must recover my self, and beg Pardon both for this Digression, and for telling you Things which you very well know already, at the same time when I offer you a Petition. But my Desire of propagating this useful Method of Philosophy will excuse my Fault, and at the same time will recommend me the more to your Favour, who are the great Patrons thereof, and in Account whereof your Name is deservedly celebrated all over Europe.

Permit me therefore to lay this Offering at your Feet, it being the Explication of one of the most noble Instruments of Experimental Philosophy. Not that I think thereby to add any think considerable to the vast Treasure of Curiosities you already possess: But that I may have an Opportunity of declaring to the World, how much I am

April 17.
1690.

Your Devoted

Humble Servant,

WILLIAM MOLYNEUX.

ADMONITION
TO THE
READER.

BEFORE the Reader proceed to the following Sheets, I deſire he may take notice;

Firſt, That he is not to expect *Geometrical Strictneſs* in ſeveral Particulars of this Doctrine. I ſay, in *ſeveral* Particulars; For many there are, which will bear the moſt *preciſe Exactneſs*: *Kepler*, *Cavallerius*, *Herigon*, *Dechales*, *Honoratus Faber*, *Gregorius*, *Barrow*, and other Authors have taken this Liberty; as being more deſirous of ſhewing in groſs the Properties of Glaſſes and their Effects in Teleſcopes, than of affecting a *Nicety*, which would be of little Uſe in Practice. Thus we ſhall find in what follows, that many Lines are ſuppoſed *equal*, which ſtrictly taken are really not ſo; but yet are ſo very little different, that for all uſe, and eaſe of Demonſtration, they may be taken as *equal*. Thus alſo we ſuppoſe very ſmall *Angles* and their *Sines* to be *proportional*; which *preciſely* is not ſo, but is ſo to the ſmalleſt and moſt inſenſible Difference. Thus likewiſe, we ſometimes conſider not the *Glaſſes Thickneſs*, but ſuppoſe it of the *leaſt Thickneſs imaginable*, or of no Thickneſs at all; which yet is falſe, but does hardly prejudice any Demonſtration. For *Dioptricks* being a part of *mixt Mathematicks*, converſant about *material Lines* (or Rays of Light) and the refractive Power of a *corporeal* Glaſs, cannot be delivered with that 'Ακρίβεια *Geometrica* requiſite in *abſtracted Mathematicks*.

Secondly, The Reader will find ſome *Corollaries* and *Scholiums* hereafter delivered, which of themſelves more properly might have conſtituted Propoſitions. But theſe did not occur ſo readily in the courſe of the Work, And therefore I choſe rather to add them as *Corollaries*

than

than change the number of Propositions and Citations. Of this kind are *Corol. ad Prop.* 8. *Corol. ad Prop.* 15. *&c.*

Thirdly, I have presumed to intitle this Work *Dioptrica Nova*, as being indeed almost *wholly new*. Very little being borrowed from others, but what is requisite to shew the former Methods of Authors in demonstrating their Propositions, and to keep up the Consecution and Series of the Propositions of this Book. Besides, I can say, that the *Geometrical Method* of calculating a Rays Progress, which in many particulars is so amply delivered hereafter, is *wholly new*, and never before publish'd. And for the first Intimation thereof, I must acknowledg my self obliged to my worthy Friend Mr. *Flamsteed. Astron. Reg.* who had it from some unpublished Papers of Mr. *Gascoignes*.

Lastly, I declare, That if in any thing hereafter delivered, I have made any Mistakes, or not so clearly expressed my self upon Intimation thereof, I shall be most ready to *retract*. And therefore if in any thing I have *slip'd*, or made a *false Step*, I desire the ingenious and candid Reader either to inform me thereof, and upon Conviction, I shall *submit*; or else that he would freely pardon the Error; *Humanum est*.

Tho I had begun, and made some Progress in this Work in *Latin*; yet I thought it convenient (at least for the present) to alter my Design: There being nothing in *this Part of Mathematicks* ever yet publish'd in *English*. And I am sure there are many ingenious Heads, great Geometers, and Masters in Mathematicks, who are not so well skill'd in *Latin*.

If Forreigners may think this Work deserves their Perusal; in time perhaps I may satisfie them.

And because I have studied to be as plain as possibly I could, I have chosen rather to be *prolix* in many Particulars, than leave any Ambiguity to the most ordinary Mathematician. And therefore in demonstrating, I often repeat several Steps tho the least altered; that all things may lye as plain as possible. And on this Account I hope, the curious and profound Geometers will pardon me; I know they are used to what is *concise* and *closely* put together, much expressed in a little: But this Method suits not every Reader: And I have chosen to accommodate mine to the plainest Capacities.

Lastly, in this practical kind of Mathematicks, I desire the Reader to have frequent recourse to Experiments, for these illustrate the Theory and make many things clear, which otherwise will pass obscurely.

I

To the READER.

I uſe the common Notes of Algebra, with two or three new ones introduced by *Branker* or Dr. *Pell,* ſuch as

$\div$ Signifies *Divided by,* $\div \bar{2}$ ſignifies *Divided by* 2.

$\bar{2}$ With a Line over it ſignifies the abſolute Number 2.

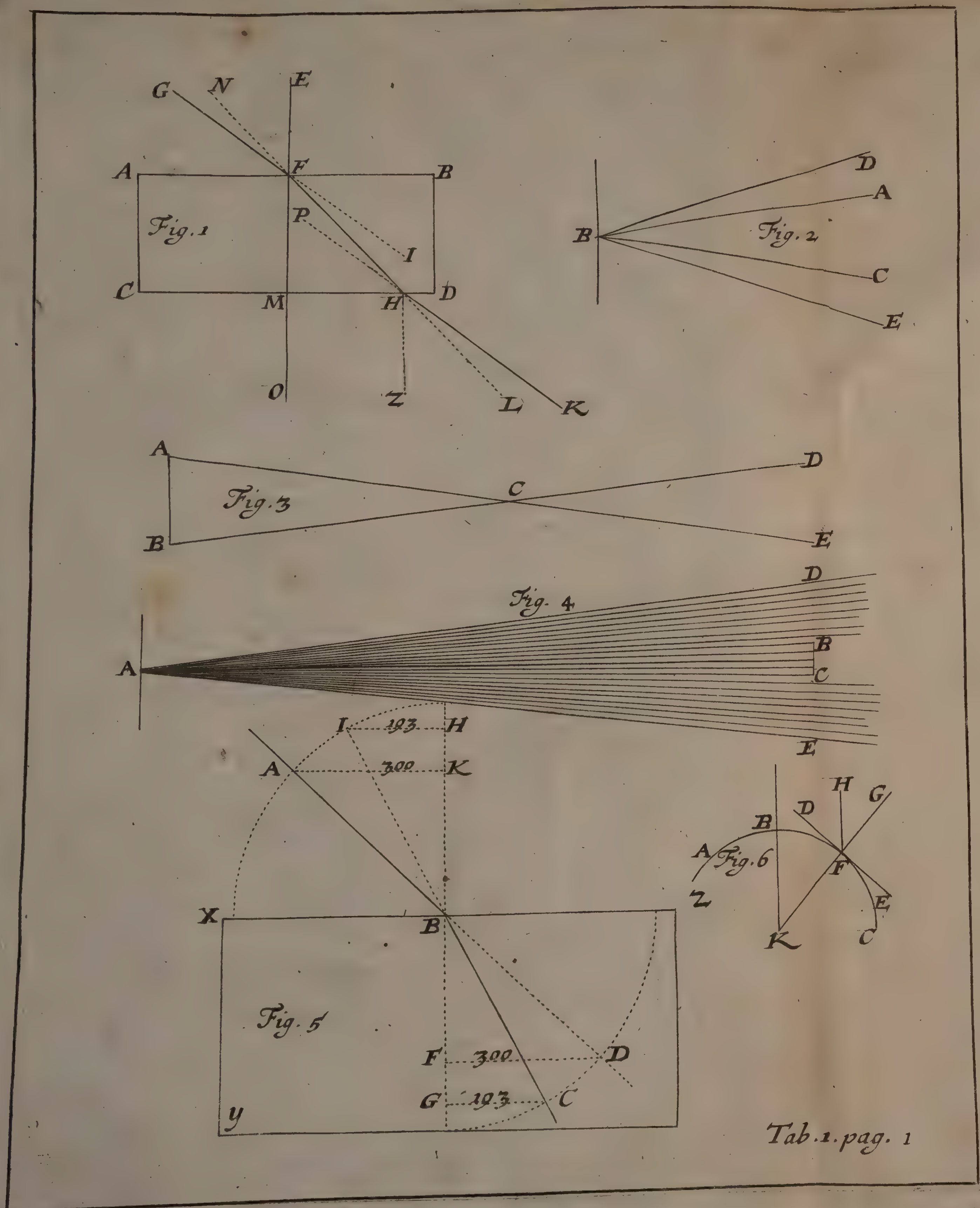
$*$ Is the Note of Multiplication or drawing into.

In the Demonſtrations the larger and leſſer Margins or Columns are of the ſame uſe as is expreſſed in Dr. *Pell's Algebra.* To which I refer the Reader.

April 17.
1690.

WILL. MOLYNEUX.

DIOPTRICKS.



Tab. 1. pag. 1

DIOPTRICKS.

Definitions.

I. TAb. 1. *Fig.* 1. ABCD is a Body of Glaſs, EFM is perpendicular to AB: ſuppoſe GF a Ray of Light falling inclined on the Glaſs AD, the Point F is called the *Point of Incidence.* *Tab.* 1. *Fig.* 1.

II. The Angle EFG comprehended between the Perpendicular and the Ray, is the *Angle of Incidence* (with *Barrow, &c.*) tho by many Dioptrick Writers 'tis called the *Angle of Inclination*; and its Complement AFG, is uſually by them called the *Angle of Incidence.* But I ſhall uſe the Terms *Inclination* and *Incidence* promiſcuouſly, always deſigning thereby the *Angle comprehended between the Perpendicular* EF *and the Ray* GF.

Experiments.

I. GF is a Ray of Light falling inclined on the Glaſs ABCD; this Ray coming out of a *Rare Medium*, as Air, into a *Denſe Medium*, as Glaſs, does not proceed on its direct Courſe in a ſtreight Line towards I; but at the Point of Incidence F 'tis bent or refracted *towards* the Perpendicular FM, and becomes the refracted Ray FH.

II. At H is its Point of *Incidence* again, from Glaſs a *Denſe Medium* to Air a *Rare*, and in this Paſſage, 'tis refracted *from* the Perpendicular HZ; ſo that inſtead of proceeding directly

ſtrait in F H L, 'tis refracted from the Perpendicular H Z, and becomes H K: So that if the Surface A B be parallel to the Surface D C, the Ray becomes again as if it had not been refracted; for now H K runs parallel to G F. The natural Reaſon of this Refraction is variouſly aſſigned by divers, and is properly of a Phyſical conſideration; and what is offered therein, is little more than Hypothetical Conjecture. I ſhall not therefore mix Gueſſes with Demonſtration: The Matter of Fact is manifeſt from Ten Thouſand repeated Experiments, and this is ſufficient to my purpoſe. But yet there is ſuch an ingenious Hypotheſis concerning this Matter, publiſhed in the *Acta Eruditorum Lipſiæ, Anno* 1682. *Menſ. Junii*, pag. 185. by the Learned and Ingenious *G. G. Leibnutzius*, that I cannot omit inſerting it in the Second Part of this Work, *Chap.* 1.

III. The Ray that falls perpendicular (as ſuppoſe E F a Ray) paſſes unrefracted, but all inclined Rays are refracted.

Definitions.

Ta.1.Fi.1. III. The Angle I F H, and K H L comprehended by the Ray directly prolonged, and the refracted Ray is the *Angle of Refraction.*

IV. The Angle H F M or Z H K compriſed between the refracted Ray and Perpendicular is the *refracted Angle.*

V. *Diverging Rays* are thoſe that ſpread and ſeparate the farther from each other, as they flow farther from the Object. Ta.1.Fi.2. *Tab.* 1. *Fig.* 2. B is a radiating Point, A B, D B, C B, E B, are diverging Rays.

VI. *Converging Rays* are thoſe that approach nigher each Ta.1.Fi.3. other, till they croſs, and then become *Diverging*. *Tab.*1. *Fig.*3. A, B, are two Radiating Points in the Object A B, the Rays C A, C B, do *Converge*, till they croſs in C, and then they become *Diverging* D C, E C.

VII. *Parallel*

VII. *Parallel Rays* are thoſe that flowing from one and the ſame Point of a *remote* Object paſs at the ſame diſtance, as to ſenſe: But this is not to be ſtrictly taken; for then the Rays flowing from one and the ſame Point of an Object, cannot be *parallel*, for they always *diverge*: Yet when an Object is at ſuch a great diſtance, and that parcel of Rays which is conſidered, is ſo ſmall, that their Divergence is little or nothing conſiderable, theſe Rays are ſaid to be parallel. *Tab*. 1. *Fig*. 4. A is a Point in an Object ſending forth its diverging Rays AD, AB, AC, AE; let BC be the breadth of the Pupil or breadth of an Optick Glaſs: Here if the Point A be ſo remote, and BC be ſo ſmall, that the Parcel of Rays BAC, do inſenſibly run parallel, then theſe are ſaid to be parallel Rays. Ta.1.Fi.4.

VIII. Thoſe radiating Points or Objects are ſaid to be *remote*, whoſe diſtance from the Eye or Glaſs is ſo great, that the breadth of the Pupil or Glaſs, in reſpect thereto, is inconſiderable.

IX. Thoſe radiating Points or Objects are ſaid to be *nigh*, when there is a ſenſible proportion between the Pupils or Glaſſes breadth, and the diſtance; ſo that the Rays flowing from any ſingle Point thereof do not run parallel to each other, but diverge conſiderably in reſpect of the Pupils or Glaſſes breadth.

Experiments.

V. *Tab*. 1. *Fig*. 5. ZXY is a Body of Glaſs, HBG is perpendicular to ZX, AB is a Ray falling on this Glaſs, and the Angle of Inclination or Incidence is HBA=DBG. *Kepler* tells us, that under 30 deg. of Inclination from Air to Glaſs, the Angle of Refraction DBC is $\frac{1}{3}$ of the Inclination, therefore the refracted Angle CBG is $\frac{2}{3}$ of the Inclination. Wherefore we lay down the following Proportions, Ta.1.Fi.5.

 as

as confirmed by *Kepler*'s Experiments, and uſually retain'd by moſt Optick Writers.

∠ Inclination DBG : refracted ∠ CBG :: 3 : 2
∠ Inclination DBG : ∠ Refraction DBC :: 3 : 1
∠ Refraction DBC : refracted ∠ CBG :: 1 : 2

V. But ſuppoſe the Ray CB to proceed from Glaſs to Air; at B 'tis refracted *from* the Perpendicular BH, and becomes BA; here the Inclination is CBG=HBI, and then

∠ Incidence HBI : refracted ∠ HBA :: 2 : 3
∠ Inclination HBI : ∠ Refraction IBA :: 2 : 1
∠ Refraction IBA : refracted ∠ HBA :: 1 : 3

Theſe Propoſitions in the 4th. and 5th. Experiments we ſhall retain in the following Demonſtrations, for the Eaſe and Plainneſs thereof. But in Calculation we ſhall obſerve the Proportion that follows in the 6th. Experiment.

VI. But the moſt Learned and Ingenious Mr. *Iſaac Newton* of *Cambridge* diſcovered by moſt accurate Experiments, that theſe Proportions of *Kepler* were not ſufficiently exact: For *Des Cartes* firſt found, that Refractions were not to be meaſured by the Proportion of Angles, but by the Proportion of Sines. See his Dioptricks *Cap. 2. Sec. 7.* And therefore Mr. *Newton* apply'd himſelf to diſcover the Proportion of the Sines, and found, *That from Air to Glaſs, the Sine* AK *or* DF *of the Angle of Incidence* ABK *or* DBG: *is to the Sine* IH *or* CG *of the refracted Angle* CBG *or* IBH :: *As* 300 *to* 193. (*or near, as* 14. *to* 9.) *And on the contrary, that from Glaſs to Air, the Sine of the Incidence : Is to the Sine of the refracted Angle* :: *as* 193 : *to* 300, *or as* 9 *to* 14. But the ſame Mr. *Newton* in his Diſſertations concerning *Colours* and *Light*, publiſh'd in the *Philoſophical Tranſactions*, has at large demonſtrated, that the Rays of Light are not all *Homogeneous*, or of the ſame ſort, but of different Forms and Figures, ſo that

that ſome are more refracted than others, tho they have the ſame or equal Inclinations on the Glaſs: And therefore there can be no conſtant Proportion ſetled between the Sines of the Incidence and of the refracted Angles. But the Proportion that comes neareſt Truth, for the middle and ſtrong Rays of Light, is nearly as 300 to 193, or 14 to 9.

VII. As the Sine of the Angle of any one Inclination: To the Sine of its refracted Angle:: So is the Sine of any other Inclination: To the Sine of its refracted Angle. Therefore by Experiment, finding the Proportion of the Sines of any one Inclination, and of its correſpondent refracted Angle, this Proportion will hold in any other Inclination or Incidence. See *Des Cartes Dioptr. Cap.* 2. *Sec.* 7. *Merſenni Optic. Lib.* 7. *Prop.* 12. *Dechales Dioptr. Lib.* 1. *Prop.* 1, 2, &c. See alſo *Cap.* 1. *Sec.* 4. *Part* 2.

Wherefore the Incidence or Inclination of a Ray being given, 'tis eaſie finding the refracted Angle; for the Proportion is, as 300: To 193 :: (or as 14 to 9) So the Sine of the Incidence: To the Sine of the refracted Angle, from Air to Glaſs.

Or Mechanically thus, A B K being the given Angle of Incidence, by a Sector make the Sine thereof K A or D F 300 parts (or 14) and take 193 (or 9) of the ſame parts, and make therewith C G or H I the Sine of the refracted Angle; then draw B C, this is the progreſs of the Ray A B after it enters the Glaſs.

VIII. *The Progreſs of Light through different Mediums is reciprocal*: That is, *Tab.* 1. *Fig.* 1. Suppoſe the luminous Point G in Air, to ſend from it the Ray G F upon the Glaſs A D; this Ray by the Glaſs is refracted into F H within the Glaſs. Let us now imagin the Point H within the Glaſs, to be a luminous Point, ſending out from it ſelf the Ray H F: This Ray upon its Emerſion from Glaſs into Air, inſtead of proceeding Ta.1.Fi.1

ceeding

ceeding directly to N, is refracted into F G. And so likewise supposing the Luminous Point G, to send its Ray G F, which after a double Refraction on each Surface of the Glass proceeds first in F H, and afterwards in H K; if we conceive K a Luminous Point, sending its Ray K H on the Glass; this Ray after a double Refraction on the two Surfaces of the Glass shall be refracted first into H F and then into F G; taking the same reciprocal Progress in both Cases.

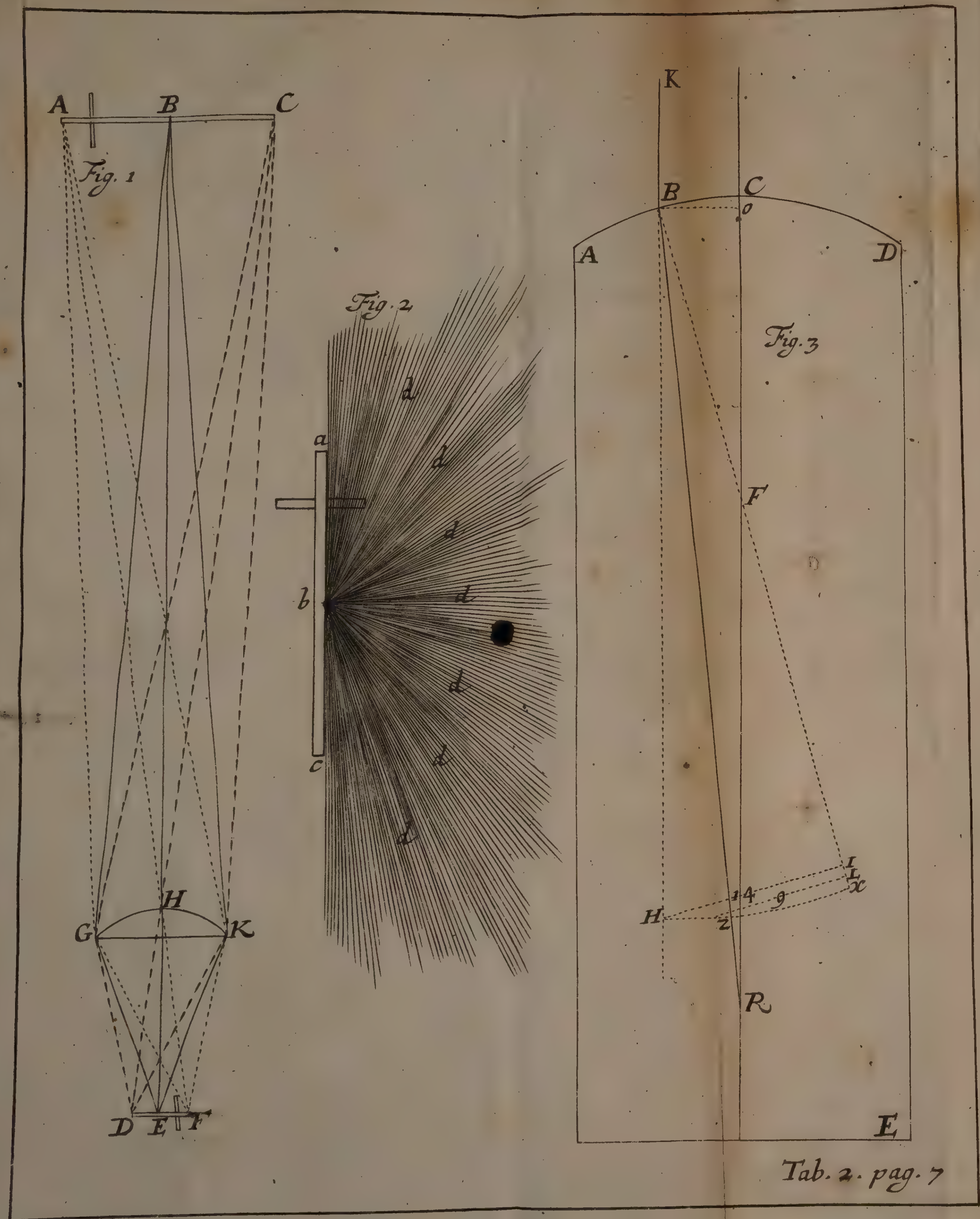
And this is not so properly proved from Experiment (tho that also does abundantly and most exactly confirm this Truth) but 'tis manifest and self-evident from the very Nature of the thing. For whatever physical Reason there is, which causes the Ray G F proceeding from Air into the Glass A D to be refracted from its direct Course F I into F H; the same reason there must needs be to cause the Ray H F, proceeding from the Glass A D into the open Air, to be refracted from its direct Course F N into F G. For doubtless the Refraction proceeds from the Disparity of the Mediums (but how or in what manner 'tis needless to enquire in this place). Now the difference of the Mediums on their two determining Surfaces, or at the Point of Incidence F, continues the same, let the Ray proceed from which of them soever. For there is as great a reciprocal Difference between Glass and Air, as between Air and Glass; and consequently the Refraction must needs be reciprocal. *Vid.* Barrow, *Lect. Opt.* 3. *Sect.* 3.

Definitions.

X. Whatever Line is perpendicular as to the Tangent of any Curve Line at the point of Contract, is said to be perpendicular to the same Curve. *Tab.* 1. *Fig.* 6. D E being a Tangent to the Arch of a Circle A B C, and F G being perpendicular to D E at the point of Contract F, the Line F G is also perpendicular to the Curve A B C.

Ta.1.Fi.6

XI. A



Tab. 2. pag. 7

XI. A Line that is inclined to the Tangent of any Curve, is inclined to the Curve; and the Angle of Inclination to the Curve, is the ſame with the Angle of Inclination to the Tangent. F H is inclined to the Curve A B C, by the Angle H F G, according to which Angle the ſame Line H F is inclined to the Tangent D E.

XII. K being the Centre of the Circle A B C, draw B K parallel to H F, the Arch B F meaſures the Inclination of the Line H F to the Curve A B C, or to the Tangent D E: For the Angle B K F is equal to the Angle H F G. (*Pr.* 29. 1. *Eucl.*

XIII. The Rays that proceed from the *middle* Point of an Object, when the Glaſs is expoſed directly before it, are ſaid to fall *directly* on the Glaſs; but the Rays that flow from the *Collateral* Points of an Object, are ſaid to fall *obliquely* on the Glaſs. *Tab.* 2. *Fig.* 1. A B C is a *diſtant* Object, ſending *Ta.*2.*Fi.*1. *parallel* Rays from each of its Points on the Glaſs G K, expoſed *directly* before it. Here the Rays flowing from the Point B, are ſaid to fall *directly* on the Glaſs, as B G, B H, B K. But the Rays flowing from the Point A, or any other *Collateral* Point, are ſaid to fall *obliquely* on the Glaſs, as A G, A H, A K. And here 'tis to be noted, that the Point B is ſaid to be *directly* expoſed to the Glaſs, when the Axis of the Glaſs E H B *directly* produced, meets this Point B: But all other Points in the Object are *Collateral* Points; for tho they may be *directly* expoſed to ſome other Points in the Glaſs, yet *direct* and *collateral*, are here to be underſtood only in reſpect of the Glaſs's *Axis* or *Vertex*. Or in ſhort, that Point in an Object is ſaid to be *direct*, the *Axis* of whoſe Cone of Rays is coincident or the ſame with the *Axis* of the Glaſs; that Point in an Object is ſaid to be *oblique* or *collateral*, the *Axis* of whoſe Cone of Rays falls *obliquely* to the *Axis* of the Glaſs. *Vid. Def.* 17. 18.

XIV. Suppoſe the Glaſs G K expoſed in the Hole of a Shutter in a dark Room, and that thereby the Object A B C were repreſented on a white Paper in F E D (how this is done we ſhall ſee hereafter.) From each Point of this Object there falls a parcel of Rays on the Surface of the whole Glaſs; thus from the Point B, there falls B G, B H, B K. This parcel of Rays is called *A Cone of Rays*, having its Vertex in the Point of the Object B, and its Baſe on the Glaſs.

XV. To this Cone of Rays there is another Correſpondent on t'other ſide the Glaſs G E K, which has its Baſe likewiſe on the Glaſs, and its *Vertex* in a correſpondent Point E of the Repreſentation.

XVI. Both theſe Cones together (that is B G E K) are called *A Pencil of Rays.*

XVII. Each Pencil of Rays has one Ray amongſt the reſt, which is called its *Axis*. The *Axis* of the direct Pencil BGEK is the Ray B H E, which falls perpendicularly on the Glaſs, and paſſes through it unrefracted. But the Axis of any of the oblique Pencils, as A G F K is ſome certain Ray, as A H F, which, tho it fall obliquely, and conſequently is refracted by the Glaſs, yet being refracted *from* the perpendicular at its Egreſs from the Glaſs, as much as it was refracted *towards* the Perpendicular, at its Ingreſs into the Glaſs; it may be taken as not refracted at all; but that part of this Axis which is before the Glaſs (*viz.* A H between the Object and Glaſs) and that part of it which is behind the Glaſs (*viz.* H F between the Repreſentation and the Glaſs) do run parallel to each other, or rather in one Right Line; that part of it only which is within the Glaſs, being bent out of its ſtrait Courſe. And that in each oblique Pencil of Rays (for of the direct Pencil 'tis manifeſt) there is ſuch a certain Ray thus affected, ſhall appear more fully hereafter. And here we are to diſtinguiſh between the *Axis of a Pencil of Rays*, and the *Axis of the Glaſs*. *See* Def. 18.

The

The Pencils of Rays from various Points are expressed differently in the Schemes (*viz.* by Lines or small Points) for better Illustration, and easier distinguishing their Progress.

XVIII. *Tab.* 2. *Fig.* 3. AE is a Body of Glass, whose Surface ACD is the Segment of a Sphere, the Centre whereof is F. KB a Ray falling on the Glass; through the Centre F draw CFR parallel to KB, CFR is the *Axis*, and the Point C the *Pole* or *Vertex* of the Glass: Or more generally thus, The *Axis* of a Glass is that *Right Line*, which, being produced both ways, falls on the *Centres* of the *Two* Spherick Surfaces (if the Glass be a double Convex, or double Concave) or which being produced both ways, falls on the *Centre* of the *Spherick* Surface, and perpendicular to the plain Surface, (if the Glass be a Plano-Convex, or Plano-Concave.) So that the Axis of a *Convex-Glass* passes perpendicular to the *thickest* part of the compleat Glass; and of a *Concave* to the *thinnest* part of the compleat Glass. Why *Compleat* is added, *vid.* Second Part, *Chap.* 4. *Sect.* 4. The *Poles* of a Glass are those two Points in the Glass through which the *Axis* passes. *Tab. 2. Fig. 3.*

'Tis needless to define what is meant by a Convex or Concave-Glass, *&c.* for every one knows this already.

XIX. The Ray KB falling on the Convexity ACD, parallel to the Axis CR, and being refracted to BR; so that now it runs no more parallel, but crosses the Axis in R: the Point R is called the *Point of Concourse, or of Convergency, or the Focus.*

1. *Supposition.*

That from every Physical Point in an Object Rays of Light are diffused in direct Lines through the Hemisphere round it. *Tab.* 2. *Fig.* 3. Thus the Point *b* in the Object *abc* projects *Ta.2.Fi.2.* from it the Rays *bd*, *bd*, *bd*, wherever they are not hindered by some Opaque Body intervening. And these Rays are

 strait

ſtrait Lines, whilſt they paſs through a Diaphanous Uniform *Medium.* But here we are to conceive a radiating or luminous Point, not in a ſtrict *Mathematical* Senſe, but in a *Phyſical*, conſidering it as the leaſt part imaginable in an Object.

2. *Suppoſition.*

That the *Axis* of the Eye, Glaſs, or Glaſſes, expoſed before an Object be *Perpendicular* to the Plane of the Object; or that the Eye, Glaſs, or Glaſſes expoſed before an Object, are ſuppoſed *directly* expoſed thereto.

3. *Suppoſition*, or *Admonition.*

Becauſe in many Authors of Opticks, much Perplexity and Error ariſes from their confuſed mentioning of Rays, and applying to them the Epithets of *Diverging*, and *Converging*, and *Croſſing*; without expreſſing clearly whether they mean Rays from *one* and the *ſame* Point in an Object, or from *Different* Points. Therefore we are to conſider in the enſuing Doctrin, whether the Name Rays do relate to thoſe from *one* and the *ſame* Point, or from different Points. And that this may be the more eaſie, I do generally expreſs the Rays from the ſame Point to be *ſuch*, that is, I call them *Rays from the ſame Point.*

PROP. I. PROBL.

Tab. 2. Fig. 3. AE *is a Body of Glaſs, whoſe Surface* ABCD *is the Segment of a Sphere, the Centre whereof is* F. KB *is a Ray of Light parallel to the Axis* CFR. *There being given* CF *the Radius of the Convexity, and* BO *the Diſtance*

Distance of the Ray from the Axis; 'tis required to find the Point R, *wherein this Ray, after one sole Refraction at the Point of Incidence* B, *crosses the Axis* CR.

Produce KB at pleasure to H, and draw BFX at pleasure.

1. In the Right Angled Triangle BOF, we have given BO and BF; whence we may find the Angle BFO=HBF, which is the Incidence.

2. Then say, as 300 : to 193 :: (or as 14 to 9) so the Sine HI of the Incidence HBF : to the Sine ZL of the refracted Angle RBF; then HBF—RBF=HBR=BRF, which is the Angle of Refraction.

3. In the Triangle RBF, all the Angles and one of the Sides BF are known, whence we may find FR: *Which was required.*

Example.

FB=FC=2 Inches, or 20000 Parts, an Inch being 10000. BO=½ an Inch, or 5000 such Parts. The Angle of Incidence or Inclination BFO=HBF will be found 14° 29′. And the refracted Angle FBR shall be 9° 15′. Wherefore BRF=HBR=5° 14′. Whence the Line FR is found 35341. Then RF+FC=35341+20000=55341=CR.

Corollary.

From this Calculation 'tis manifest, That in this Case the Point of Concourse R is distant from the Pole of the Glass C, almost a Diameter and half of the Convexity: For a Diameter and half is 60000, and CR is 55341, which wants only about $\frac{1}{12}$ of 60000.

Scholium 1.

This Propoſition is the 34th. of *Keplers Dioptricks*, and in him and other Opticians, 'tis uſually thus expreſſed and demonſtrated.

The Rays that are parallel to the Axis, and fall ſo on the Convex Surface of a Glaſs, and at their Ingreſs are refracted, and ſo paſs forward in Glaſs only, are united with the Axis at the diſtance of almoſt a Diameter and half of the Convexity, ſuppoſing the Glaſs to be a Portion of no more than about 30 *degrees of its Convexity, or (which is the ſame thing) ſuppoſing the Inclination of the moſt extreme Ray to be but* 15 *degrees.*

This Limitation of 15 degrees Inclination is added, becauſe above 15 degrees Sines and Angles are not proportional, but under 15 degrees they are nighly proportional, a double Angle having a double Sine, a treble Angle a treble Sine, &c.

Ta 2. Fi. 3. Wherefore they thus demonſtrate this Propoſition. *Tab.* 2. *Fig.* 3. BFC=HBF is the Inclination, HBR=BRF is the Angle of Refraction, RBF the refracted Angle. But by *Experiment* 4. RBF the refracted Angle : is to BRF the Angle of Refraction : : as 2 to 1. Therefore the Sines of theſe Angles, being under 15 degrees, ſhall be as 2 : to 1. But in plain Triangles the Sines of the Angles and the Sides ſubtending theſe Angles are proportional; therefore the Side RF is double FB or FC. Wherefore RC is thrice FC, or equal to three Semidiameters: *Which was to be demonſtrated.*

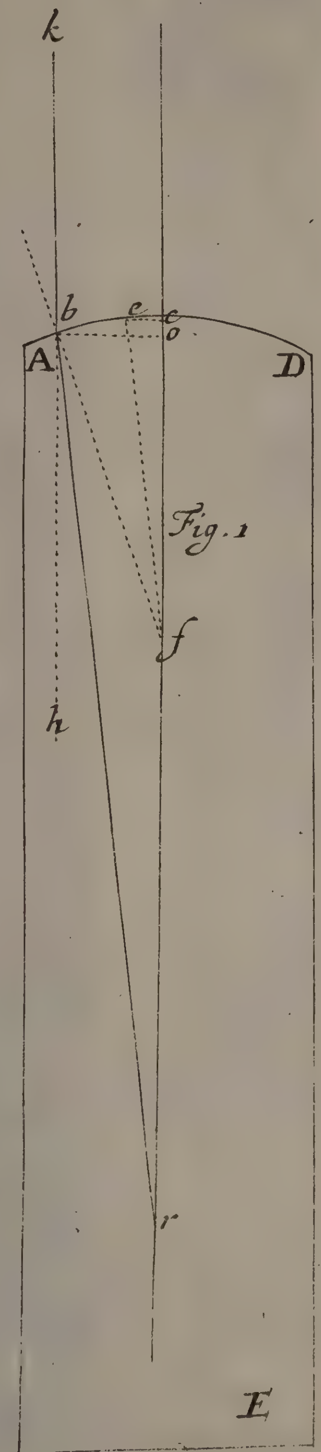
Scholium 2.

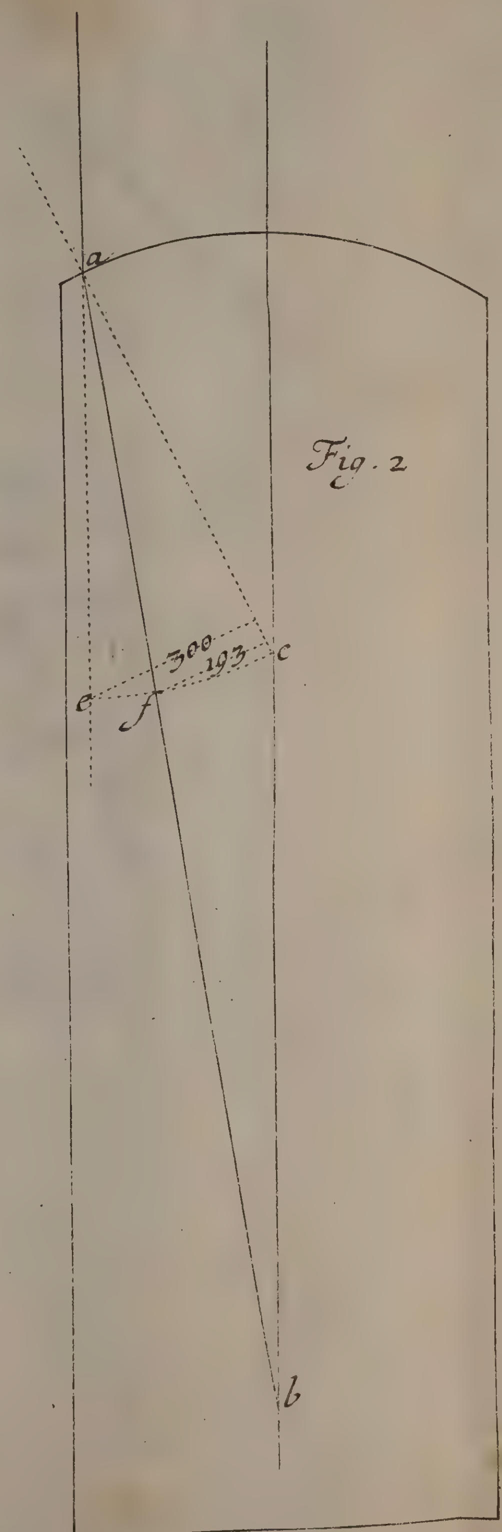
But a neater way of Expreſſing and Demonſtrating this Propoſition is thus.

When

Fi. 1.

2.





Tab. 3. pag. 12

When a Ray falls on a Spherick Glaſs, and afterwards it being but once refracted, croſſes the Axis. It ſhall be, as the Sine of the Angle of Refraction : to the Sine of the Angle of Inclination :: ſo the Radius of the Glaſs's Sphere : to the refracted Ray.

Tab. 3. *Fig.* 1. Let bec be the Convex Surface of the Glaſs ADE, kb a Ray of Light, f the Centre of the Convexity, bfc the Angle of Inclination, bo the Sine of the Inclination, br the refracted Ray meeting the Axis in r, $hbr=brc=$ to the Angle of Refraction; draw fe parallel to br, then efc is equal to brc, and ec is the Sine of the Angle of Refraction. And by reaſon of the ſimilar Triangles bro, efc, it ſhall be $ec : bo :: ef : br$. *Which was to be demonſtrated.* Ta.3. Fi.1.

Wherefore ſuppoſing that from Air to Glaſs, the Sine of the Angle of Inclination : be to the Sine of the refracted Angle :: as 300 : to 193 : then the Sine of the Inclination ſhall be : to the Sine of the Angle of Refraction :: as 300 : to 300—193 = 107. Becauſe the Angle of Refraction is equal to the difference between the Angle of Inclination and the refracted Angle: And Sines and Angles being (in theſe ſmall Angles) proportional. It ſhall be, As 107 : 300 :: ſo Radius of the Convexity : to the Concourſe of the refracted Ray with the Axis.

It ſhall alſo be, As 107 : 193 :: ſo the Radius of the Convexity : to the Concourſe of the refracted Ray beyond the Centre of the Sphere.

For *Tab.*3.*Fig.*2.	1	$eac : fac :: 300 : 193$ Ta.3. Fi.2.
Dividing the Firſt	2	$eac-fac : fac :: 300-193 : 193$
That is	3	$eaf : fac :: 107\ 193$
Moreover,	4	$eaf=fbc=$ Angle of Refraction.
From 3 and 4	5	$fbc : fac :: 107 : 193$
But in the $\triangle\ abc$	6	$fbc : fac ::$ Rad.$=ac : cb$
From 5 and 6	7	$107 : 193 :: ac : cb=$ To the Concourſe of the Ray beyond the Centre of the Sphere c. *Which was to be demonſtrated.*

PROB.

Prop. II. Probl.

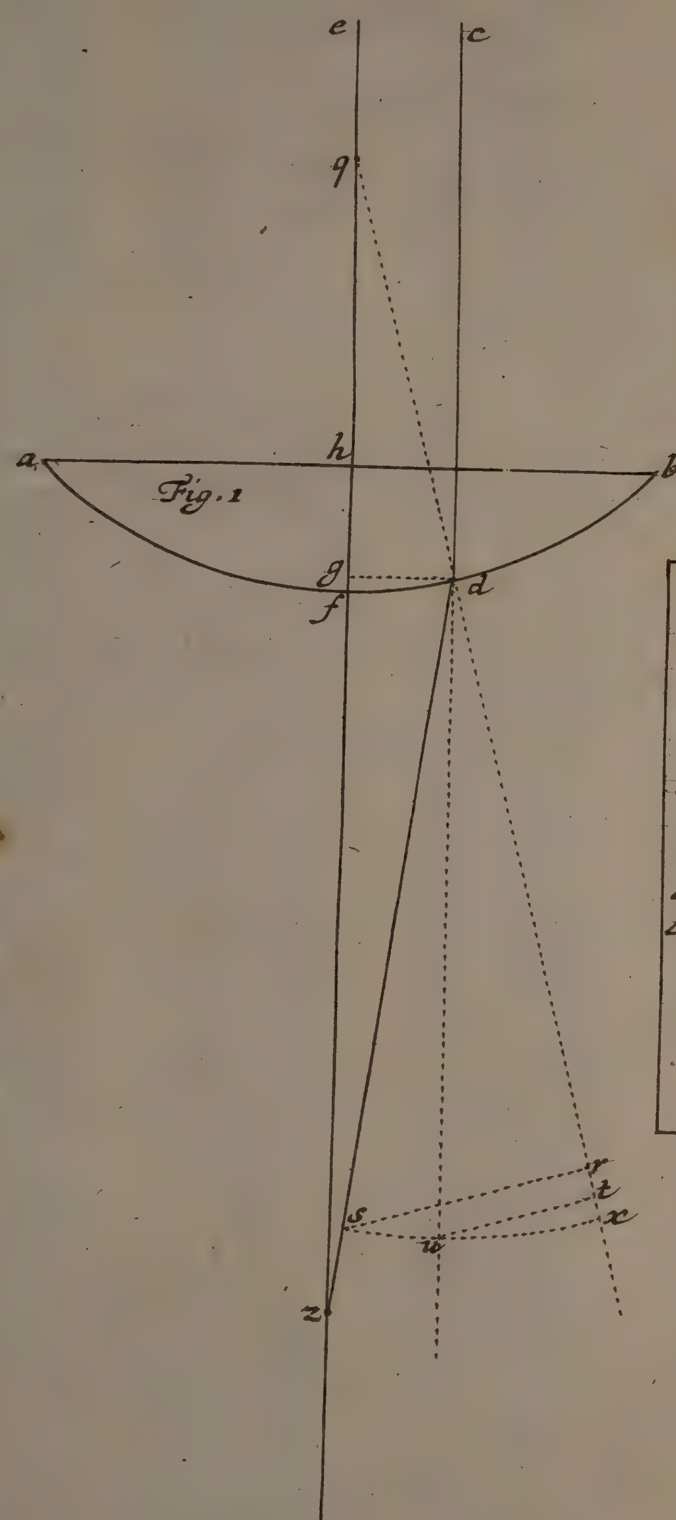
Ta.4.Fi.1. *A Plano-Convex Glaſs* afb Tab. 4. Fig. 1. *being expoſed to a diſtant Object with its Convex ſide towards it, and receiving the Ray* cd *parallel to the Axis* ez, *'tis required to determin the Point of Concourſe* z, *from theſe* Data, fh *the Glaſſes thickneſs,* qf *the Radius of the Convexity, and* dg *the diſtance of the Ray* cd *from the Axis* ez.

The ſame Problem is propoſed from the ſame *Data*, for the plane Side towards the Object.

But firſt for the Convex-Side to the Object: The Mechanical Conſtruction is thus, *Tab.* 4. *Fig.* 1. Produce the Ray *cd* through the Glaſs at pleaſure to *m*. And from *d* the Point of Incidence draw the Line *dq* to the Centre of the Convexity; this is the Perpendicular, and the Angle *qdm* the Incidence or Inclination. On *d* as a Centre at any diſtance ſtrike the Arch *moq*, in which make *mn* the Sine of the Angle of Incidence, to *op* the Sine of the refracted Angle, as 14 to 9 (or *op* 9 ſuch parts as *mn* is 14, which is eaſily done by the Sector) the Line *dko* ſhall be the Line in which the Ray would proceed; if the Medium of Glaſs were continued ſo far.

But becauſe in this Line at *k* the Ray emerges from the plain Surface of the Glaſs a denſe Medium into Air a rare Medium, from *k* raiſe *kx* perpendicular to the plain Surface *ahb*, and on *k*, as a Centre at any diſtance ſtrike the Arch *xus*; then the Angle *xku* is the Incidence of the Ray paſſing from Glaſs to Air.

Making therefore *tu* the Sine of this Angle, to *rs* the Sine of the ſecond refracted Angle from the Perpendicular as 9 to 14, the Line drawn from *k* through the Point *s*, and produced to the Axis at *z*, ſhews the Progreſs of the Ray in the Air after its Emerſion from Glaſs: Wherefore the Point *z* is determined in the Axis.



Data

$qf = qd = 20000$

$gd = 5000$

$hf = 5858$

unde

$\angle dqg = udx = 14^{\circ}.28'.40''$

$qg = 19365$

$qf - qg = gf = 635$

$\angle sdx = 22^{\circ}.52'.10''$

$\angle sdx - udx = sdu = dzg = 8^{\circ}.23'.30''$

$gz = 33894+$

$gz - gf = fz = 33259$

Tab. 5. pag. 15

Trigonometrical Calculation.

But this Point z, or the Meaſure of the Line fz may be found more accurately by Calculation thus. In the Triangle qdg Right Angled at g are given qd and dg, to find the Angle of Incidence dqg, and the Side qg. Then $qf-hf=qh$, and $qg-qh=hg=di$.

Firſt then, becauſe the Ray paſſes at d from a rare Medium into a denſe, it ſhall be refracted towards the Perpendicular dq. Wherefore the Analogy is, As 14 : to 9 :: ſo the Sine of the Angle of Incidence dqf $(=mdq)$: to the Sine of the refracted Angle odq (that is kdl). But mdq $(=idl)$ $-kdl=idk=xku$, dm and kx being parallel :) And this is the Angle of Incidence as the Ray paſſes from Glaſs to Air.

Secondly, In the little Right Angled Triangle idk, the Side di, and the Angle idk are known; whence the Side ik may be found, then ih $(=dg)$ $-ik=kh$.

Thirdly, Becauſe at k the Ray emerges from a Denſe into a Rare *Medium*, it ſhall be refracted from the Perpendicular kx; and the Analogy ſhall be, as 9 : to 14 :: ſo the Sine of the Angle of Incidence xku : to the Sine of the Second refracted Angle xks $(=kzh$, kx, and hz being Parallel.)

Fourthly, In the Right Angled Triangle khz, the Side kh, and all the Angles are known, whence the Side hz, and conſequently fz $(=fh+hz)$ the Diſtance of the Point of Concourſe from the Pole of the Glaſs f is diſcovered.

When the plain Side is turned to the Object, the Method is the ſame, only ſhorter; for *Tab.* 5. *Fig.* 1. in the Right Angled Triangle qdg, we have qd and dg to find $dqg=udx$, the Incidence, and the Side qg: Then $qf-qg=gf$. Next, As 9 : to 14 :: (or as 193 : to 300 ::) ſo the Sine of udx : to *Ta.5.Fi.1.*

the

Ta.4 Fi.1. the Sine of $s d x$. But $s d x - u d x = s d u = d z g$. Then in the Right Angled Triangle $d z g$ we have $d g$, and the Angles to find $g z$. And $g z - g f = f z$. Which was to be found.

Scholium.

In the little Tables inserted in *Tab.* 4. and *Tab.* 5. I have given Examples at large of the Calculation of the Progress of the Ray $c d$, which I presume is pretty truly wrought: But if perhaps any Errors do intervene, they are easily corrected.

By these Tables we may perceive, that the Point of Concourse z is distant from the Pole of the Glass f about a Diameter of the Convexity, but not fully so much.

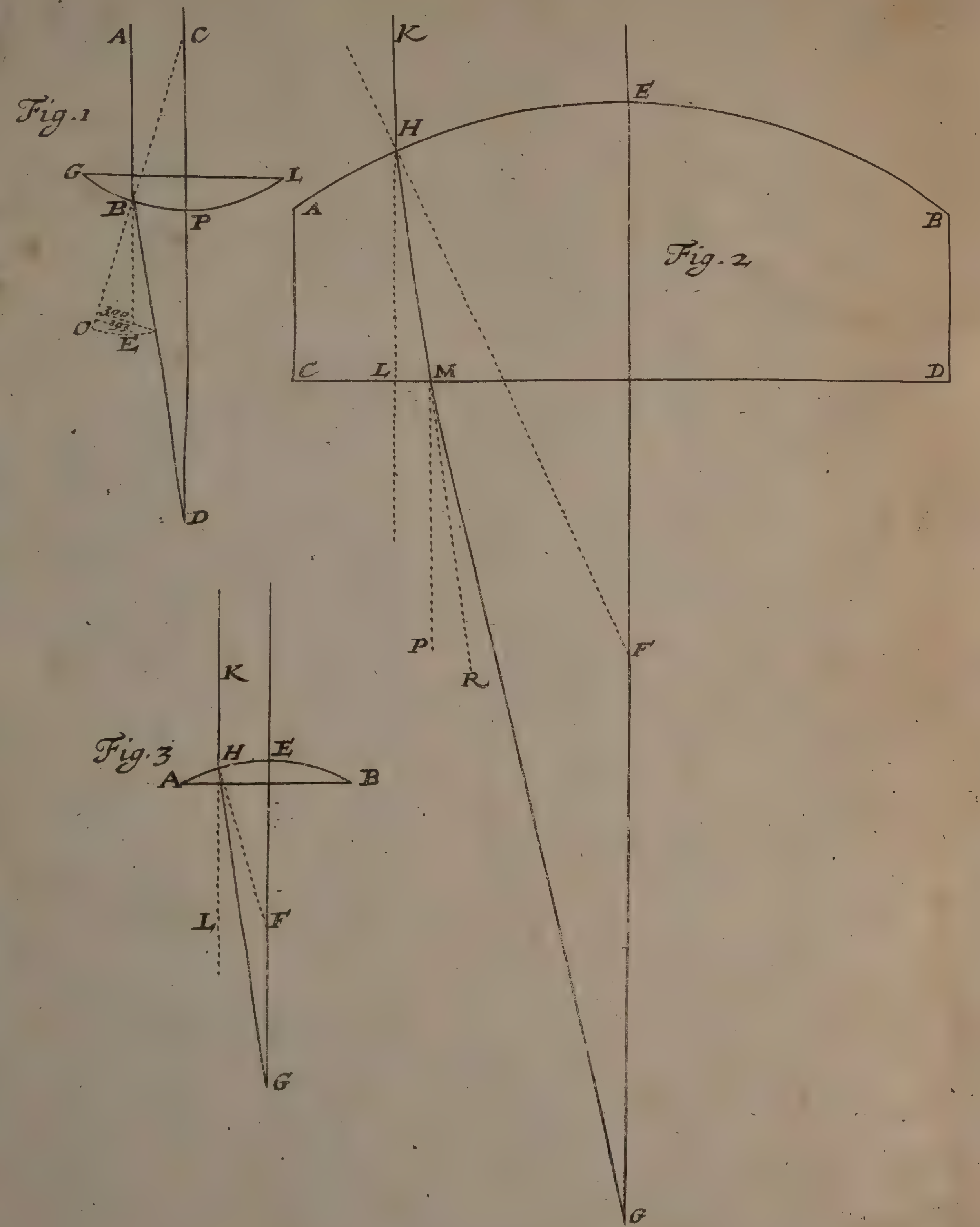
'Tis also to be noted in *Tab.* 4. *Fig.* 1. That the Point of Concourse z, shall be less or more removed from the Pole f, according as we give the Glass less or more thickness. For 'tis manifest, that the farther we let the Ray run in the *Medium* of Glass, after it enters it at d, before it emerges into Air at k, the farther the Point of Concourse z shall be protracted from

Ta.6.Fi.1. f. For we may imagin the *Medium* of Glass continued below the Diameter of the Convexity, and then 'tis evident that the Point of Concourse z shall be much more below it. But it can be no more below it than a Diameter and half. By the First Proposition hereof.

This our Second Proposition is usually found in Dioptricks thus expressed and demonstrated.

Parallel Rays falling on a Plano-*Convex-Glass, are united with the Axis about the distance of a Diameter of the Convexity from the Pole of the Glass, if the Segment be but* 30 *Degrees.*

Demon-



Tab. 6. pag. 16

Demonstration.

First for the plain Side towards the Object. *Tab.6. Fig.1.* AB is a Ray of Light falling on the Glaſs GL parallel to the Axis CD. C is the Centre of the Convexity GPL. Joyn CB; the Angle of Incidence or Inclination is BCD, the Angle of Refraction is EBD or BDC. *Tab. 6. Fig. 1.*

Wherefore in the Triangle BCD, the Angle BCD being double the Angle BDC (For by *Kepler's* Experiments, which are follow'd by moſt Authors, from Glaſs to Air, the Angle of Incidence : is to the Angle of Refraction :: as 2 : to 1.) the Side BD ſhall be double the Side BC; the Angles and Sides or Sines under 15 Degrees being proportional; but DP is almoſt equal to BD (for the Thickneſs and Breadth of the Glaſs, in Optick Glaſſes eſpecially in Glaſſes of Long *Foci*, are inconſiderable), therefore DP is double CP. *Which was to be Demonſtrated.*

Secondly, for the Convex-ſide towards the Object. *Tab.6. Fig.2.* ABCD is a Plano-Convex Glaſs (I have here expreſſed it of this thickneſs, to ſhew the Angles and Progreſs of the Ray more plainly, tho really this great Thickneſs hinders the Exactneſs of the Demonſtration; for the Glaſs is to be ſuppoſed of the moſt inconſiderable Thickneſs) F is the Centre of the Convexity. KH is a Ray falling thereon, parallel to the Axis EG. This Ray, (after it has ſuffered a Double Refraction, one at its Ingreſs from Air into Glaſs at H towards the Perpendicular HF, by which it becomes the Refracted Ray HM, and another Refraction at its Egreſs from Glaſs to Air at M from the Perpendicular PM, by which, inſtead of going forward towards R, it becomes the Refracted Ray MG) at G croſſes the Axis, ſo that GE is the Diameter of the Convexity AEB. *Ta.6.Fi.2.*

D For

For the Incidence or Inclination of the Ray K H is L H F, of which Angle there is taken off $\frac{1}{3}$ L H M, for the Angle of Refraction from Air to Glaſs is $\frac{1}{3}$ of the Incidence (by *Exper.* 4.) If therefore the Ray had proceeded onwards in Glaſs towards R, it had croſſed the Axis at a Diameter and half (by the foregoing Propoſition), but at its egreſs into Air at M, it is again refracted from the Perpendicular P M by the Angle R M G $= \frac{1}{2}$ P M R. The Angle of Refraction R M G from Glaſs to Air, being $\frac{1}{2}$ the Incidence P M R (by *Exper.* 5.) But P M R is equal to L H M. Wherefore the firſt Angle of Incidence LHF by this Double Refraction, has loſt firſt $\frac{1}{3}$ of it ſelf L H M, and again $\frac{1}{2}$ of this Third R M G: But $\frac{1}{3}$ and $\frac{1}{2}$ of $\frac{1}{3}$ is equal to $\frac{1}{2}$, wherefore the Angle of Incidence is diminiſhed by its Half.

Ta.6.Fi.3. Let us therefore conſider the Glaſs A B, *Tab.* 6. *Fig.* 3. without reſpect to its Thickneſs, or to the Refraction the Ray ſuffers at its egreſs on the lower ſide of the Glaſs into Air. But let us imagin, that by the firſt Refraction, the Ray were brought nigher the Perpendicular H F by half the Angle of Incidence L H F, and we ſhall find, that then the Ray ſhall croſs the Axis at G; ſo that G E ſhall be the Diameter of the Convexity. For L H G or G H F is equal to $\frac{1}{2}$ L H F or HFE the Incidence; but H F E is equal to G H F + H G F (32. 1. *Eucl.*) therefore G H F and H G F are equal, each of them being $\frac{1}{2}$ H F E, and conſequently their oppoſite Sides H F and G F are equal. Wherefore G E is the Diameter of the Convexity A E B. *Which was to be Demonſtrated.*

Corollary.

In Plano-Convex Glaſſes, As $300 - 193 = 107$: To 193 :: So Radius of the Convexity : To the refracted Ray taken to its Concourſe with the Axis; which in Glaſſes of large Spheres and ſmall Segments is almoſt equal to the Diſtance of the Focus taken in the Axis. *Tab.*6. *Fig.* 1.

Ta.6.Fi.1.

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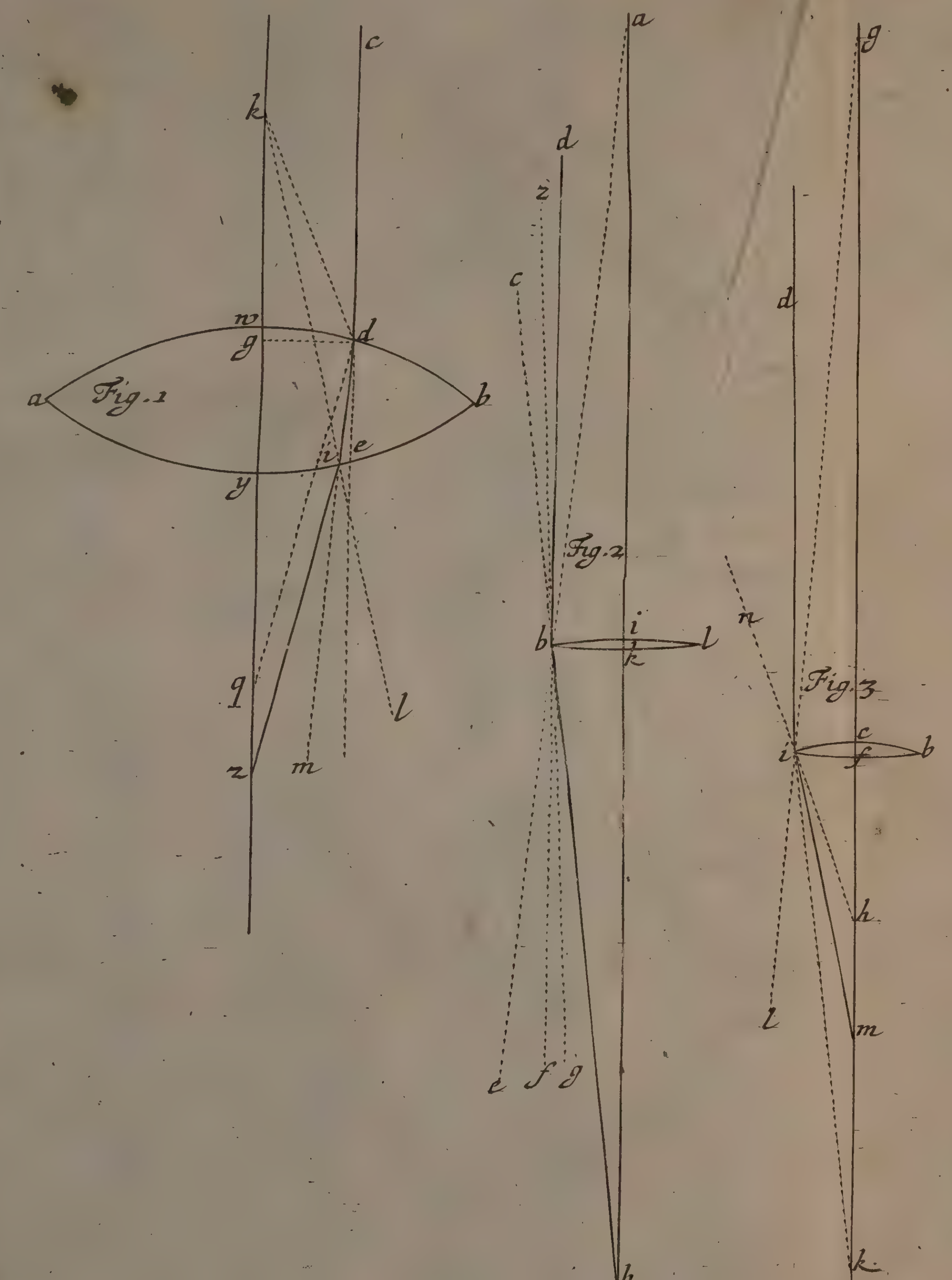


Tab. 7. pag. 19

Demonstration.

The Inclination $=ABC=OBE=BCD=193$.
Refracted Angle $=OBD=300$
Angle of Refraction $=EBD=BDP=OBD-OBE$
$=300-193=107$.
In the $\triangle$ BDC : $s\,\angle BDC : s\,\angle BCD :: Rad.=BC : BD$.
That is,
$107 : 193 :: BC : BD=PD$ in Glasses of large Spheres.
Which was to be Demonstrated.

Porp. III. Probl.

In a Double Convex-Glass ab, Tab. 7. Fig. 1. *of equal or unequal Convexities; Let the Centres of the Spherical Surfaces be* k *and* q; cd *a Ray falling parallel to the Axis* kqz, *and being Refracted at its Ingress at* d, *and at its Egress at* i; *'Tis required to determin the Point of its Concourse with the Axis at* z, *from these Data, the Radii* ky, qw, *of the Convexities, and* gd *the Distance of the Point of Incidence from the Axis.* Ta.7.Fi.1.

This is so easily done by Scale and Compass from what foregoes, that I shall not insist on a farther Explication thereof, but shall shew the more certain Method of tracing the Progress of this Ray by Calculation.

Produce cd directly to e, and di to m. We have $ky+qw-wy=kq$.

First therefore in the Right-angled Triangle qdg, gd and qd being given, we may find the $\angle gqd=qde$ and the side qg. Then $kq-qg=kg$.

Secondly, In the Right-angled Triangle kgd, there being given kg and gd, we may have the $\angle gkd=kdc$, and the side kd.

 Thirdly,

Thirdly, As 300 : to 193 :: ſo S. $\angle qde$: to S. $\angle qdi$.

Fourthly, $qde - qdi = edi$. And $180' - kdc - edi = kdi$.

Fifthly, In the Triangle kdi, the Angle kdi and the Sides ki, kd, being known, we have the Angle $kid = mil$. And the Angle ikd. Then $gkd - ikd = qki$.

Sixthly, As 193 : to 300 :: ſo S. $\angle kid$: to S. $\angle zil$, then $zil - qki = kzi$.

Laſtly, In the Triangle zki, ki and all the Angles being known, we find kz, from which ſubtracting ky, there remains zy, which was required.

If by this Method we calculate the Progreſs of a Ray through a Double Convex-Glaſs of equal Convexities; and the Thickneſs of the Glaſs be little or nothing in compariſon of the Radius of the Convexity; and the Diſtance of the Point of Incidence from the Axis be but ſmall, we ſhall find the Point of Concourſe to be diſtant from the Glaſs about the Radius of the Convexity nearly.

This our Third Propoſition in moſt Dioptrick Writers is uſually thus expreſſed.

Parallel Rays falling on a Double Convex-Glaſs of equal Convexities on both ſides, are united with the Axis, about the Diſtance of the Radius of the Convexity from the Pole of the Glaſs, if the Segment be but 30 Degrees.

But their Demonſtrations of this Propoſition are uſually perplexed enough, by reaſon of the ſmallneſs and multiplicity of Angles, which are expreſſed in their Figures, as alſo by the Thickneſs of the Glaſs, which they are forced to repreſent, and yet is to be neglected in their Demonſtrations. I ſhall give as ſhort a Demonſtration as I can, after their Method, as follows.

Ta.7.Fi.2. *Tab.* 7. *Fig.* 2. Let a be the Centre of the Convexity bkl, h the Centre of the Convexity bil, db a Ray of Light produced directly to f parallel to the Axis ah. abe is a Right Line, and hbc is a Right Line. The firſt Angle of Inclination

tion is $c\,b\,d = h\,b\,f = e\,b\,f$, and the perpendicular is $h\,b$. By the first Refraction the Ray is deflected from its straight course $b\,f$ and becomes $b\,g$, approaching the perpendicular $b\,h$ by the Angle $f\,b\,g$, which is $\frac{1}{3}$ of the Inclination $e\,b\,f$. Produce $g\,b$ directly to z. Now the Angle of Incidence on the lower Convexity $b\,k\,l$ from Glass to Air is $a\,b\,z = e\,b\,g$, and the Perpendicular is $a\,b$ or $b\,e$: Wherefore the Ray instead of going forwards in $b\,g$ is now, by its Emersion into Air, refracted from the Perpendicular $b\,e$, by the Angle $g\,b\,h$, which is therefore to be equal to $\frac{1}{2}$ the second Angle of Incidence $e\,b\,g$. by Exper. 5. and that $g\,b\,h$ is equal to $\frac{1}{2}\,e\,b\,g$, I thus demonstrate;

By the Suppofals in the Prop. and Common Geometry.	1	$e\,b\,f = f\,b\,h$
By the Fig. — — — — —	2	$g\,b\,h = f\,b\,h - f\,b\,g$
From 1 and 2 — — — --	3	$g\,b\,h = e\,b\,f - f\,b\,g$
Doct. Refract. Exper. 4. - —	4	$f\,b\,g = \frac{1}{3}\,e\,f\,b$
From 3 and 4 — — —	5	$g\,b\,h = e\,b\,f - \frac{1}{3}\,e\,b\,f = \frac{2}{3}\,e\,b\,f$
Moreover by the Scheme —	6	$\frac{1}{2}\,e\,b\,g = \frac{1}{2}\,e\,b\,f + \frac{1}{2}\,f\,b\,g$
$4 \div 2$ —————————	7	$\frac{1}{2}\,f\,b\,g = \frac{1}{6}\,e\,b\,f$
From 6 and 7 — — ——	8	$\frac{1}{2}\,e\,b\,g = \frac{1}{2}\,e\,b\,f + \frac{1}{6}\,e\,b\,f = \frac{2}{3}\,e\,b\,f$
From 5 and 8 -———-———	9	$g\,b\,h = \frac{1}{2}\,e\,b\,g$

Which was to be Demonst.

Therefore it follows that the Ray by this second Refraction proceeds in $h\,h$, crossing the Axis in h the Centre of the Convexity. *Which was to be demonstrated.*

See the Demonstrations of this Proposition *Kepleri* Dioptri. Prop. 39. *Herigon.* Prop. 15. *Dechales* Pr. 21. lib. 13. *Hon. Faber* Prop. 43. Sec. 8. *Cherubin de Orleans la Dioptr. Oculair*, Prop. 2. *Zahn Telescop.* Fund 2. Syntag. 1. Cap. 4. Prop. 8.

Of Glasses of unequal Convexities.

As for double Convex Glasses of unequal Spheres on each Side, the foregoing Method of Calculation determines the Point

of

of Concourſe exactly in them alſo: But the ſhorter Rule laid down by moſt Optick Writers, is this.

As the Sum of the Radii of both Convexities: to the Radius of either Convexity :: So the double Radius of the other Convexity: to the Diſtance of the Focus. Vid. Corol. Pr. 16.

Demonſtration.

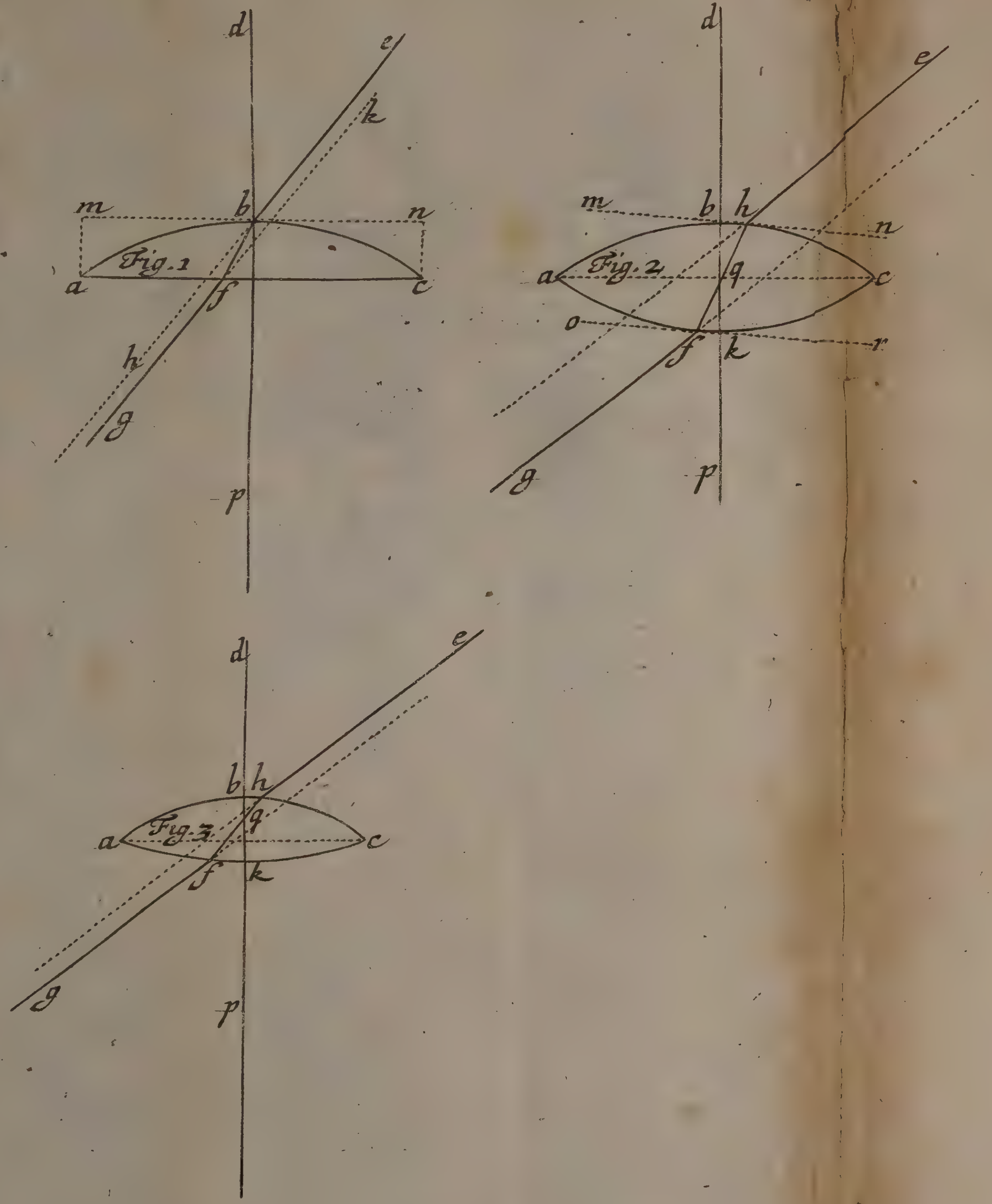
T. 7. F. 3. *T. 7. F. 3.* Let the Axis of the Glaſs ib be gk, di a Ray of Light Parallel thereto, g the Centre of the Convexity ifb, h the Centre of the Convexity icb, gil and hin are right Lines. The firſt Angle of Inclination is $din = ihc$, and by the firſt Refraction the Ray is deflected into ik, croſſing the Axis in k, ſo that kc is equal to $3ch$. The ſecond Angle of Incidence from Glaſs to Air on the Surface ifb is the Angle lik: I ſay kim being made $\frac{1}{2}$ of lik, the Ray croſſes the Axis in m, and it ſhall then be (according to the Rule) $gh : 2fg :: ch : mc$. Which I thus Demonſtrate.

In the $\triangle\, kim$	1	$\angle kim : \angle k :: km : mi = mf$ in large Spheres.
Double the Antecedents	2	$2\angle kim = lik : \angle k :: 2km : mf :: km : \frac{1}{2}mf = \frac{1}{2}mc$:
Divide the ſecond Step.	3	$\angle lik - \angle k = \angle g : \angle k :: km - \frac{1}{2}mc : \frac{1}{2}mc$.
Then in $\triangle\, gik$ it ſhall be	4	$\angle g : \angle k :: ik = kc : ig = fg$.
From 3 and 4	5	$kc : fg :: km - \frac{1}{2}mc : \frac{1}{2}mc$
Compound the 5th.	6	$kc + fg : fg :: km : \frac{1}{2}mc$
Double the Conſequents	7	$kc + fg : 2fg :: km : mc$.
Compound the 7th.	8	$kc + 3fg : 2fg :: km + mc = kc = 3ch : mc$
That is otherwiſe	9	$3ch + 3fg : 2fg :: 3ch : mc$.
Divide the Ant. by 3.	10	$ch + fg = gh : 2fg :: ch : mc$.

Which was to be Demonſtrated.

Vid. *Cherubin* Dioptr. Ocul. pag. 61, 62. *Hon. Faber* Synop. Opt. Prop. 43. Sec. 8, 9. *Dechales* Prop. 23, 24. *Zahn* Teleſcop. Fund. 2. Cap. 9. Prop. 38.

But



Tab. 9. pag. 25

But this Rule will need but little farther Demonstration, if we consider, that it holds even in double Convexes of equal Convexities; for in them, as the Sum of the Radii of both Convexities: to the Radius of the Convexity:: so the double Radius of the Convexity: to the Distance of the Focus. And 'tis the same in Glasses of unequal Convexities, for these refract in the same Proportion to their Curvities as the other.

Wherefore from these three foregoing Propositions we lay these down as General Rules, *That the Point of Convergency for Parallel Rays falling on a Plano Convex-Glass, is distant about the Diameter of the Convexity. On a double Convex of equal Convexities, 'tis about the semi-Diameter of the Convexity. And on a Double Convex of unequal Convexities, the Rule for finding it is what we have laid down before.*

Scholium.

Before I proceed, it will be requisite here to note, *That the Rays which fall nigher the Axis are not united thereto so near the Pole of the Glass, as those that fall farther from it*; and the difference of the Points of Concourse is not so great, when the Convex side of a Plano-Convex Glass is turned to the Object, as when the Plain side is towards the Object.

For this observe *Tab.* 8. *F.* 1, 2, 3. in each of which, the Radius of the Convexity $c v$ is 2 Inches, or 20000 Parts (an Inch being 10000 Parts) the Arch $r v r$ in the first and second Figures is a Quadrant, and consequently the Thickness of the Glass $v z$ in each is 5858, being the versed Sine of 45° to the Radius 20000. But in the double Convex *Fig.* 3. the thickness of the Glass $v z$ is $\frac{8}{10}$ of an Inch or 8000 such Parts $a b$ in each is a Ray falling on the Glass Parallel to the Axis $c v$, at the distance of $\frac{1}{10}$ of an Inch or 1000 such Parts as $c v$ is 20000. $d e$ a like Ray falling $\frac{1}{2}$ or $\frac{5}{10}$ of an Inch, or 5000 such *Ta.*8.*Fi.* 1, 2, 3.

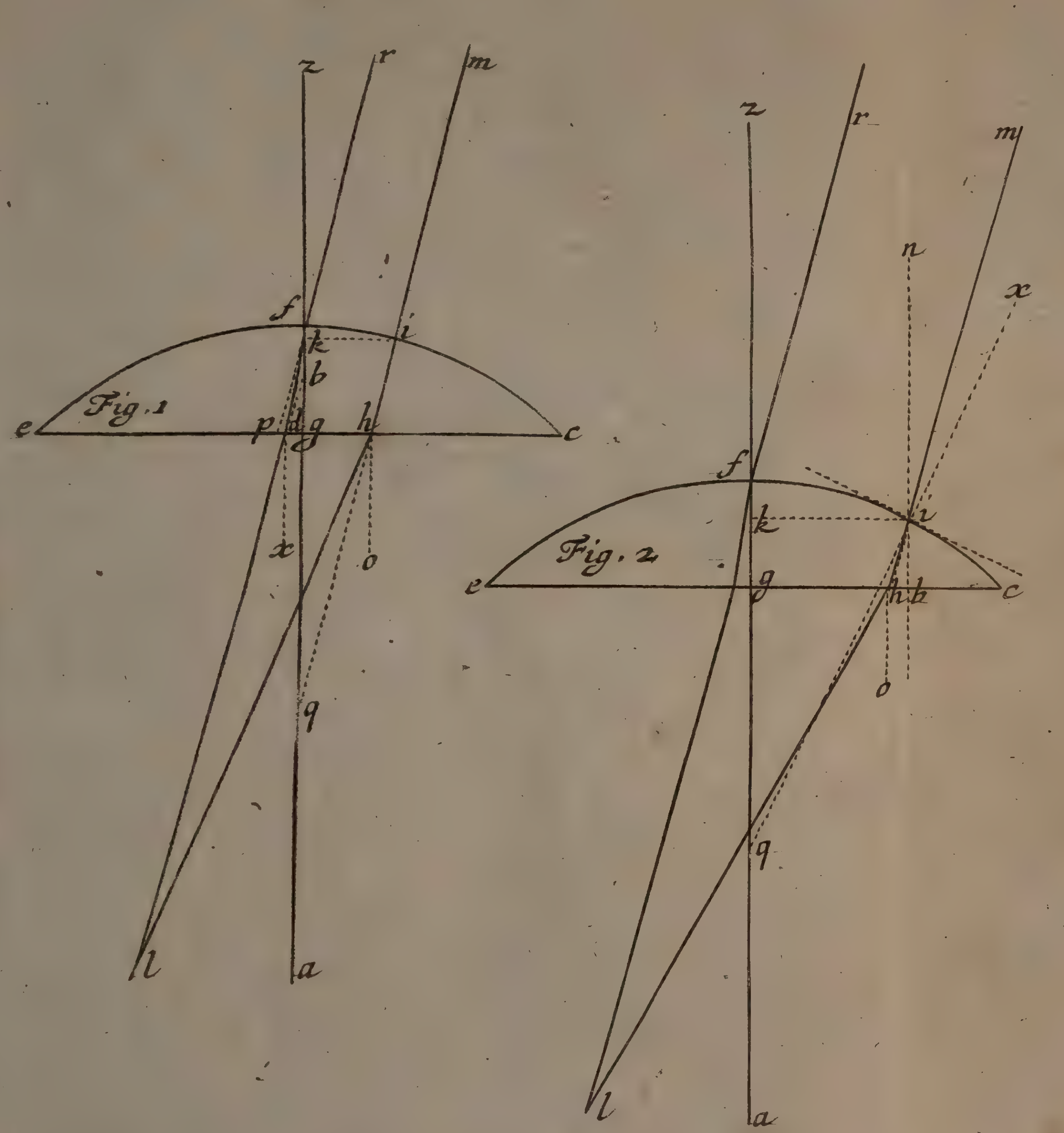
Parts

Parts diſtant from the Axis, i is the Point wherein the Ray $a\,b$ meets the Axis, h the point in which $d\,e$ croſſes the Axis. Here I call *$h\,i$ the Depth of the Focus*, which we ſee is greater in the ſecond Figure, wherein the plain ſide is towards the Object, than in the firſt *Fig.* wherein the Convex Surface is towards the Object. For Proof of this, let the Angles of Incidence be to the refracted Angle (according to *Kepler*) as 3 to 2, Expreſſed in the firſt Column of the following Table; or let the Sines of the Incidence be to the Sines of the refracted Angles, as 300 to 193 (according to Mr. *Newton*) as is expreſſed in the ſecond Column of the Table. Then by the foregoing Method of Calculation, delivered in the ſecond and third Propoſitions, we ſhall find as in the following Table.

	1ſt. Column.	2d. Column.
	3 : 2 :: *Inclin. Ref.*	300 : 193 :: *Inc. Ref.*
For the Plano-Convex with the Convex Side to the Object.	$v\,i = 4{,}1930$	$v\,i = 3{,}4762$
	$v\,h = 4{,}1230$	$v\,h = 3{,}4112$
Tab. 8. Fig. 1.	$h\,i =$ 700	$h\,i =$ 650
For the Plano-Convex with the plain ſide to the Object.	$v\,i = 3{,}9910$	$v\,i = 3{,}5957$
	$v\,h = 3{,}7140$	$v\,h = 3{,}3259$
Tab. 8. Fig. 2.	$h\,i =$ 2770	$h\,i =$ 2698
For the Double Convex of equal Convexities.	$v\,i =$ 1,841	$v\,i =$ 1,762
	$v\,h =$ 1,685	$v\,h =$ 1,653
Tab. 8. Fig. 3.	$h\,i =$ 156	$h\,i =$ 109

From this Table 'tis manifeſt, firſt that the Ray $a\,b$, which falls nigher the Axis $v\,c$, is not united thereto ſo near the Pole of the Glaſs v, as the Ray $d\,e$, which falls farther from the Axis; for $a\,b$ is united to the Axis at the Diſtance $v\,i$, and $d\,e$ is united to the Axis at the Diſtance $v\,h$, but $v\,i$ is greater than $v\,h$.

Secondly,



Tab. 10. pag. 28

Secondly, 'Tis evident that the *Focal Depth h i* is not so great in a Plano-Convex Glass, when the Convex Side is towards the Object, as in *Fig.* 1. *Tab.* 8. as when the plain Side is towards the Object, as in *Fig.* 2. *Tab.* 8. for by the Table *h i* is in the former but 650, but in the latter posture 'tis 2691, that is more than four times greater.

Whence I deduce this, that in viewing an Object by a Plano-Convex Glass, 'tis best turning the Spherical Side to the Object; and so likewise for burning by such a Glass, 'tis best turning the Convex Side towards the Sun.

Of OBLIQUE RAYS.

Premises to the Fourth Proposition.

Hitherto I have consider'd only the *Direct* Rays. I come now to the Consideration of *Oblique* Rays: And for this we must have Recourse to our foregoing 13*th.* and 17*th.* Definitions; as also to the first and second Experiments: By the 13th Definition, we know what *Oblique* Rays are: And by the 17th Definition, we learn that in every Parcel of *Oblique* Rays, flowing from the same Point of an Object upon a Glass; there is one certain Ray, which may be called the *Axis*, and that this *Axis* is as it were not at all refracted. But for a farther Declaration hereof, *Tab.* 9. *Fig.* 1. Let *a b c* be *Ta.9.Fi.1.* a Plano-Convex Glass, with the Convex Side towards the Object, *d b* the Axis of the *Direct* Cone of Rays, *e b* a Ray of the *Oblique* Cone, falling on the Vertex of the Glass at *b*, I say, this is the Axis of an *Oblique* Cone, and after its Double Refraction; first at its Ingress at *b*, and then at its Egress at *f*, it becomes *f g* Parallel to *e b*. This is Evident, if we imagine the Ray *e b* to fall on the Plane Glass *a m n c*, for 'tis refracted the same way in one Case as in t'other. Wherefore the Ray *e b f g*, is as it were strait, as *e b h* or *k f g*; so likewise

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likewiſe we may imagine the Plane Side of this ſame Glaſs *a c* turn'd towards the Object, then *p b d* is the Axis of the *Direct* Cone, and *g f* the Axis of an *Oblique* Cone. We ſee therefore that a Plano-Convex Glaſs, being expoſed with its *Convex Side* to the Object, the Rays that fall from the ſeveral Points of the Object, whoſe *Ingreſs* is at the Glaſſes *vertex b*, are the *Axes* of the reſpective Cones. But the *Plane Side* being towards the Object, the Rays from the ſeveral Points, which have their *Egreſs* at the Glaſſes *vertex b*, are the Axes of the reſpective Cones. And that amongſt the Infinite Rays that proceed from a *Collateral* Point of an Object, and form an *Oblique* Cone, falling on the Plano-Convex Glaſs; there is ſome one certain Ray, that falls on the Vertex, and has its *Ingreſs* at *b*; Or (if the Plane Side be towards the Object) has its Emerſion at *b*, is manifeſt from this Reaſon only, that the Rays compounding ſuch a Cone, are ſpread over the *Whole* ſurface of the Glaſs, and are as it were *Infinite* or *Indefinite*.

Ta.9.Fi.2. In like manner, *Tab.* 9. *Fig.* 2. *a b c k* is a Double Convex of Equal Convexities, *d b k p* the Axis of the *Direct* Cone of Rays, *e h* a Ray of an Oblique Cone, which at its Ingreſs at *h* is refracted into *h f*, in ſuch manner that it makes the Arches *b h*, *k f*, equal; then at its egreſs it becomes *f g*, Parallel again to *e h*, as if it had not been at all refracted, but had paſſed directly forward, or through a Plane Glaſs. This alſo will be manifeſt, if at the Point of Incidence *h*, and at the Point of Emerſion *f*, we draw the Tangents *m n*, *o r*; for theſe Tangents are Parallel when *h b* and *f k* are equal Arches; and the Ray *e h* is no otherwiſe refracted by the Convex Glaſs, than if it had fallen on the Plane Glaſs *m n o r*.

And that amongſt the Infinite Rays that proceed from a Collateral Point of an Object, and form an *Oblique* Cone, falling on the Double Convex *a b c k*; there is ſome one certain Ray, that has its Incidence ſo, that after its Refraction at

its

its Ingreſs, it may proceed ſo in the Glaſs, as to cut off equal Arches *b h*, *k f*, may be eaſily conceived; becauſe the whole ſurface of the Glaſs is occupy'd by each Cone from each Point; ſo that the Point *h* (for inſtance) cannot miſs of having amongſt the reſt its Ray, which muſt neceſſarily be ſo refracted.

What has been ſaid concerning a Double Convex of Equal Convexities *Tab*. 9. *Fig*. 2. may be accommodated to a Double Convex of unequal Convexities *Tab.* 9. *Fig*. 3. For as in *Fig*. 2. the Arches *b h*, *k f*, are to be equal (the Convexities being Equal) ſo in *Fig*. 3. the Arches *b h*, *k f*, are to be *Similar*; that is to ſay, *b h* is to be as many degrees of its Circumference, as *k f* is of its Circumference. Thus ſuppoſe in the ſame *Fig*. 3. The Arch *a b c* to be ſtruck with a Radius of two Inches, and the Arch *a k c* to be ſtruck with the Radius 4 Inches; theſe Radii being to each other as 2 to 1, *k f* muſt be to *b h* as 2 to 1, for then they ſhall be of equal degrees, each in its own reſpective Circumference, for here likewiſe in this third *Fig*. the Tangents at *h* and *f*, run Parallel to each other. Ta.9.Fi.3.

The Axes of all the Radious Cones, falling on a Plano-Convex with its Convex Side to the Object, Croſs each other in the Vertex of the Convex ſurface, at their *Ingreſs* into the Glaſs, as in *Tab*. 9. *Fig*. 1. at *b*; but if the Plane Side be turned towards the Object, they Croſs in the Vertex of the Convex ſurface at their Egreſs from the Glaſs, as in the ſame *Fig*. 1. at *b*. Ta.9.Fi.1.

The Axes of all the Radious Cones, falling on a Double Convex of Equal Convexities, Croſs each other in the Middle or Centre of the Glaſſes Axis. Thus in *Tab*. 9. *Fig*. 2. *d b p* is the Line, which produced, would paſs through the Centres of the two Convex ſurfaces; this Line I call the *Axis of the Glaſs*, let *b q* be equal to *k q*, then *q* is the Point Ta.9.Fi.2.

 wherein

wherein the Axes of all the Radious Cones Croſs each other.

The Axes of all the Radious Cones, falling on a Double Convex of unequal Convexities, Croſs each other in a Point within the Glaſs in its Axis, which divides the ſaid Axis in the Proportion, that the Radius of one Convexity has to the Ta.9.Fi.3. Radius of t'other. *Tab.* 9. *Fig.* 3. Let the Radius of the Convexity *a k c* be to the Radius of the Convexity *a b c*, as 2 to 1. Let *b k* the Axis of the Glaſs be divided in *q*, ſo that *k q* may be to *b q* as 2 to 1, then *q* is the Point wherein the Axes of all the Radious Cones Croſs each other.

The ſame may be underſtood of Concave Glaſſes.

And this is ſufficient for explaining what I mean by the *Axis of an Oblique Cone of Rays.*

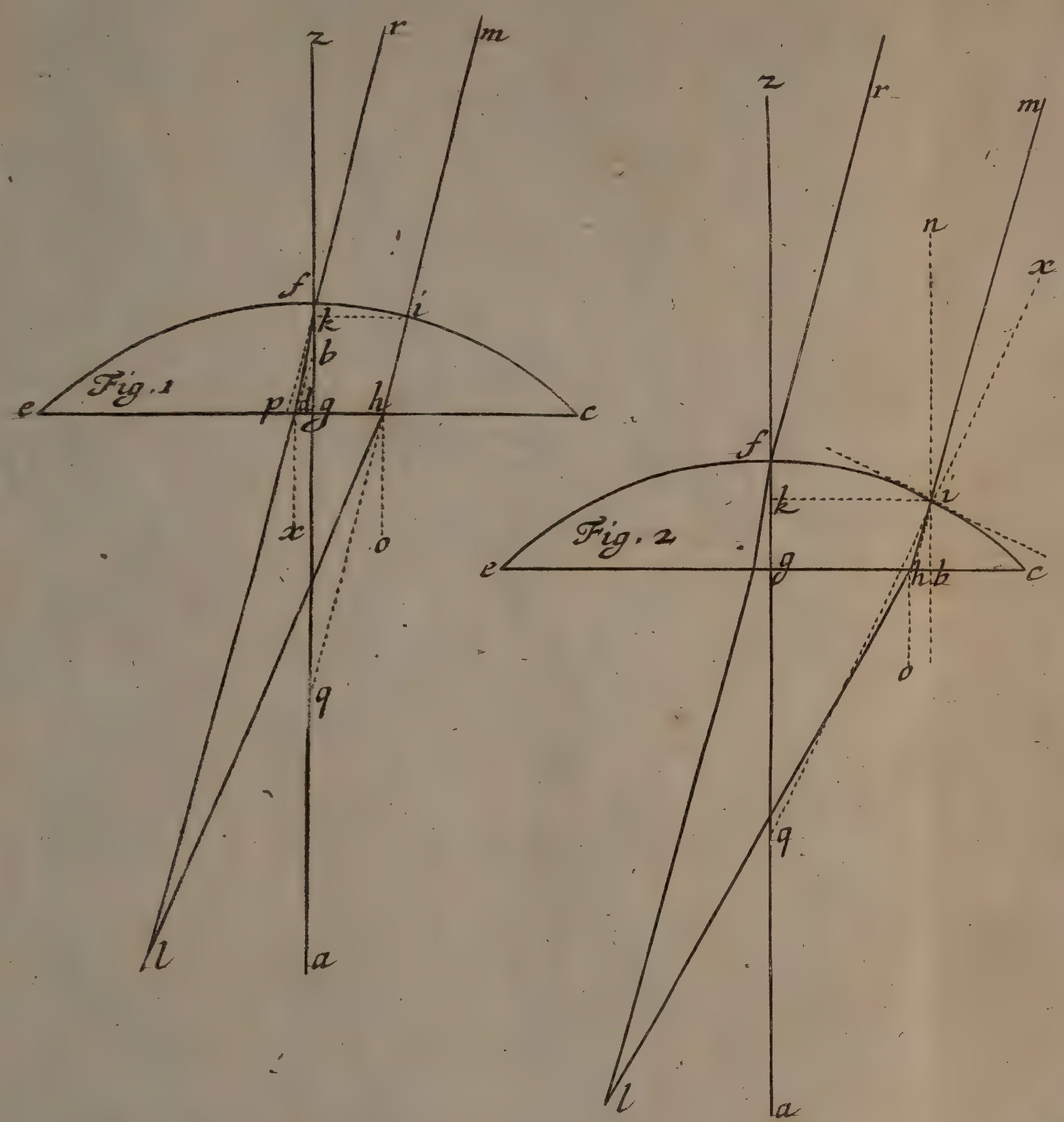
I come now to the Fourth Propoſition.

Prop. IV.

The Parallel Rays that proceed from each Collateral Point of an Object, and fall Obliquely on a Glaſs, are united with their Axis at the ſame Diſtance, as the Direct Parallel Rays are united with their Axis, viz. *By a Plano Convex about the Diſtance of the Diameter; by a Double Equal Convex about the Diſtance of the Radius; and by a Double Unequal Convex, as is before determin'd.*

I have hitherto propoſed the Properties of Glaſſes Problematically, to be Demonſtrated or rather traced out by a Geometrical Calculation of the Rays Progreſs. This Method, as 'tis the moſt Artificial, ſo 'tis the moſt Legitimate and Exact, and *wholly New* and *Diſtinct from that of other Optick Writers.* I ſhall firſt obſerve the ſame Method in this Propoſition, Tab. 10. Fig. 1. propoſing it Problematically thus, *Tab.* 10. *Fig.* 1. *e f c* is a Plano Convex Glaſs, with the Convex Side towards the Object.

Let



Tab. 10. pag. 28

Let there be given $q f$ the Radius of the Convexity (as in the Table after the foregoing *Scholium*) 20000, and $f g$ the Glaſſes thickneſs 5858, $z f q a$ is the Axis of the Glaſs it ſelf, or of the Direct Cone of *Rays*, the Point of whoſe Concourſe is in a, ſo that $f a$ (as we have found in the foregoing Propoſitions) is 34112. Let $r f$ be the Axis of an Oblique Cone of Rays, falling on the Vertex f; and making the Angle of Incidence $r f z$ ($= x d l$) of any given Quantity (as in this Example ſuppoſe 14°. 28′. 40″.) Let us ſuppoſe an other Ray $m i$ Parallel to $r f$, and its Point of Incidence i at ſuch a Diſtance from the Axis $z f a$, that being produced directly forwards, it may paſs through the Centre q of the Convexity; that is to ſay, let us ſuppoſe $k q i$ to be equal to $z f r$. Thus I propoſe it for eaſe of Illuſtration, for the Calculation may as truly be performed by ſuppoſing the Ray $m i$ at any other Diſtance, as I ſhall declare preſently.

From theſe things given, 'tis required to find the Point l in the Axis, where the Ray $m i$ croſſes it. For the Diſcovery whereof we proceed thus.

Draw $i k$ Perpendicular to $z q$, then in the Right-angled Triangle $k q i$, we have $q i$, and the Angle $k q i$ to find $k q$ and $k i$; then $q f - f g = q g$. And $k q : k i :: q g : g h$.

Produce $r f$ to p, then in the Right-angled Triangle $f g p$, we have $f g$, and the Angle $g f p = r f z$ to find the Side $p g$; or thus, the Triangles $g f p$, $g q h$ are Similar, then $g q : g h :: g f : g p$. Say then, as 300 : to 193 :: ſo S. $\angle g f p$: to S. $\angle g f d$.

In the Right-angled Triangle $f g d$, we have $f g$, and the $\angle g f d$, to find the Side $d g$, then $d g + g h = d h$.

Draw $h o$ and $d x$ Perpendiculars to $e c$, ſay then, as 193 : to 300 :: ſo S. $\angle o h q = g q h$: to S. $\angle o h l$. But $90° - o h l = d h l$. And $90° + x d l$ or $z f r = h d l$.

Laſtly,

Laſtly, In the Triangle $h\,d\,l$, we have the two Angles $d\,h\,l$ and $h\,d\,l$, and the Side $h\,d$ to find the Side $d\,l$; which was required.

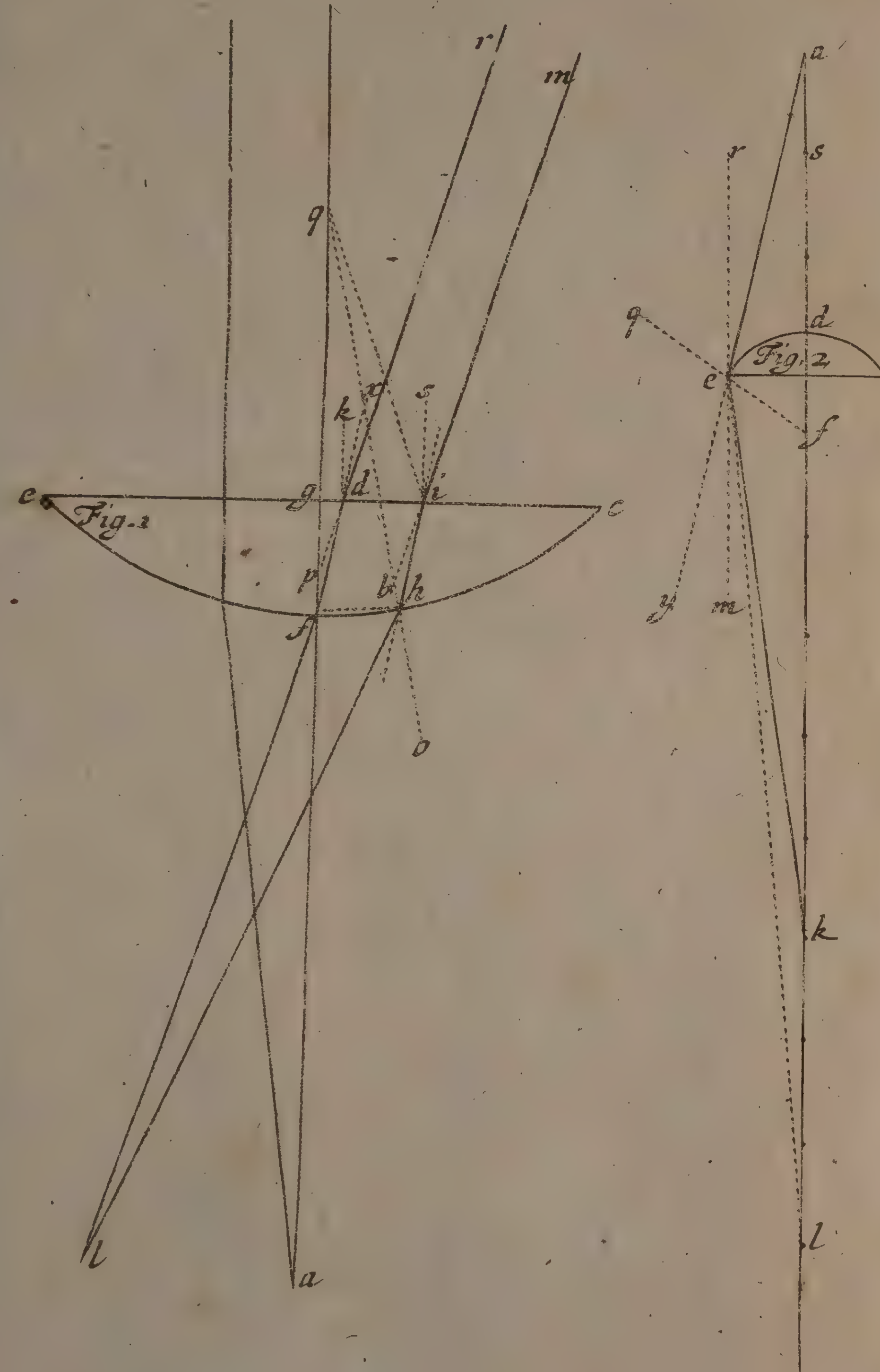
I have ſaid before, that the ſame Calculation may be performed for any other Ray Parallel to $r\,f$, falling at any diſtance $k\,i$ from the Direct Axis $z\,f\,q\,a$; ſuppoſing with the former Data (inſtead of the Angle $f\,q\,i$) (*Tab.* 10. *Fig.* 1.) the Diſtance $k\,i$ (*Tab.* 10. *Fig.* 2.) be given. For then in

T.10.Fi.2. *Tab.* 10. *Fig.* 2. draw $n\,i\,b$ Parallel to $z\,q$, and produce $q\,i$ to x, then $n\,i\,m = r\,f\,z$. In the Right-angled Triangle $q\,k\,i$, having $k\,i$ and $q\,i$, we get the Angle $k\,q\,i = n\,i\,x$, and the Side $k\,q$. And $n\,i\,x - n\,i\,m = m\,i\,x =$ to the Angle of Incidence. And $q\,f - q\,k = f\,k$, $f\,g - f\,k = k\,g = i\,b$, by which in the ſeveral ſmall Triangles within the Glaſs, we may find the Angle of the Rays Incidence from Glaſs to Air, as in the immediate foregoing Example, and ſo onwards in like manner.

I ſhall propoſe one Caſe more in this Doctrine of *Oblique*

Tab. 11. Fig. 1. *Rays*, and that is in *Tab.* 11. *Fig.* 1. where the Plane Side of the Glaſs is turn'd towards the Object. And that we may trace a Ray in this Caſe by Calculation, and find out a proper Ray for our purpoſe, we are to work a little backwards, thus, $q\,f = q\,h$ the Radius of the Convexity is given 20000, $f\,g$ the thickneſs of the Glaſs is given 5858; And $q\,f - g\,f = q\,g = 14142$. Let $d\,f$ be a Ray within the Glaſs, whoſe Ingreſs is at d, ſo that $d\,g$ is given 1000; and its Egreſs is at the Glaſſes Vertex f: In the Right-angled Triangle $d\,g\,f$, we have two Sides given to find the Angle $d\,f\,g$. Let $k\,d$ be Perpendicular to $e\,c$, and produce $f\,d$ directly to x: Then ſay, as 193 : to 300 :: ſo Sine of the Angle $k\,d\,x = d\,f\,g$: to Sine $\angle\,k\,d\,r$. And $k\,d\,r - k\,d\,x = x\,d\,r$. Wherefore $r\,d$ is a Ray of Light falling on the Glaſs at d *obliquely* by the Angle $k\,d\,r$, which after its Immerſion into the Glaſs proceeds onwards, and Emerges at the Vertex of the Glaſs f. This Ray at its

Emer-



Tab. 11. pag 30

Emerſion at f ſhall again run Parallel to $r\,d$, by what foregoes; wherefore the Angle $l\,f\,a$ is equal to the Angle $k\,d\,r$.

And thus much for the backward Operation, which we have Inſtituted to find a proper Ray to work with, that is, to find the Axis of an Oblique Cone, and its Angle of Inclination.

Let $m\,i$ be another Ray Parallel to $r\,d$, and falling on the Glaſs at i, ſo that $g\,i$, may be given 5000. 'Tis required to determine the Point l, where the Ray $m\,i$, after a Double Refraction at its Ingreſs at i, and at its Egreſs at h, meets the Axis $r\,d\,f\,l$.

Produce $r\,d$ directly to p, and $m\,i$ to b, and make $s\,i$ Perpendicular to $e\,c$; Seeing therefore that $m\,i$ is Parallel to $r\,d$, the refracted part $i\,h$, ſhall be Parallel to the refracted part $d\,f$, and the Angle $x\,d\,r$ $(=p\,d\,f)$ ſhall be equal to $b\,i\,h$, and the Angle $k\,d\,r$ is equal to $s\,i\,m$. Joyn $f\,h$.

Then in the Right-angled Triangle $q\,g\,i$, we have $q\,g$ and $g\,i$ to find the Angle $g\,q\,i = q\,i\,s$, and the Side $q\,i$: But $q\,i\,s + s\,i\,m = m\,i\,q$, and $180° - m\,i\,q = q\,i\,b$. And $q\,i\,b + b\,i\,h$ is equal to $q\,i\,h$.

Therefore in the Triangle $q\,i\,h$ we have the Sides $q\,i$ and $q\,h$, and the Angle $q\,i\,h$, to find the Angles $i\,q\,h$ and $q\,h\,i$. But $g\,q\,i - i\,q\,h$ is equal to $g\,q\,h$ or $f\,q\,h$.

Then in the Iſoſceles Triangle $f\,q\,h$, we have $f\,q$ and $q\,h$ equal to each other, and the Angle $f\,q\,h$ to find the Side $f\,h$, and the Angle $f\,h\,q = q\,f\,h$.

Produce $q\,h$ directly to o, then as 193 : To 300 :: So Sine Angle $q\,h\,i$: To Sine of the Angle $o\,h\,l$. But $180° - o\,h\,l - f\,h\,q = f\,h\,l$. Alſo $180° - q\,f\,h = h\,f\,a$. And $h\,f\,a + l\,f\,a = h\,f\,l$.

Laſtly, In the Triangle $h\,f\,l$, we have $f\,h$ and all the Angles to find the Side $f\,l$, which was required.

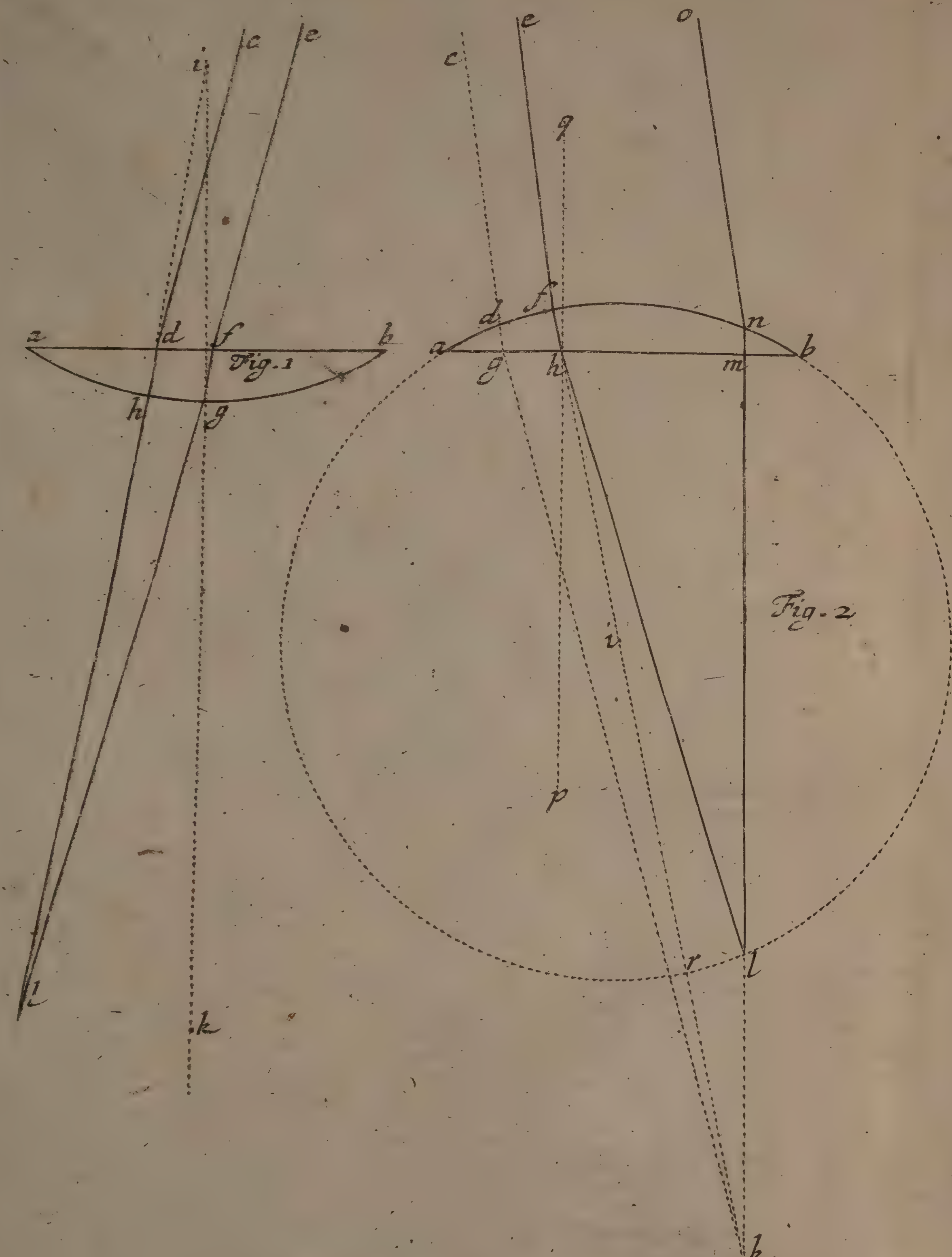
Common

Common Demonſtration.

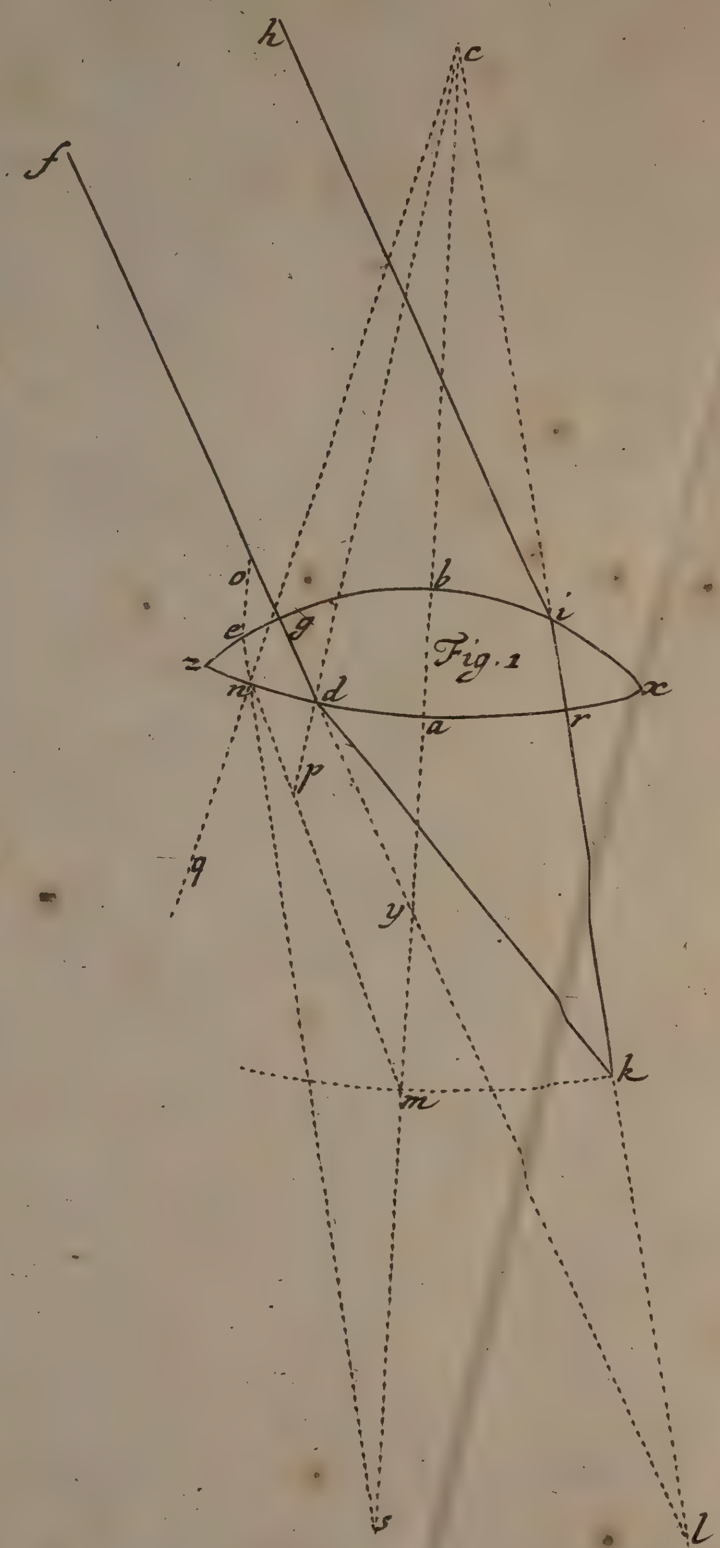
I ſhall now Demonſtrate this Propoſition the uſual Way. *Tab.* 12. *Fig.* 1. *a g b* is a Plano-Convex, let the Plain ſurface *a b* be firſt turn'd to the Object : And let the two Parallel Rays *c d*, *e f*, fall Obliquely on the Plane ſurface *a b*, I ſay, that theſe Rays ſhall be united in *l*, ſo that *h l* ſhall be near equal to the Diameter of the Convexity, which we'll ſuppoſe *g k*.

Tab. 12. Fig. 1.

Seeing the Angles *e f b*, *c d b*, are equal, theſe two Rays have equal Angles of Inclination, and conſequently ſhall have equal Correſpondent Angles of Refraction within the Glaſs,and therefore the refracted Rays *d h*, *f g*, ſhall alſo be Parallel within the Glaſs: And becauſe from the ſame Point in the Object, that tranſmits theſe two Rays *c d*, *e f*, there proceed Infinite Rays, which falling on the Glaſs, and being refracted therein, do occupy the whole Glaſs; ſome one of theſe Rays after its firſt Refraction, being produced backwards, ſhall paſs through the Centre of the Convexity *i*. Let *h d* be this Ray produced directly both ways to *i* and *l*: Seeing therefore *h d* is Perpendicular to the ſurface *a g b*, at *h* it ſuffers no Refraction in its Egreſs from the Glaſs. Wherefore let us conſider the other refracted Ray *f g*, and what Refraction it ſuffers at the Point *g* on its Egreſs from the Glaſs: Here the Angle of Inclination is *f g i* equal to the Altern *g i h*; Let us ſuppoſe the refracted Angle *k g l*, which (by Experiment V) ought to be $\frac{1}{3}$ more than the Inclination $fgi = gih$. Wherefore in the Triangle *i g l*, $li : lg :: \text{S.} \angle lgi = \text{S.} \angle lgk : \text{S.} \angle lig$. But in theſe ſmall Angles, as the Sines, ſo the Angles, and therefore if the Angle *l g k* be $\frac{1}{3}$ more than the Angle *l i g*, the Side *l i* is $\frac{1}{3}$ more than the Side *l g* or *l h* (for *l g* and *l h* are inſenſibly equal in Glaſſes of ſmall Segments of large Spheres)



Tab. 12. pag. 32.



Tab. 13. pag 33

Spheres) wherefore *h i* being the Semidiameter, *h l* ſhall be the Diameter. *Which was to be Demonſtrated.*

Secondly in *Tab.* 12. *Fig.* 2. Let the ſame Glaſs *a b* be turned with its Convex-Side to the Object, and thereon let the Parallel Rays *c d*, *e f*, fall obliquely to the Axis. Theſe Rays ſhall be united together behind the Glaſs almoſt at the diſtance of the Diameter of the Convexity. T.12.Fi.2.

For amongſt the parallel Rays that fall on the Glaſs, let us conceive one *e f*, that after its firſt Refraction at *f*, paſſing in the Glaſs in the Line *f h*, and being produced directly, would paſs through the Centre of the Convexity *i*. If we conceive the Medium of Glaſs continued, we know that theſe two Rays *e f*, *c d*, would be united in *k* at the diſtance of a Diameter and half from the Glaſs (by *Prop.* I.). From the point *k* draw *k n* perpendicular to the plane ſurface of the Glaſs *a b*, and let the Ray *n o* be drawn parallel to *e f*, *c d*; this Ray *n o* ſhall alſo, by its firſt Refraction, be directed towards the Point *k*; and becauſe within the Glaſs the Ray *n m* is perpendicular to the plain Surface *a b*, it ſhall proceed unrefracted through *m l k*. I ſhall now ſhew that *f h* by its ſecond Refraction at its egreſs at *h* is refracted to *l*. Draw *h l*, and through *h* draw *q h p* perpendicular to *a b*. The Angle of Inclination within the Glaſs is $fhq=phk=hkl$. And now I ſhall prove that the Angle *k h l* is $\frac{1}{2}$ the Angle of Inclination *p h k* or *h k l*. In the Triangle *h k l*, $s.\angle hkl : s.\angle khl :: hl : kl$ (or *k r*, for they are nearly equal), but *h l* is almoſt double *k l* or *k r*, therefore the Angle *h k l* is almoſt double *k h l*. *Which was to be Demonſtrated.* And therefore the parallel Oblique Rays have the Point of Concourſe at the diſtance of a Diameter almoſt from a Plano-Convex Glaſs. *Which was to be Demonſtrated.* Vid. *L'Optique de Pierre Ango Livre* 3. *Sec.* 69. *&c.*

Thirdly, In *Tab.*13. *Fig.*1. *z x* is a double Convex, *c* is the Centre of the Convexity *z a x*, *y* the Centre of the Convexity T.13.Fi.1.

$z\,b\,x$, $c\,y\,s$ the Axis of the Glaſs. 'Tis evident by what foregoes, that whatever Rays fall on this Glaſs parallel to the Glaſs's Axis are united thereto, at the diſtance of the Focus, (*viz.* about the Semidiameter in equal Convexities) as ſuppoſe at *m*. Wherefore let us conſider two other Rays, parallel between themſelves, but oblique to the Glaſs's Axis $c\,s$, ſuch as $f\,g$, $h\,i$, I ſay theſe likewiſe, after a Double Refraction do concur about the diſtance of the *Principal Focus* (for ſo I call the Focus of the Rays parallel to the Glaſs's Axis.)

Seeing there are infinite parallel Rays, that fall thus obliquely on the Surface $z\,b\,x$, moſt certainly there muſt be one amongſt the reſt, which ſo falls thereon, that being produced directly, ſhall paſs through the Centre *y* of the Convexity $z\,b\,x$: Let us conceive $f\,g$ to be this Ray, ſo that $f\,g\,y$ is a Right Line; let it be produced to *l*, ſo that $y\,l$ may be equal to double $g\,y$.

From the Point *l* to *c* the Centre of the Convexity $z\,a\,x$ draw the Right Line $l\,c$, and where this Line cuts the Surface $z\,b\,x$, as in *i*, let the Ray $h\,i$ fall parallel to $f\,g$. 'Tis evident, that by the firſt Refraction the Ray $h\,i$ ſuffers on the Convexity $z\,b\,x$, it will concur with the Ray $f\,g$ in *l* at the diſtance of a Diameter and half, for now $f\,g\,d\,y\,l$ may be conſidered as the principal Axis.

Let us ſuppoſe the principal Focus of this Glaſs to be at *m*. From the Centre *c* ſtrike the Arch $m\,k$ cutting $c\,l$ in the Point *k*. The Ray $h\,i$ ſhall be refracted at *i*, but the refracted Ray $i\,r$ (ſeeing it is perpendicular to the Surface $z\,a\,x$ by Conſtruction) ſhall proceed unrefracted directly to *k*; but the Ray $f\,g\,d$ being refracted in the Point *d* ſhall proceed refracted, and ſhall be united with the Ray $i\,k$ in *k*. Which I thus prove,

From *c* draw the Line $c\,d\,p$, the Angle $p\,d\,y$ is equal to the Angle of Inclination $g\,d\,c$ in the Glaſs; and therefore if $d\,k$ be

be truly the refracted Ray, the Angle pdy ought to be double the Angle ydk, and that it is so I thus prove.

Let us suppose the Ray oe Parallel to the principal Axis cs, this Ray by the first Refraction it suffers at e is directed to the Point s, so that bs is a Diameter and half, or triple by, and therefore equal to gl. The same Ray oe being refracted at e to n, and at n suffering a second Refraction is brought to concur with the Axis cs in m, the principal Focus by Supposition. In which Case, the Angle of Inclination in the Glass shall necessarily be enc, which is equal to qns, and the Angle of Refraction shall be snm; and therefore the former qns shall be double the latter snm. But I say, as qns is double of snm :: so pdl is double of ldk. Which I thus prove,

In the Triangle ncm	1	$s.\angle cnm = s\angle qnm : s\angle nmc :: cm = ck : nc = cd$
Also in the Triang. dck	2	$s.\angle cdk = s\angle pdk : s\angle dkc :: kc : cd.$
From 1 and 2	3	$s\angle qnm : s\angle nmc :: s\angle pdk : s\angle dkc.$
Then Sines and Angles being proportional	4	$qnm : nmc :: pdk : dkc$
By the Scheme	5	$qnm = qns + snm.$ Also $pdk = pdl + ldk.$
From 4 and 5.	6	$qns + snm : nmc :: pdl + ldk : dkc.$
Alternando 6.	7	$qns + snm : pdl + ldk :: nmc : dkc.$
Moreover in $\triangle\ nms.$	8	$s.\angle smn = s.\angle nmc : s.\angle snm :: ns = dl : ms = kl$
Also in Triang. dkl	9	$s.\angle dkl = s.\angle dkc : s.\angle ldk :: dl : kl$
From 8 and 9	10	$s.\angle nmc : s.\angle snm :: s.\angle dkc : s.\angle ldk$
Sines and Angles prop.	11	$nmc : snm :: dkc : ldk$
Alternando 11	12	$snm : ldk :: nmc : dkc$
From 7 and 12	13	$qns + snm : pdl + ldk :: snm : ldk$
Alternando 13	14	$qns + snm : snm :: pdl + ldk : ldk.$
Dividendo 14	15	$qns : snm :: pdl : ldk.$

From which 15th Step it follows, that if *q n s* be double of *s n m*:: ſo *p d l* is double of *l d k*. *Which was to be Demonſt.*

Note, In this Demonſtration we aſſume *n s* and *d l* to be equal, and *m s* and *k l* to be likewiſe equal, whereas they are not ſo exactly, for the Thickneſs and Breadth of the Glaſs cauſes ſome little difference, but in Glaſſes of ſmall Segments and large Spheres, and where the Rays do not fall very obliquely, this Difference is ſo very inconſiderable, that in a Demonſtration relating to a Phyſico-Mathematical Matter, they may well be conſidered as equal.

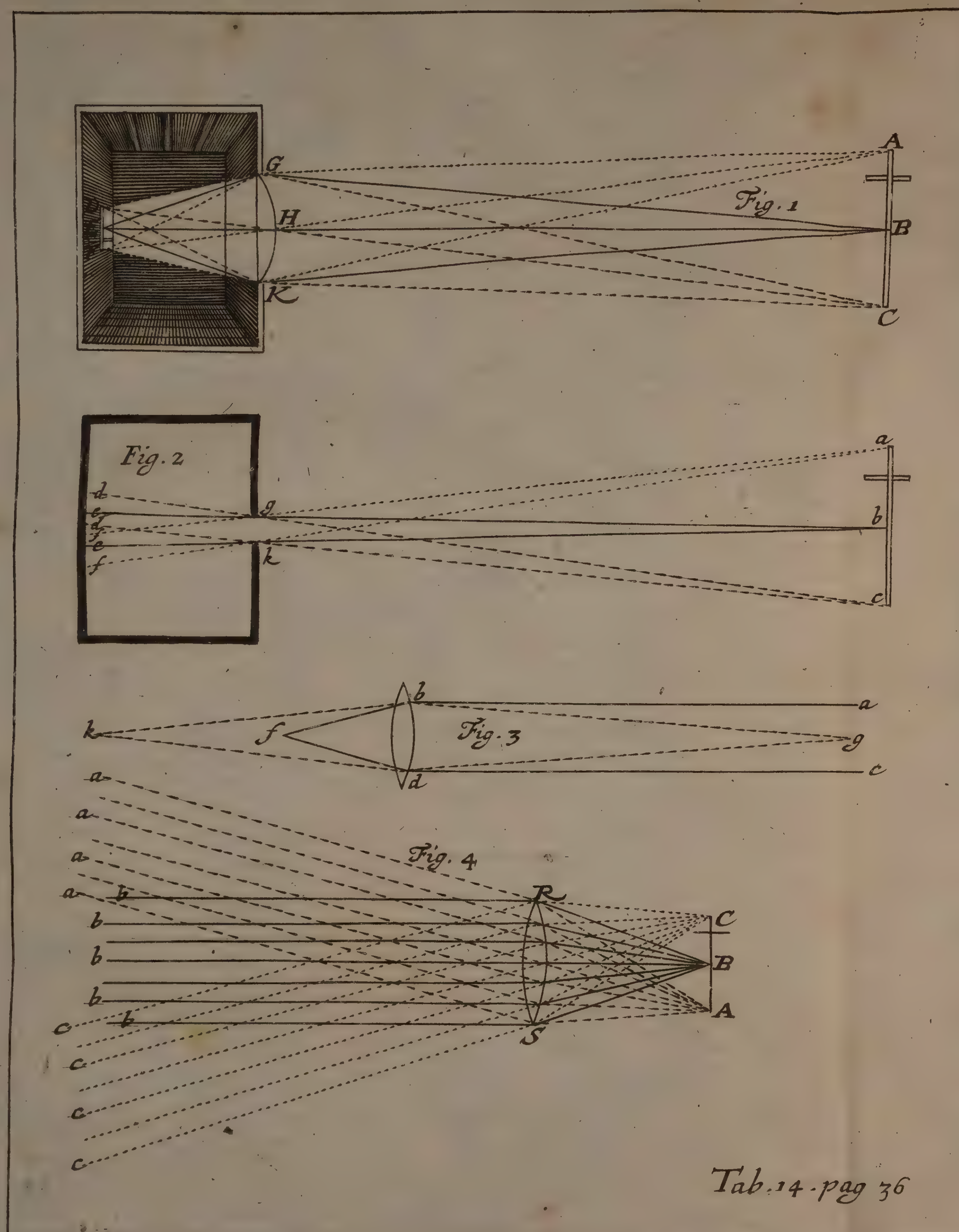
Scholium.

But tho the Propoſition expreſſes, that the Parallel Rays which proceed from each collateral Point of an Object, and fall *obliquely* on a Glaſs are united with their Axis at the ſame diſtance, as the direct Parallel Rays; yet this is to be underſtood with ſome Limitation: For if the Rays fall *very obliquely*, their Union with the Axis is not ſo exact, but they are ſcattered and the Focal Depth is very great. As will appear by Calculation.

Of the Repreſentation of outward Objects in a Dark Chamber; by a Convex-Glaſs.

From what has been hitherto laid down reſults that admirable Appearance, and Effect of a Convex-Glaſs duly placed in a ſmall Hole in a Dark Chamber; Which therefore we ſhall
Tab. 14. *Fig.* 1. here conſider, *Tab.* 14. *F.* 1. A B C is a diſtant Object, which we are now to conſider as very remote, ſo that tho the Figure, for want of Room in the Paper, expreſſes the Rays flowing from every ſingle Point, as very much diverging; yet we are to imagine, that the Rays (for Inſtance) from the Point A run as it were Parallel to each other; the diſtance of the Object

B H



Tab. 14. pag 36

B H being ſuppoſed vaſtly great, in Compariſon to the Glaſſes Breadth G K (But after the 5th. Propoſition following, this Advertiſement of the great diſtance of the Object will be needleſs, as therein ſhall appear) G H K is a Plano-Convex-Glaſs, receiving on its Surface a Cone of Rays from each ſingle Point of the Object, but here for avoiding Confuſion in the Scheme, we have only expreſſed the Cones flowing from the uppermoſt, middle, and lower Point of the Object, that is, from A, C, and B; and in each of theſe Cones neither have we expreſſed any more Rays than the Axis of the Cone (as for inſtance) A H, and the two extreme Rays of the ſame Cone A G and A K.

First therefore for the Rays that flow from the Point A. By what forgoes we know that the Axis of this Cone A H, after it has paſſed the Glaſs proceeds directly towards F, as if it were not at all refracted; and that the Rays A G, A K, are united with this Axis in a Point (ſuppoſe at F) diſtant from the Glaſs about the Diameter of its Convexity (the Glaſs being a Plano-Convex) and what is ſaid of the Rays A G, A K, being united with their Axis, muſt be conceived of all the Rays that make up the whole Cone flowing from this ſingle Point A: and they are all united likewiſe with the Axis in the Point F, forming another correſpondent Cone G F K, having its Baſe on the Glaſs, and its Vertex in the Point F, making the Pencil of Rays F G A K F.

And ſo likewiſe, what is declared of the Point A, and the Rays that flow from it, and of their being united together in F, may be eaſily conceived of the Rays flowing from the middle Point B, and their being united in E; and may be alſo underſtood of the Rays flowing from C, and their union in D; and ſo of the Rays proceeding from any other Point in the Object A B C, and their being united in a correſpondent Point in F E D. And this F E D we call the *Diſtinct Baſe, Focus,* or *Burning Point.* 'Tis called the *Diſtinct*

ſtinct Baſe, becauſe therein is a *Diſtinct* Repreſentation of the Object. And ſuppoſing the Glaſs GK in the Hole of a dark Room, ſo that the Repreſentation FED may be diſturbed by no other luminous Rays from outward Objects, and a Paper were placed at DEF, there to receive the Image, we ſhould thereon ſee the Object moſt livelily painted in its exact Shape and Colours. 'Tis called the *Focus*, or *Burning Point*, becauſe the Glaſs being expoſed to the Sun, it there collects the Rays from each Point of the Sun's Body, and paints its Image there moſt vividly, exciting a violent Heat, even to the inflaming of combuſtible Bodies.

But whereas in a *Scholium* after the Third *Propoſition*, it is obſerved, that the Rays falling nigh the Axis are not united thereto ſo near the Glaſs, as the Rays that fall farther from the Axis. And conſequently that all the Rays from one Point in the Object, do not unite in a correſpondent Point in the *Diſtinct Baſe*, but that (as we have noted before) the Focus has ſome Depth, and conſiſts not in an Indiviſible Point. Yet this hinders not the Repreſentation in the *Diſtinct Baſe* to be very lively and exact. For tho all the Rays from each Point are not united in an anſwerable Point in the Image, yet there are a ſufficient quantity of them to render the Repreſentation very perfect. And tho ſome Rays from other adjacent Points may a little intermix with thoſe from its neighbouring Parts, yet theſe are ſo few, that they make no great diſturbance.

N. B. Moreover in a *Scholium* following the Fourth *Propoſition*, 'tis obſerved, that when the Incidence of the oblique Rays is very oblique, that then their Concourſe with their Axis is not ſo very regular. From hence it proceeds, that the Repreſentation in the *Diſtinct Baſe* is not ſo clear and exact towards the Extremes, as towards the Middle.

As for the reaſon of this ſurprizing Appearance in a dark Chamber, wherein there is a ſmall Hole armed with a Convex-

vex-Glaſs; it will be manifeſt, if we conſider, that from every Point in the inverted Repreſentation on the Paper, the Rays proceed to the Eye that looks at it, exactly in the ſame manner, as from the Object it ſelf. For let us imagin the Point A to be *Blue*, B to be *Yellow*, and C *Red*. Becauſe all the Rays flowing from A are ſeparated by themſelves, and fall in the Image on the Point F, where no other Rays intermix with them to diſturb them, they muſt needs there repreſent *Blue*. For paſſing through the Glaſs, it being a Diaphanous Colourleſs Medium, does not at all alter them (notwithſtanding the Appearance of the *Priſme*,) and the Point F in the Paper being enlightened only with Rays of *Blue* Light, and of it ſelf being *White*, indifferent to all Colours, or rather indeed of no Colour, muſt needs repreſent *Blue*; and ſo E *Yellow*, and D *Red*. As to the *Shape* of the Object, it muſt needs be expreſſed exact likewiſe; for every Phyſical Point in the Object ſending out a Cone of Rays falling on the Glaſs, and theſe being formed into another Correſpondent Cone on the other ſide the Glaſs, determining their Vertices in Correſpondent Points of the Image; the Eye muſt neceſſarily perceive on the Paper the lively Image of the Object, being it is affected in the ſame way by *This*, as by the Object it ſelf. For from the Object it ſelf the Eye receives only a Cone of Rays from each of its Points, and ſo it receives a Cone of Rays from each Point in the Image on the Paper; each Point in the Image on the Paper being enlightened by the Rays it receives through the Glaſs; and conſequently the Paper, being an Impolite Surface, reflects theſe Rays into the whole circumjacent Medium. *Vid. Schol. Prop.* 52. *Gregorii Optic. Promot.*

And now that I am on the Conſideration of this Appearance, it may not be improper to conſider the Effect of a Plain ſmall Hole without a Glaſs in a dark Room. *Tab.* 14. *Fig.* 2. *Tab.* 14. *Fig.* 2.
repreſents a dark Chamber, in whoſe ſide there is a ſmall Hole

g k, ½ an Inch wide; *a b c* is a diſtant Object. Now becauſe there is no Glaſs in the Hole *g k*, the Rays from each Point of the Object paſs through the Hole directly ſtrait, and repreſent on the Wall within, at *d e d f e f*, a faint and confuſed Image of the Object: Becauſe here by the narrowneſs of the Hole, the Rays from ſome Points are hindred from intermixing with the Rays from ſome other Points; as here the Rays from the Point *a* do not intermix on the Wall with the Rays from the Point *c*; for we ſee *d d* do not intermix with *f f*. But then the Repreſentation is confuſed; for tho ſome Rays are hereby hindred from intermixing with ſome other Rays, yet other Rays from nigh adjoyning Points do intermix with their neighbouring Rays, as here the Rays from *a*, and thoſe from *b* are blended together in *f e*, and ſo thoſe from *c* and *b* in *d e*. And if the Hole be made ſo narrow as to hinder this communication of Rays in a great meaſure (for 'tis abſolutely impoſſible to hinder it altogether) then the opening admits ſo few Rays from each Point, that the Image is obſcure and imperceivable.

There is another Particular relating to this Appearance in a dark Chamber, which ſeems not ſo clearly treated of in thoſe Optick Writers which I have conſulted. Particularly *Zahn* in

Tab. 14. *Fig.* 2. his *Ocul. Artif. Fund.* 1. *Synt.* 3. *Cap.* 2. *Sec.* 8. gives no ſatisfactory Account thereof. The Matter is thus; in *Tab.* 14. *F.* 1. if the Image D E F in the dark Chamber be received on a *Speculum* or Looking-Glaſs inſtead of a White Paper, the Eye perceives a moſt dilute and faint Image on the *Speculum*: But if the *Speculum* be placed any where between G K and D E F, and a Paper placed as far diſtant from the *Speculum* before it, as the diſtinct Baſe D E F is behind the *Speculum*; then the *Speculum* ſhall reflect on the Paper the diſtinct Image of the Object.

The dilute and very faint Image, which the Eye perceives on the *Speculum* in the firſt Caſe, proceeds from the Imperfect Politure of the *Speculum*; For if it were a moſt exquiſitely ſmooth

ſmooth Surface, even that faint Image it ſelf would not appear. But as 'tis impoſſible by the Induſtry of Man to procure ſuch, and all Miroirs partake in ſome meaſure of a little Roughneſs, therefore it is that we have that dilute Image. And the reaſon we have no ſuch vivid Repreſentation from a Speculum (conſidered as a perfectly poliſh'd Surface) as from a Paper, as alſo the reaſon of the appearance in the ſecond forementioned poſture of the Speculum, does ſo depend on the Doctrine of Reflection or Catoptricks, that I ſhall paſs it over in this place, it being manifeſt to any one meanly verſed in that Doctrine. *Vid. Schol. Prop.* 52. *Gregorii Optic. Promot.* ×

Of Nigh Objects, or Diverging Rays.

We have hitherto conſidered the Radiating Points of *Diſtant* Objects both *Direct* and *Oblique*, the Rays from each ſingle Point whereof do proceed as it were Parallel. I come now to the Conſideration of *Nigh* Objects or *Diverging* Rays, and for theſe let us look back on the 5th, 6th, 7th, 8th, and 9th Definitions.

When the Rays from each Point of an Object run as it were *Parallel*, we may imagin that they may be *ſooner* or *eaſier* brought together, than when they *Diverge* conſiderably, by coming from a *nigh* Object. And that therefore a Convex Glaſs unites the *Parallel* Rays *nigher* to it ſelf, than *Diverging* Rays. *Tab.* 14. *Fig.* 3. *a b*, *c d* are Parallel Rays flowing from one T.14.Fi.3. Point of a diſtant Object, theſe are united by the Convex-Glaſs *b d* in its Focus at *f*. Let *b g*, *d g* be Diverging Rays flowing from the Point *g*, theſe are united in the Point *k*, farther from the Glaſs than the Point *f*.

For we may conceive (to omit the farther Proof of this by Geometrical Calculation), that the Parallel Rays have not ſo great a Reluctancy to be brought together, as the Diverg-

ing Rays; and therefore the Refractive Power of the Glaſs has not ſo much to do to bring *thoſe* together, as to bring *theſe*. And conſequently it performs the *Firſt ſooner* than the *Latter*, or in a *ſhorter* ſpace, after the Ray's Emerge from behind the Glaſs. And this the *more*, according as the Object *g* approaches *nigher* and *nigher* to the Glaſs, till at laſt it be ſo nigh, that the Rays, after they have paſſed the Glaſs, become *Parallel* or *Diverging*, as I ſhall ſhew in the following Propoſitions.

Definitions.

The Focus wherein the Parallel Rays of a *Diſtant* Object are united, I call the *Focus*, ſimply without any Addition, ſometimes the *Abſolute Focus*, or *Solar Focus*.

The Focus wherein the Rays from a *Nigh* Object, or the Diverging Rays are united, I call the *Reſpective Focus*.

The foregoing Precepts, which I have given for calculating the Progreſs of a Ray through a Spherical Glaſs, are abundantly ſufficient to find the Point of Union of Rays from Nigh Objects, or of Diverging Rays; The Diſtance of the Object or point of Divergency from the Glaſs (together with the foregoing Data) being given. I ſhall not therefore inlarge thereon, but Inſtead thereof I ſhall give theſe following ſhort Rules in this Matter.

Prop. V.

In Convex-Glaſſes, When a Nigh *Object is Placed more Diſtant than the Focus, The Rule for Determining the Diſtinct Baſe, or Reſpective Focus is This,*

As the Difference, between the Diſtance of the Object, and Focus:
Is To the Focus, or Focal Length::
So the Diſtance of the Object from the Glaſs :
To the Diſtance of the Reſpective Focus or Diſtinct Baſe from the Glaſs.

Demon-

Demonſtration.

In *Tab.* 11. *Fig.* 2. Let *e d* be a Plano-Convex Glaſs, whoſe abſolute Focus we know is about a Diameter of the Convexity, Let *s d* be this Diameter. *a* is a Radiating Point, *f* the Centre of the Convexity, *f e* the Radius of the Convexity produced directly to *q*. *a e* a Ray falling on the Glaſs, produced directly to *y*. Here we ſee the Angle of Inclination, or Incidence of the Ray *a e* is *q e a*.

Let it then be made — — — — 1 $as : sd :: ad : dk$

I ſay *k* is the reſpective Focus of the Ray *a e*.

Let *k l* be made $= \frac{1}{2}$ *k d*.

I ſhall firſt Demonſtrate, that by virtue of the firſt Refraction which the Ray ſuffers at its entrance on the Convex-Side of the Glaſs at *e*, 'tis directed as if it proceeded ſtrait towards *l*.

For to the Conſequents of the Analogy in the firſt ſtep add their Halfs, and it ſhall be — — — — 2 $as : sf :: ad : dl$

And compounding the 2d — 3 $af : sf :: al : dl$

Here we ſee *s f* is equal to three Semidiameters of the Convexity, that is, to thrice *f e*.

The Angle of Refraction is *y e l*, if therefore we prove that *y e l* is $\frac{1}{3}$ of *q e a* the Angle of Incidence, it will be manifeſt, that by the firſt Refraction the Ray is directed towards *l*.

In order to the Proof hereof, we lay down theſe Suppoſitions.

1. In the Triangle ael we ſuppoſe le and ld equal, becauſe we ſuppoſe the Glaſs of the leaſt Thickneſs imaginable, and the Segment of a large Sphere.

2. We ſuppoſe likewiſe, that the Angles of Incidence are all very ſmall, that ſo Sines and Angles may be proportional. Tho we could not expreſs this truly in the Figure.

Wherefore, for the Demonſtration of the forgoing Poſition, in the Triangle ael it is — — — —

4 $al:le=ld::s\angle ael:s\angle a$

But the Angle ael is the Complement of the Angle yel to 180° Therefore the Sine of the Angle ael is equal to the Sine of the Angle yel.

Wherefore the 4th Step runs thus

5 $al:ld::s.\angle yel:s.\angle a$

But in theſe ſmall Angles, as the Sines are, ſo are the Angles. Therefore the 5th ſtands thus —

6 $al:ld::\angle yel:\angle a$

Then from 3 and 6 it follows —

7 $\angle yel:\angle a::af:sf=3ef$

Then in the 7 Triple the Antecedent on this ſide, and ſubtriple the Conſequent on the other ſide, and it will be — — — — — — — —

8 $3\angle yel:\angle a::af:ef$

Moreover, in the $\triangle aef$ it is —

9 $af:ef::s\angle aef:s\angle a$

But the Sine of the Angle aef is equal to the Sine of the Angle qea, being Complements to 180°.

Where-

Wherefore the 9 may ſtand thus | 10 $af : ef :: s\angle qea : s\angle a$

But Sines and Angles in theſe ſmall Angles being proportional, it follows from the 8th and 10th. Steps, That | 11 $3\angle yel : \angle a :: \angle qea : \angle a.$

Wherefore from the Analogy in the 11th, it is evident, that the Angle of Inclination or Incidence qea is thrice the Angle of Refraction yel; ſeeing three times the Angle yel, and the Angle qea, bear the ſame Proportion to the ſame Angle a. And this was the firſt thing to be proved; and conſequently the Ray ae by its firſt Refraction at its Point of Incidence is directed towards the Point l.

It remains to be Demonſtrated ſecondly, that the Ray, at its Eruption on the plain ſide of the Glaſs into Air, is refracted into ek.

For the Proof of this draw rem Parallel to the Axis. Now the Angle of Incidence from Glaſs to Air ſhall be mel, the Angle of Refraction lek, which we ſhall prove to be half the Angle of Inclination mel; or we ſhall prove that the Angle mek is thrice the Angle of Refraction lek. Which being Demonſtrated, 'tis certain the Ray is refracted into ek.

For the Demonſtration hereof we retain the Series of our former Steps, and in the Triangle lek we have it | 12 $le = ld : lk :: s.\angle ekl : s\angle lek$

But $s.\angle ekl$ is equal to $s.\angle ekf$ being Complements to 180°. Therefore the 12th. Analogy may ſtand thus | 13 $ld : lk :: s\angle ekf : s\angle lek$

And the Angle ekf is $= \angle mek$

Wherefore | 14 $ld : lk :: s\angle mek : s\angle lek$

Now the Angles being as the Sines, and lk being by Conſtruction the third Part of ld, it follows from the 14th Step, that

that the Angle *l e k* is the third Part of the Angle *m e k*. Wherefore the Angle *l e k* is the Angle of Refraction agreeable to the Angle of Inclination *m e l*. *Which was to be Demonstrated.*

Example of this Rule.

Of this Rule I ſhall give this ſhort Example. Let the Focus of a Glaſs be 48 Inches; the Diſtance of the Object 156, their Difference is 108. Then 108 : 48 :: 156 : 69⅓ = to the reſpective Focus.

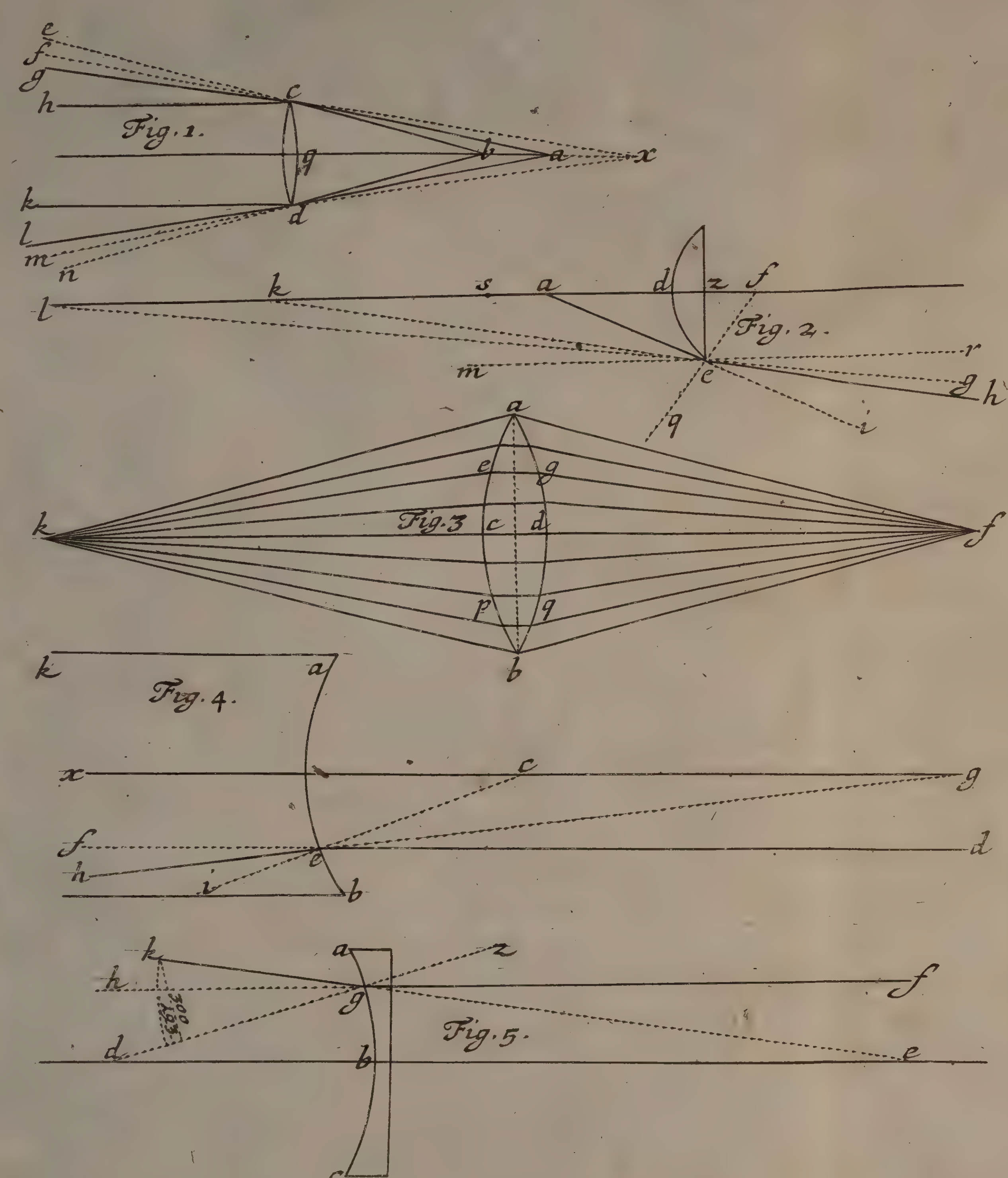
By this Rule we may view very nigh Objects with long Teleſcopes, converting them as it were into Microſcopes, only by lengthning them. But of this more hereafter.

Alſo from hence I ſhall deduce a Method of meaſuring diſtances at one Station: but hereof alſo more hereafter.

Prop. VI.

An Object being placed in the Focus of a Convex-Glaſs, the Rays from each Point thereof, after paſſing the Glaſs become Parallel;

Tab. 14. *Fig.* 4. *Tab.* 14. *F.* 4. A B C is an Object placed in the Focus of the Glaſs R S. The Rays from each Point of this Object flow upon the Glaſs, as here we have expreſſed thoſe from the Points A B C; I ſay the Rays from each of theſe Points after paſſing the Glaſs become Parallel, thoſe from A being *a a a*, from B *b b b*, from C *c c c*. For let us imagine a diſtant Object to ſend its Parallel Ray, *a a a* from its upper Point, *b b b* from its middle Point, *c c c* from its lower Point: Theſe (by what foregoes) ſhall be formed by the Glaſs into the diſtinct Baſe A B C. Let us now conceive this diſtinct Baſe, A B C, to be made the Object; and the Caſe is moſt plain, that its Rays muſt needs be remitted back again in the ſame manner as they were received



Tab. 15. pag 47

ceived from the diſtant Object on the ſide *a b c*, that is, Parallel. For the Progreſs of a Ray is reciprocal. *per Exp.* 8. *Vid. Prop.* XXVII.

Prop. VII.

If an Object be placed nigher a Convex-Glaſs than its Focus, the Rays from each ſingle Point thereof after they have paſſed the Glaſs, do proceed onwards diverging; But do not diverge ſo much as before they entred the Glaſs.

For if it be as much as the Glaſs can do, by its refractive Power to reduce to a Paralleliſm the diverging Rays from an Object placed in its Focus; it muſt needs be more than it can do, to reduce to a Paralleliſm the Divergency of the Rays, from an Object placed nigher to it than its Focus; The Divergence in this latter Poſture being much greater than in the former, and conſequently not ſo eaſily reduced by the refractive Power of the Glaſs. In *Tab.* 15. *Fig.* 1. *c d* is a Convex-Glaſs in whoſe *Focus* at *a* there is a radiating Point, from which proceed the Rays *a c f*, *a d m*, which meeting with the Glaſs, inſtead of going onwards to *f* and *m*, are reduced thereby to *c h*, *d k*, Parallel to each other. *b* is another radiating Point nigher the Glaſs than its Focus *a*, we may plainly perceive the Rays *b c*, *b d*, diverge more than the Rays *a c*, *a d*. Wherefore *b c*, *b d*, inſtead of proceeding directly onwards to *e* and *n*, are reduced to *c g*, *d l*; But *c g* and *d l* do yet diverge, but not ſo much as *b c*, *b d*. *Tab.* 15. *Fig.* 1.

Definition.

Imaginary Focus.

In the ſame Figure, draw *g c*, *l d*, directly to Croſs in *x*; I call *x* the *Imaginary Focus*; which is determined by the following Propoſition.

Prop.

Prop. VIII.

In Convex Glaſſes when an Object is placed nigher the Glaſs than the Focus, the Rule for finding the Imaginary Focus is this,

As the Difference between the Diſtance of the Object from Glaſs, and the Glaſſes Focus:
Is to the Glaſſes Focus ::
So the diſtance of the Object from the Glaſs:
To the Diſtance of the Imaginary Focus from the Glaſs.

Demonſtration.

In *Tab.* 15. *F.* 2. Let $e\,d$ be a Plano-Convex-Glaſs, whoſe abſolute Focus is a Diameter of the Convexity, let $s\,d$ be this Diameter; a is a radiating Point, f the Center of the Convexity $f\,e$ $(=f\,d)$ the Radius of the Convexity produced to q, $a\,e$ a Ray falling on the Glaſs produced directly to i.

We here ſuppoſe the Thickneſs of the Glaſs to be inconſiderable; and alſo, that the Breadth of the Glaſs is ſo little, that the Angles of Incidence ſhall be ſo ſmall, that Sines and Angles may be proportional.

Let it then be made according to the Rule, $a\,s : s\,d :: a\,d : d\,k$. I ſay $k\,d$ is the Diſtance of the Imaginary Focus of this Ray $a\,e$, after it has paſſed the Glaſs: That is, it ſhall be refracted, and proceed onwards in $e\,h$, as if it came directly from the Point k, $k\,e\,h$ being a ſtrait Line.

Let $k\,l$ be made $= \frac{1}{2}\,k\,d$. I ſhall firſt Demonſtrate, that by virtue of the firſt Refraction the Ray ſuffers at its Entrance on the Convex-Side of the Glaſs at e, 'tis refracted ſo as if it proceeded directly from l, that is to ſay, 'tis refracted into $e\,g$, $l\,e\,g$ being a Right Line.

'Tis

'Tis first manifest, that the Angle of Incidence is qea; if therefore we prove, that the Angle of Refraction $ieg = lea$ is $\frac{1}{3}$ of qea, we shew hereby, that by the first Refraction, the Ray is broken as if it proceeded directly from l.

That we may prove this, add to the Consequents of the

foresaid Analogy their Halfs,	1 $as : sd :: ad : dk$
And it shall be	2 $as : sf :: ad : dl$
Inverting, and Dividing this Second	3 $sf - as : sf :: dl - ad : dl$
That is by the Scheme	4 $af : sf :: al : dl$
Moreover in the Triangle aef it is	5 $af : ef :: s\angle fea : s\angle eaf$
But $s\angle fea$ is $= s\angle qea$ being Complements to 180°	6 $af : ef :: s\angle qea : s\angle eaf$
And in these small Angles, Sines and Angles being proportional, the 6th shall be	7 $\angle qea : \angle eaf :: af : ef$
Subtriple the Antecedent on this side, and triple the Consequent on t'other	8 $\frac{1}{3}qea : eaf :: af : 3ef = sf$
Then from the 4th and 8th Steps it follows	9 $\frac{1}{3} qea : eaf :: al : dl$
Moreover in the Triang. ela it is	10 $la : le = ld :: s\angle lea : s\angle lae$
But $s\angle lae = s\angle eaf$ being Complements to 180°	11 $la : ld :: s\angle lea : s\angle eaf$
And Signs and Angles being proportional	12 $al : dl :: \angle lea : \angle eaf$
From the 12th and 9th Steps 'tis manifest that	13 $\frac{1}{3} qea : eaf :: lea : eaf$

Wherefore from the 13th Step 'tis evident that $\frac{1}{3}$ qea is equal to lea, they having the same Proportion to the same Angle eaf.

Which was the first thing to be proved: and consequently 'tis manifest that the Ray *a e*, by its first Refraction at its Incidence on *e*, is refracted into *e g*, as if it came directly from *l*, *l e g* being a Right Line.

It remains to be Demonstrated secondly, that the Ray upon its Eruption on the plain side of the Glass into Air is refracted into *e h*, as if it proceeded directly from *k*, *k e h* being a Right Line.

Draw *m e r* Parallel to the Axis *l f*, Now the Angle of Incidence from Glass to Air is $ger = mel = ela$. The Angle of Refraction is $geh = lek$, which I shall prove to be half the Angle of Inclination *m e l*; Or I shall prove that the refracted Angle $mek = reh$ is thrice the Angle of Refraction *l e k*.

Which being Demonstrated, 'tis certain the Ray is refracted into *e h*, as if it came directly from *k*, and consequently *k* is the *Imaginary Focus* of the Ray *a e*. But this we shall thus Demonstrate.

In the Triangle *l e k*	14	$le = ld : lk :: s\angle lke : s\angle lek$
But $s\angle lke = s\angle ekf$ being Complements to 180.	15	$ld : lk :: s\angle ekf : s\angle lek$
But $\angle ekf = \angle mek$ therefore, the 15th is	16	$ld : lk :: \angle mek : lek$

And by Construction *l d* was put equal to 3 *l k*, therefore from this 16th step 'tis manifest, that the Angle *m e k* is equal to Thrice the Angle *l e k*. *Which was to be Demonstrated.*

Corollary.

Concerning *Convex-Glasses* exposed to Converging Rays.

Hitherto we have spoken of *Convex-Glasses* exposed either to *Parallel* or to *Diverging* Rays. Let us now consider their
T. 15. F. 2. Property exposed to Converging Rays. *Tab.* 15. *Fig.* 2. Let *h e* be a Ray of Light falling on the Convex-Glass *e d*, and con-

verging

verging directly towards the point k; $d\,k$ the distance of this Point of Convergence from the Glaſs, and $s\,d$ the Focal length of the Glaſs being given, 'tis required to find $d\,a$, the distance at which this Ray converges after paſſing the Glaſs.

We know that the Progreſs of a Ray is *reciprocal* (by *Exp.* 8.) and therefore, that if the Ray $a\,e$ be refracted into $e\,h$, the Ray $h\,e$ ſhall be refracted into $e\,a$.

Wherefore it being by *Prop.* VIII. — — $ds - da = as : sd :: ad : dk$ †
Alternating we have it — — — — $sd : dk :: ds - da : ad$
And Compounding — — — — — $sd + dk : dk :: ds : ad$ †

From which laſt Analogy this Rule ariſes, for ſolving the Problem in the Corallary.

As the Sum of the Focal Length of the Glaſs and the diſtance at which the ſtrait Ray converges directly — — — $ds + dk$:
To the ſaid Diſtance at which the ſtrait Ray converges dk ::
So the Focal Length of the Glaſs — — ds :
To the Diſtance at which the refracted Ray converges — — ad :

Obſervation.

Dechales in his firſt Book of Dioptricks, *Prop.* 48. gives a Rule for ſolving this 8th Propoſition: But therein he is the moſt egregiouſly miſtaken as ever Man was, that pretended to Demonſtration, and commits the moſt notorious Error that can be imagined. And yet herein he wants not his Followers, for *Zahn* in his *Teleſcopium Fundam.* 2. *Syntag.* 1. *Cap.* 4. *Prop.* 13 tranſcribes *Dechales*, and copies out even his *Errata Typographica*, beſides the chief and great Error of the whole Solution. Which ſhews that *Zahn* was either very careleſs, or elſe that he underſtood nothing of the Matter, as indeed he ſeems to do very little, for he is a mere blind Tranſcriber from others.

Dechales Rule in his fore-cited Propofition is this. *As the Sum of the Diftance of the Object from the Glafs, and half the Glafs's, Focus: Is to the Glafs's Focus* :: *fo the Diftance of the Object from the Glafs: to the Diftance of the Imaginary Focus beyond the Diftance of the Object*: That is, in *Tab.* 15. *F.*2. $af : sd :: ad : ak$. But how falfe this is, fhall appear not only from his miftaking one Angle for another in his Demonftration, but from the following Examples, which I have taken the pains to calculate for a more full Confutation of *Dechales*, and Illuftration of my own Rule.

Tab. 15. *Fig.* 2.

1. *Example.*

Inch.

Given in *T.*15. *F.*2. { Plan.-Conv. Gl. Foc. 144,00 $= sd = 2\ ef$
Rad. of the Convexit. 72,00 $= ef = fd$
Diftance of the Object 48,00 $= ad = az$
$\frac{1}{2}$ Breadth of the Glafs 1,00 $= ez$

By my Rule { $dk = 7200$
$kl = 3600$
$dl = 10800 = dk + kl$
$al = 6000 = dl - da$,

By *Dechal. Prop.*48. { $dk = 10560$
$kl = 5280$
$dl = 15840 = dk + kl$
$al = 11040 = dl - da$

Let us now try by Calculation, which of thefe is the Truth. *Tab.* 15. *Fig.* 2.

Tab. 15. *Fig.* 1.

In the Right-angled Triangle efz, we have the Sides ef and ez, to find the Angle $efz = 0^\circ\ 47'\ 40''$ and its Complement to 90° $fez = 89^\circ\ 12'\ 20''$.

Then in the Right-angled Triangle eaz, we have az, and ez to find the Angles and Side ae.

Thus

Thus $\angle eaz = 1^\circ\ 11'\ 40''$
$\angle aez = 88\ 48\ 20$
$ae = 4801$

Then $aez + fez = aef = $ — — — $178\ 0\ 40$
And Compl. aef to $180^\circ = qea = $ Incl. $= 1\ 59\ 20$
Then as $3 : 1 :: s\angle qea : s\angle ieg = lea = 0^\circ\ 39'\ 50''$ Ang. of Ref.
Moreover, $eal = 180 - eaz = $ — — $178\ 48\ 20$
And $ela = 180 - eal - lea = mel = ger = 0\ 31\ 50$

Then in the Triangle eal, we have two Angles, lea and ela, and the Side ae, to find the Side al. Thus,

As $s\angle ela = 0^\circ\ 31'\ 50''$	*Log. C. Ar.*	2.0333979726
To $ae = 4801$	— — — — — — —	3.6813317060
So $s\angle lea = 0^\circ\ 39'\ 50''$	— — — —	8.0639630497
To $al = 6007$	— — — — — —	3.7786927283

which is sufficiently agreeable to my $al = 6000$, but differs vastly from *Dechales* $al = 11040$.

I now proceed to find the Side dk, which by my Rule is 7200, but by *Dechales* is 10560.

$\angle qez = qea + aez = 1^\circ\ 59'\ 20'' + 88^\circ\ 48'\ 20'' = 90^\circ\ 47'\ 40''$
$\angle qem = qez - 90 = $ — — — — — — — — $0\ 47\ 40$
$\angle mel = ale = ger = $ — — — — — — — $0\ 31\ 50$,
This is the Angle of Inclination or Incidence from Glaſs to Air, on the plane ſide of the Glaſs.

Then, As $2 : 3 :: s\angle ger = 0^\circ\ 31'\ 50'' : s\angle her = mek = ekd = 0^\circ\ 47'\ 40''$.

And $kez = 90 - ekd = 89^\circ\ 12'\ 20''$.

Then in the Triangle kez, all the Angles, and the Side ez are given, to find kd or kz (we ſuppoſing the Thickneſs of the Glaſs inconſiderable). Thus,

Rad. — — — — — — —		
To $ez = 100$	Log.	2.0000000000
So Tang. $\angle kez = 89^\circ\ 12'\ 20''$		11.8580312668
To $kd = 7208$		3.8580312668

Which

Which $k\,d$ anſwers ſufficiently exact to what my Rule gives it 7200, the Eight Parts coming in for the Thickneſs of the Glaſs, and the Angles not being calculated more accurately than to 10″ Seconds *Numero rotundo*. But this is much different from what *Dechales* Rule gives it 10560.

2. *Example.*

Given in *Tab.* 15. *F.* 2. $\begin{cases} \text{Glaſs's Focus} = 14400 = s\,d = 2\,e\,f \\ \text{Rad. of the Conv.} = 7200 = e\,f = f\,d \\ \text{Diſt. of the Object} = 5763 = a\,d = a\,z \\ \tfrac{1}{2}\text{Bread. of the Glaſs} = 100 = e\,z \end{cases}$

By my Rule $\begin{cases} d\,k = 9608 \\ k\,l = 4808 \\ d\,l = 14412 = d\,k + k\,l \\ a\,l = 8549 = d\,l - a\,d \end{cases}$

By *Dechales*, 48 *Prop.* $\begin{cases} d\,k = 12165 \\ k\,l = 6082 \\ d\,l = 18247 = d\,k + k\,l \\ a\,l = 12484 = d\,l - a\,d \end{cases}$

The Thickneſs of the Glaſs $z\,d$ is but $\frac{1}{7200}$ and therefore inconſiderable, it being the Verſed Sine of the Angle $e\,f\,z$. We ſuppoſe therefore $k\,d$ and $k\,z$ equal.

Then by Calculation, *ex Datis*, we find as follows,

1	$e\,f\,z=$	0°	47′	40″
2	$f\,e\,z=$	89	12	20
3	$e\,a\,z=$	0	59	40
4	$a\,e\,z=$	89	00	20
5	$a\,e$	=5764		
6	$a\,e\,f=$	178	12	40

7	$q\,e\,a =$ 1° 47′ 20″	= Inclination
8	$l\,e\,a =$ 0 35 40	= Refraction
9	$e\,a\,l =$ 179 0 20	
10	$e\,l\,a =$ 0 24 0	
11	$a\,l = 8566$	

Which $a\,l = 8566$ is but 17 more than my Rule gives it, which (confidering the Inaccuracy of the Calculation of the Angles, being only *Numero rotundo* to about 10″ Seconds) proves my Rule fufficiently true, and fhews *Dechales* as falfe, which makes $a\,l = 12484$.

Moreover, By my Rule $d\,k$ is $= 9608$, and by *Dechales* 12165. The Angles requifite to find $d\,k$ we have thus,

	1	$q\,e\,z = 90°$ 47′ 40″
	2	$q\,e\,m =$ 0 47 40
Before found--	3	$a\,l\,e =$ 0 24 0 $= g\,e\,r = m\,e\,l =$ Inclin.
	4	$m\,e\,k =$ 0 36 0 $= h\,e\,r = e\,k\,d.$
	5	$k\,e\,z = 89$ 24 0
	6	$k\,d = k\,z = 9550$

Which $k\,d = 9550$ is but 58 lefs than my Rule gives it; and this fmall Difcrepancy may eafily arife from the Error of 10″ Seconds in the Calculation of the Angles. But *Dechales* gives it 12165.

I have chofen a Plano-Convex Glafs to demonftrate this Rule, which holds as true in double Convexes, but would in them be more difficult and intricate to demonftrate.

PROP.

Prop. IX.

In double Convex-Glaſſes of equal or unequal Convexities, the Rays proceeding from the Diſtance of a Diameter of one Convexity, are united at the Diſtance of a Diameter of t'other Convexity.

T.15. F.3. In *Tab.* 15. *Fig.* 3. Let us conceive the Glaſs *a b* divided by the Pointed Right Line *a b* into two Plano-Convex Glaſſes *a c b* and *a d b*. Let the Point *k* be diſtant from the Glaſs *a c b* the Diameter of the Convexity *a c b*. By what foregoes, *Prop.*II. The Rays *k a*, *k e*, *k c*, *k p*, *&c.* ſhall be ſent parallel through the Glaſs *a c b*, ſo that they ſhall become *e g*, *c d*, *p q*. Then falling Parallel on the Plano-Convex Glaſs *a d b*, they are united thereby in *f*, at the diſtance of its Diameter: By the foregoing Doctrine. *Prop.* II.

Of Concave-Glaſſes.

Hitherto we have treated of *Spherical Convex Glaſſes* only. We now proceed to the Conſideration of *Concave-Glaſſes*.

Definition.

Vertual Focus, *or* Point of Divergence.

T.15. F.5. In *Tab.* 15. *Fig.* 5. *a b c* is a Plano Concave Glaſs, whoſe Axis is *d e*, *f g* is a Ray falling thereon parallel to the Axis *d e*, *d* is the Centre of the Arch *a b c*. This Ray after it has paſſed the Glaſs at its Emerſion at *g*, inſtead of proceeding directly to *h*, is refracted from the Perpendicular *d g*, and becomes *g k*. Draw *g k* directly to croſs the Axis in *e*. I call the Point *e* the *Virtual Focus*, or *Point of Divergence.*

Concerning

Concerning Concaves, there is little to be ſaid after our foregoing Method for Calculating the Progreſs of a Ray falling on a Spherical Convex; for this may eaſily be accommodated to Concaves.

I ſhall therefore ſlightly paſs theſe over, and ſhall only lay down their Properties in brief, with their uſual Demonſtrations, referring to Calculation for a more accurate Scrutiny.

Prop. X.

A Ray falling parallel to the Axis from Air, on the Concave Surface of a Glaſs Medium, has its Virtual Focus by its firſt Refraction at the Diſtance of a Diameter and half of the Concavity.

Tab. 15. *Fig.* 4. *a b k h* is a Glaſs Medium terminated by the Concave Surface *a b*, *d e* a Ray of Light falling parallel to the Axis *g c x*. I ſay the Ray *d e*, by its Refraction at the Point of Incidence *e*, is ſo refracted into *e h*, as if it proceeded directly from *g* the diſtance of a Diameter and half of the Convexity. T. 15. F. 4.

Let *c* be the Centre of the Concave-Arch *a e b*, draw *c e* directly to *i*, produce *d e* directly to *f*, and *h e* directly to *g*. The Angle $ced=fei$ is the Angle of Inclination, the Perpendicular is *ei*. By what foregoes, the Ray paſſing from Air a Rare Medium into Glaſs a Denſe Medium, is refracted towards the Perpendicular by the Angle *f e h*, which muſt be $\frac{1}{3}$ of the Inclination *f e i*. Therefore $feh=deg=egc$ is half $hei=gec$. Wherefore in the Triangle *c e g*, the Angle *c e g* being double the Angle *e g c*, the Side *c g* that ſubtends *c e g*, ſhall be double the ſide *c e* that ſubtends *e g c*. For we ſuppoſe theſe Angles to be ſo ſmall, that Sines and Angles, or Sides and Angles are proportional. *Which was to be Demonſtrated.*

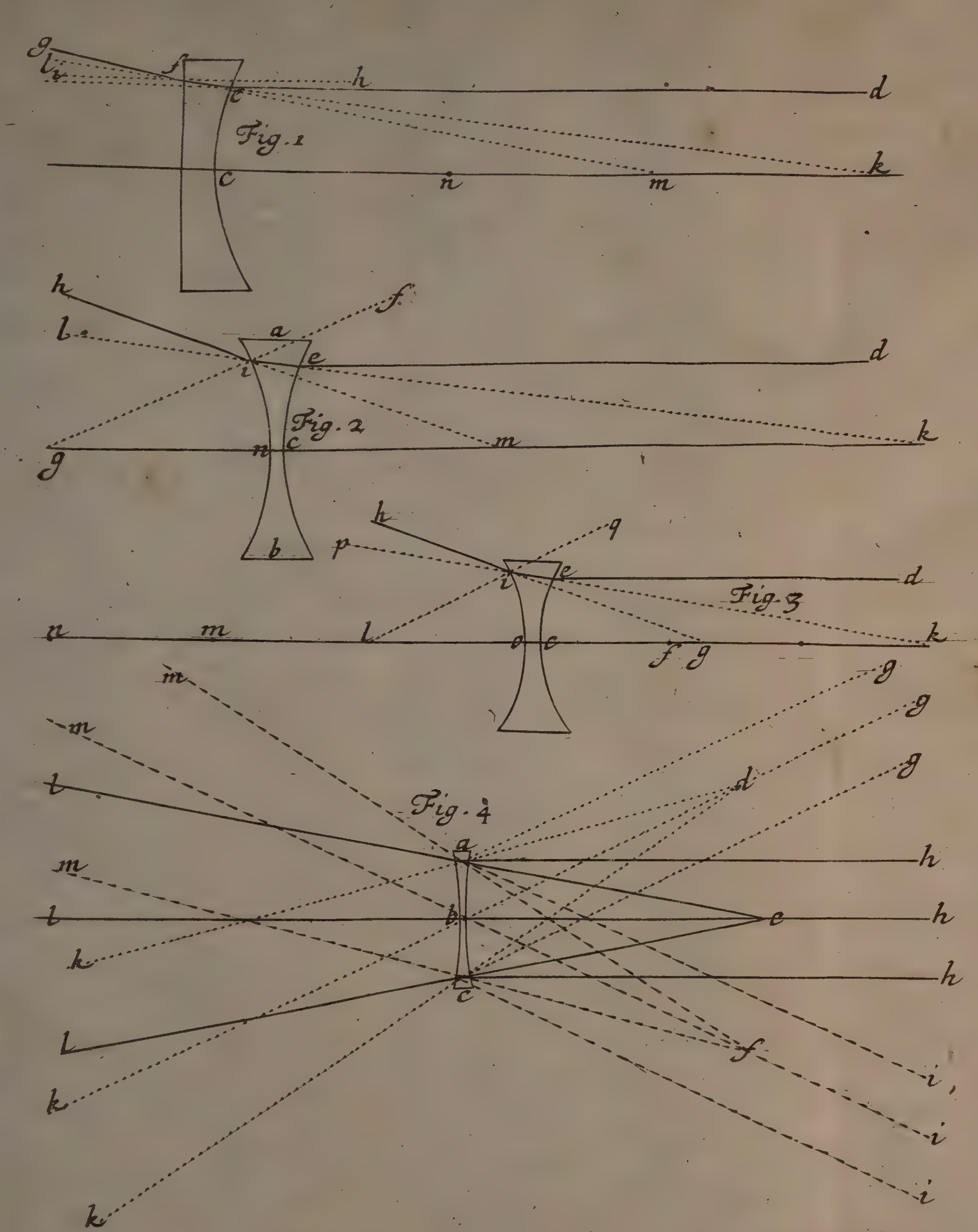
Prop. XI.

In Plano Concave Glaſſes, when the Rays fall parallel to the Axis, the Point of Divergence or Virtual Focus is diſtant from the Glaſs the Diameter of the Concavity.

T.15. F.5. *Tab.* 15. *Fig.* 5. *a b c* is a Plano-Convex Glaſs, whoſe Axis *e d*, *f g* is a Ray of Light falling on this Glaſs parallel to its Axis. This Ray after it has paſſed the Glaſs at *g*, is ſo refracted into *g k*, that *g k* being produced directly backwards, it ſhall interſect the Axis in *e*, ſo that *e b* ſhall be the Diameter of the Cavity *a b c*.

Firſt, Let the plain Side be turned towards the Object, as in *Tab.* 15. *Fig.* 5. Becauſe the Ray falls perpendicular on the plain Superficies, it is not at all refracted at its Immerſion into the Glaſs. But at *g* it emerges from the Concave Side of the Glaſs into Air; and its Inclination is *g d b* (*d* being the Centre of the Cavity *a b c*) and 'tis now refracted from the Perpendicular *d g*, by the Angle *h g k*, which is half the Inclination *g d b*. But *h g k* is equal to *f g e*=*g e d*. Wherefore in the Triangle *g d e*, the Angle *g e d* is half the Angle *g d e*, and therefore the Side *g d* is half the Side *g e*=*e b* in Glaſſes of large Spheres.

T.16. F.1. Secondly, *Tab.* 16. *Fig.* 1. Let the Concave Side *e c* be turned to the Ray *d e*. By the firſt Refraction the Ray ſuffers at its Immerſion into Glaſs at *e*, 'tis refracted from *e* to *f*, and directed to the Point *k*, ſo that *c k* is thrice the Semidiameter *n c* (by the Xth hereof). Through the Point *f* draw *i h* perpendicular to the plain Side of the Glaſs, and continue *e f* directly to *l*. The Angle of Inclination of the Ray within the Glaſs is *h f e*=*i f l*= *d e k* = *e k c*. Now the Ray emerging from Glaſs to Air, the ſecond refracted Ray ought



Tab. 16. pag. 58

ought to recede from the Perpendicular $i\,f$, ſo that the Angle $g\,f\,i$ muſt be thrice the Angle $g\,f\,l$. Produce $g\,f$ directly to m. Then $g\,f\,i$ ſhall be equal to $f\,m\,c$. Wherefore in the Triangle $k\,f\,m$; As the Sine of the Angle $k\,m\,f$, or the Sine of the Angle $f\,m\,c$: To the Sine of the Angle $f\,k\,m$ :: So $f\,k$: To $f\,m$, and (neglecting the Thickneſs of the Glaſs) So is $k\,c$: To $m\,c$. And Angles and Sines being proportional, $k\,c$ ſhall be to $m\,c$ as 3 to 2; but $k\,c$ is Thrice the Semidiameter, therefore $m\,c$ is the Diameter. *Which was to be Demonſtrated.*

Corollary.

In Plano-Concave Glaſſes, as $300 - 193 = 107 : 193$ :: So the Radius of the Concavity : To the Diſtance of the Virtual Focus.

Demonſtration.

Tab. 15. *F.* 5. The Inclination is $h\,g\,d = g\,d\,b = f\,g\,z = 193$. T. 15. F. 5.
Refracted Angle is $z\,g\,e = k\,g\,d = 300$.
Ang. of Refr. $kgh = kgd - hgd = 300 - 193 = 107 = fge = ged$.
In the Triang. gde. As $s\angle ged$: To $s\angle gde$ :: So Rad. $= gd$: To ge.
That is——— ——— $107 : 193 :: g\,d : g\,e = e\,b$ in Glaſſes of large Spheres and ſmall Segments.

Prop. XII.

In Double Concaves of equal Cavities, Parallel Rays have their Virtual Focus, or Point of Divergence at the Diſtance of the Radius of the Concavity.

Tab. 16. *F.* 2. $a\,b$ is a Concave of equal Cavities, that is, *Tab.* 16. *Fig.* 2.
$m\,c$ the Radius of the Cavity $e\,c$, is equal to $g\,n$ the Radius of the Cavity $n\,i$. Let the Ray $d\,e$ fall thereon parallel to the Axis $k\,c$. This Ray, after it has ſuffered a

double Refraction, ſhall proceed in ih, as if it came directly from m, the Centre of the Cavity.

Let ck be made three times cm, draw $keil$. By the firſt Refraction the Ray is propagated into ei, as if it proceeded from k (by *Prop.* X.). From the Centre g draw the Perpendicular gif. The Angle of Inclination of the Ray within the Glaſs is $eif=lig$. The Ray muſt therefore at its Egreſs at i be refracted, ſo that the Angle of Refraction $hil=kim$ muſt be half the Inclination lig. Wherefore in the Triangle kig, As the Sine of the Angle kig, or of the Complement to 180° *viz.* lig : To the Sine of the Angle at g : : So $kg=4$: To ik or $nk=3$. And as the Sines, ſo are the Angles, ſeeing they are ſuppoſed very acute: Therefore, As 4 : To 3 : : So $\angle lig$: To the $\angle g$: Add $hil=2$ to $lig=4$, we have $hig=6$. Wherefore $\angle hig$: Is to the Angle g : : As 6 : To 3 : : or 2 : To 1. But the Sine of the Angle hig is equal to the Sine of the Angle gim; and conſequently in the Triangle gim, As Sine of the Angle gim : To the Sine of the Angle g :: So gm : To im. Therefore gm : Is to im : : As 2 : To 1. So that neglecting the Thickneſs of the Glaſs, mi may be taken for the Radius of the Cavity, and gm is double thereto. Wherefore 'tis Demonſtrated, that m is the Virtual Focus. *Which was to be Demonſtrated.*

Prop. XIII.

In double Concaves of equal or unequal Concavities, the Virtual Focus or Point of Divergence of the Parallel Rays is determined by this Rule:

As the Sum of the Radii of both Concavities :
Is to the Radius of either Concavity : :
So the Double Radius of t'other Concavity :
To the Diſtance of the Virtual Focus.

Tab.

Tab. 16. *Fig.* 3. fc is the Radius of the Concavity ec, make kc thrice fc, lo is the Radius of the Cavity oi; let lm be put equal to lo, and mn be put $=lm$, that is, on ſhall then be equal to thrice ol. T.16. F.3.

I ſay the Ray de, after a double Refraction, ſhall proceed in ih, as if it came directly from g, and that it ſhall be, As fl the Aggregate of the Semidiameters (neglecting the Thickneſs of the Glaſs) : To fc the Radius of one Concavity :: So om the Diameter of t'other Concavity : To gc the Diſtance of the Point of Divergence.

By the firſt Refraction the Ray proceeds in ei, as if it came directly from k, $keip$ being a Right Line (*Prop.* X.).

Then the Inclination within the Glaſs is $qie=pil$. And the ſecond Angle of Refraction is $pih=gik$, which is to be half the Angle of Inclination pil.

Wherefore Sines and Angles being proportional, and the Thickneſs of the Glaſs neglected, we thus proceed in the Demonſtration.

In the Triang. gik we have it — — — — —	1	$gik=hip : gki :: kg : gi=gc$
Double the Antecedent on this ſide, and halve the Conſequent on t'other —	2	$2gik=pil : ikl :: kg : \frac{1}{2}gc$
Dividing the Second —	3	$pil-ikl$ ($=ilk$ 32. 1 *Eucl.*) $: ikl :: gk-\frac{1}{2}gc : \frac{1}{2}gc$
Then in the Trian. kli-	4	$ilk : ikl :: ki=kc : il=lo$
From the Third and Fourth it follows — — —	5	$kc : ol :: kg-\frac{1}{2}gc : \frac{1}{2}gc$
Compounding the Fifth	6	$kc+ol=kl : ol :: kg : \frac{1}{2}gc$
Double the Conſequents	7	$kl : 2ol=om=ln :: kg : gc$
Compound. the Seventh	8	$kl+ln=kn : om :: kg+gc=kc : gc$
Alternate the Eighth —	9	$kn : kc :: om : gc$
But kn is equal to $3fl$,		

that

that is, thrice the Aggregate of the Semidiameters fc and lo. Alſo kc is equal to thrice the Semidiameter fc: and as the Triples are, ſo are their Thirds. Wherefore from the 9th. Step it follows — | 10 $fl : fc :: om : gc$.

Which was to be Demonſt.

Example.

Let fc be $=$ 8 Feet or 96 Inches
$lo =$ 10 Feet, or 120 Inches.
Then fl ſhall be $=$ to 216 Inches.
Say then $fl = 216 : fc = 96 :: 240 = 2\,lo : 107 = gc$.
Or thus, $fl = 216 : lo = 120 :: 192 = 2\,fc : 107 = gc$.

Corollary.

It follows from the four laſt Propoſitions, that ſuppoſing a Ray falling on a Concave Glaſs, and tending towards the virtual Focus, after it has paſſed the Glaſs, it ſhall proceed Parallel to the Axis. Becauſe the Progreſs of Light through Glaſſes is reciprocal. *Vid. Prop.* VI.

Scholium.

It is before Demonſtrated in *Prop.* IV. that the Focus or Point of Convergence of a Convex-Glaſs, for the Parallel Rays that fall *obliquely* thereon is at the ſame Diſtance, as the Focus of the *direct* Parallel Rays. The ſame may be Demonſtrated concerning the Virtual Focus or Point of Divergence in Concave Glaſſes; but tis ſufficient only to illuſtrate this Matter by *Tab.* 16. *Fig.* 4. Let us imagine the uppermoſt Point of a diſtant Object

Tab 16. *Fig.* 4.

Object to send the Parallel Rays *g a*, *g b*, *g c*, on the Concave-Glass *a b c*, and its middle Point to send the Rays *h a*, *h b*, *h c*; and the lowermost Point to send the Rays *i a*, *i b*, *i c*. Here are therefore three Radious Cones, having their Vertices in their respective Points of the Object, and their Bases on the Surface of the Concave-Glass: the Axes of these Cones, are *g d b k*, *h e b l*, *i f b m*: These, by what foregoes preliminary to the *Prop.* IV. may be conceived to pass the Glass as it were unrefracted. But the other Rays of each Cone are refracted, that is, *g a* is so refracted at *a* into *a k*, as if it proceeded directly from *d* the Virtual Focus or Point of Divergence for this oblique Cone. And so *g c* is refracted at *c* into *c k*, as if it proceeded directly from *d*. And in like manner *h a*, *h b*, *h c*, are refracted into *a l*, *b l*, *c l*, as if they came from *e*. And the Rays *i a*, *i b*, *i c*, are propagated in *a m*, *b m*, *c m*, as if they proceeded directly from *f*. So that *d e f* is the Virtual Focus of the Glass exposed to the Object, that sends these direct and oblique Cones of Rays.

Converging Rays.

The Properties of Concaves exposed to *Converging* Rays are shewn in these two next following Propositions.

Prop. XIV.

In Concave Glasses, if the Point to which the incident Ray converges, be distant from the Glass farther than the Virtual Focus of Parallel Rays, the Rule for finding the Virtual Focus of this Ray, is this:

As the difference between the distance of this Point from the Glass, and the distance of the Virtual Focus from the Glass:
Is to the distance of the Virtual Focus ::
So the distance of this Point of Convergence from the Glass:
To the distance of the Virtual Focus of this Converging Ray.

Tab.

T.17.F.1. *Tab.* 17. *Fig.* 1. *a b* is a Plano Concave Glaſs, *d* the Centre of the Concavity, *d a o* a right Line, *c f* the length of the Virtual Focus, which (by what foregoes, *Prop.* XI.) is equal to 2 *a d* or 2 *d c*; ſo that *f d* is equal to 3 *d c* or 3 *a d*. Let *n a* be a Ray of Light converging directly towards the Point *g*. If it be made according to the Rule in the Propoſition, *g f*: *f c* :: *g c* : *c h*. Then *h* ſhall be the Virtual Focus of the Ray *n a*, that is the Ray *n a* after the double Refraction it receives by the Glaſs ſhall proceed onwards in *a m*, as if it came directly from *h*, *m a h* being a Right Line.

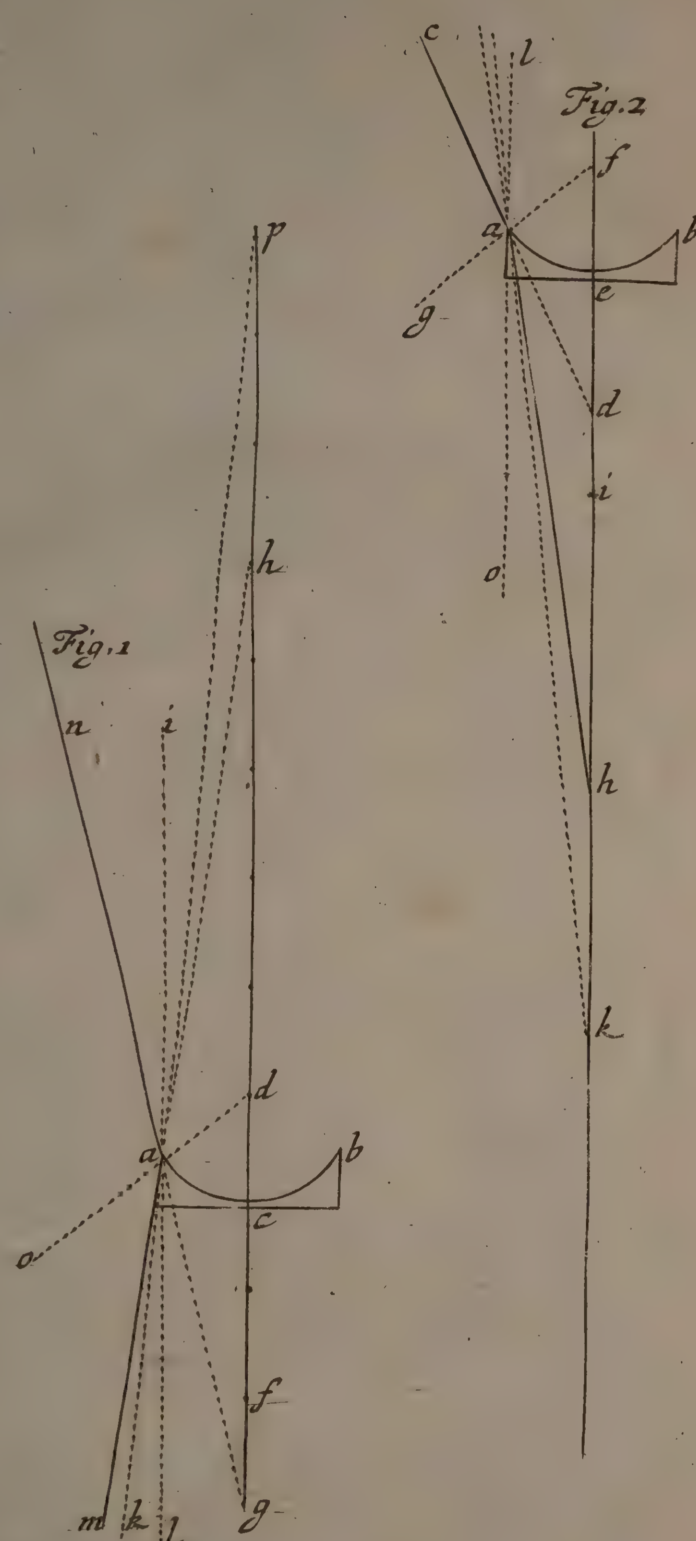
The Proof hereof is after the ſame manner with the *Prop.* V. beforegoing; and we ſuppoſe (as in that) the Thickneſs of the Glaſs to be nothing in Compariſon to the Focal Length; and likewiſe that the Angles of Incidence are ſo ſmall, that they are as the Sines; alſo that the Breadth of the Glaſs is ſo little, that it makes the Angles *a g c*, *a d c*, *a h c*, *a p c*, &c. very ſmall and Proportionable to their Sines.

Demonſtration.

Let *p h* be made half *h c*, and conſequently the third Part of *p c*. I ſay by virtue of the firſt Refraction which the Ray ſuffers at its entrance into Glaſs at *a*, 'tis refracted into *a k*, as if it proceeded directly from *p*, *p a k* being a Right Line.

To prove this we muſt obſerve that the Angle of Inclination or Incidence is $nad = oag$. If therefore it be proved, that the Angle of Refraction $gak = nap$ is $\frac{1}{3}$ of the Angle *n a d*, our firſt Poſition will be manifeſt, *viz.* That the Ray by the firſt Refraction is bent into *a k*.

First



Tab.17.pag 64

First therefore it is by Position	1	$gf : fc :: cg : ch$
Add to the Consequents their Halfs — — — — — — — —	2	$gf : fd :: cg : cp = pa$
Compounding the second —	3	$gf + fd = gd : fd :: cg + cp = gp : pa$
Also in the Triangle gad —	4	$gd ; da = dc :: s\angle dag = s\angle nad : s\angle g$
And Sines and Angles being proportional, it follows from the 4th. — — — — — — — —	5	$\frac{1}{3} nad : \angle g :: gd : 3\,ad = fd$
It then follows from the 5th. and 3d. — — — — — — — —	6	$\frac{1}{3} nad : \angle g :: gp : pa$
Moreover in the Triangle gap	7	$gp : pa :: s\angle gap = s\angle nap : s\angle g$
And Sines and Angles being proportional — — — — — —	8	$gp : pa :: \angle nap : \angle g.$
From the 6th and 8th it follows	9	$\frac{1}{3} nad : \angle g :: \angle nap : \angle g.$

From which 9th Step 'tis manifest that the first Angle of Refraction, nap, is equal to $\frac{1}{3}$ of the first Angle of Incidence nad. Which was first required to be Demonstrated.

We are secondly to Demonstrate, that by virtue of the second Refraction the Ray receives on the plane side of the Glass at its Egress into Air, 'tis refracted into ma, as if it came directly from h. Draw ial Parallel to the Axis hg, and Perpendicular to the plane side of the Glass; The Angle of Incidence is $lak = iap$; the Angle of Refraction is $mak = hap$; the refracted Angle $mal = hai$.

We are therefore to prove, that the refracted Angle mal is equal to triple the Angle of Refraction mak.

In the Triangle pah we have this Analogy $pa = pc : ph :: s\angle ahp = s\angle ahc = s\angle hai = s\angle mal : s\angle hap = s\angle mak$. But pc is thrice ph, therefore (Sines and Angles being proporti-

onal) the Angle $m\,a\,l$ is thrice the Angle $m\,a\,k$. *Which was to be Demonstrated.*

PROP. XV.

Tab. 17. Fig. 2. *In Concave Glasses* a b (T. 17. F. 2.) *if the Point* d *to which the Incident Ray* c a *converges, be nigher to the Glass than the Virtual Focus of Parallel Rays* i; *the Rule to find where it crosses the Axis at* h, *is this,*

As the Excess of the Virtual Focus more than this Point of Convergency i d:
To the Virtual Focus i e::
So the distance of this Point of Convergency from the Glass d e:
To the distance of the Point where this Ray crosses the Axis h e.

In the forementioned Figure $a\,b$ is a Plano-Concave Glass, f the Center of the Cavity $i\,e$ the Length of the Virtual Focus of Parallel Rays, which we know is a Diameter of the Cavity, that is, $=2fe$. On this Glass there falls the Ray $c\,a$ converging directly to the Point d. If it be made $i\,d : i\,e :: d\,e : h\,e$, I say h is the Point in the Axis, where the Ray $c\,a$, after its double Refraction, crosses it.

Demonstration.

For the Proof thereof we suppose all that we laid down as supposed in the last Proposition. Draw $f\,a$ directly to g, and make $h\,k$ equal to $\frac{1}{2}$ $h\,e$. I shall first shew, that by virtue of the first Refraction, which the Ray suffers at a, 'tis refracted directly towards k. Here the first Angle of Incidence is $c\,a\,f$, $=g\,a\,d$. If therefore we prove that $k\,a\,d$ is equal to $\frac{1}{3}$ $c\,a\,f$, our first Position shall be manifest, that the Ray is refracted first towards k.

For

For by the Rule it is — — —	1	$id : ie :: ed : eh$
Add to the Consequents their Halfs — — — — — — — — — —	2	$id : if :: ed : ek$
Inverting and dividing the Second	3	$if - id = df : if :: ek - ed = dk : ek$
Moreover in the Triang. dfa	4	$df : af :: s\angle fad = s\angle caf : s\angle adf$
From the 4th Step it follows	5	$\frac{1}{3}\angle caf : \angle adf :: df : 3af = if$
By the 3d and 4th it is — —	6	$\frac{1}{3} caf : adf :: dk : ek$
Also in the Triangle akd —	7	$kd : ka = ke :: s\angle kad : s\angle kda = s\angle adf$
Sines and Angles being prop.	8	$kd : ke :: \angle kad : \angle adf.$
From the 6th and 8th it follows	9	$\frac{1}{3} caf : adf :: kad : adf$

Wherefore from this 9th Step 'tis manifest that kad is equal to $\frac{1}{3}\ caf$, which was first to be Demonstrated.

I shall next demonstrate, that by virtue of the second Refraction the Ray proceeds in ah.

Draw oal Parallel to the Axis fk, and therefore (by Supposition 2d) perpendicular to the plane side of the Glass. Here the Angle of Incidence from Glass to Air shall be oak. I am therefore to prove that the Angle of Refraction kah is equal to $\frac{1}{3}\ oah$ the refracted Angle. Thus,

In the Triangle kah it is $ka = ke : kh :: s\angle ahk = s\angle ahe = s\angle oah : s\angle kah$

And Sines and Angl. being prop. $ke : kh :: oah : kah$

But kh is $\frac{1}{3}$ of ke and therefore the Angle kah, is $\frac{1}{3}$ of the Angle oah. *Which was to be Demonstrated.*

DIVERGING RAYS.

Corollary.

Concerning *Concave-Glasses* exposed to *Diverging Rays.*

The Virtual Focus of a Concave Glass exposed to *Parallel* Rays is found by *Prop.* X, XI, XII, XIII. And the Properties of a Concave exposed to *Converging* Rays are shewn in *Prop.* XIV. and XV.

Let us now consider a Ray *Diverging* from the Axis, as
T.17. F.2. suppose a nigh Object (*T.* 17. *F.* 2.) in *h*, whose Ray *h a* diverges from the Axis *h e*, and so falls on the Concave Glass *a b*. Let us conceive this nigh Object *h* either farther from, or nigher to the Glass, than the Glasses Virtual Focus. This Ray after passing the Glass diverges more, than before it entred it. And the Rule for determining the Point *d*, from which this Ray after passing the Glass does as it were directly *Diverge*, is this,

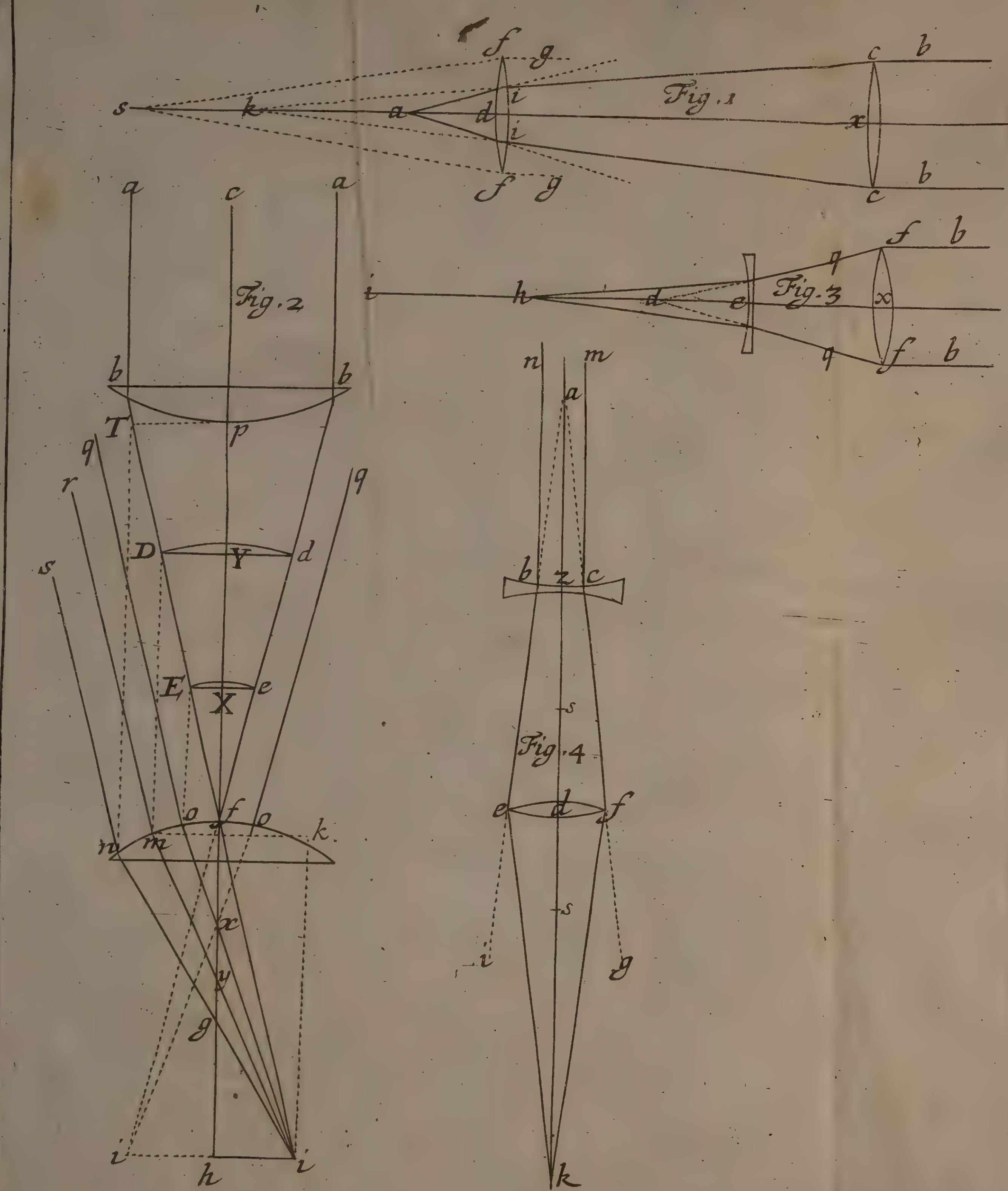
As the Sum of the distance of the Object from the Glass and the Glass's Virtual Focus :
To the Distance of the Object from the Glass ::
So the Glasses Virtual Focus :
To the distance of this Point of Divergency from the Glass :

Demonstration.

By the foregoing XVth *Prop.* — $id : ie :: de : eh$
And Alternating — — — — — $ie : eh :: id\ (= ie - de) : de$
And Compounding — — — — $ie + eh : eh :: ie : de$

Which last Analogy gives the forementioned Rule in Words as is manifest by comparing it with the Scheme.

PROP.



Tab.18.pag 69

Prop. XVI. Probl.

The Focal Lengths of two Convex-Glaſſes being given, the longer being placed before the ſhorter at any given Diſtance leſs than the longer Focal Length; 'Tis required to determin the Diſtance of the Diſtinct Baſe from the ſhorter Glaſs.

This is ſolved from the VIII. *Propoſition* hereof.

Tab. 18. *Fig.* 1. $c\,x\,c$ is a Convex-Glaſs uniting the Parallel Rays $b\,c$, $b\,c$, in its Focus at k; $f\,d\,f$ is a ſhorter Convex uniting the Parallel Rays $g\,f$, $g\,f$, in its Focus at s. In the Problem, $k\,x$ the longer Focal Length, and $s\,d$ the ſhorter Focal Length, and $d\,x$ the Diſtance of the Glaſſes, are given; To find $d\,a$, the Point wherein the Rays $c\,i$, $c\,i$, are united with the Axis by the Glaſs $f\,f$. T. 18, F. 1

By the 8th. *Propoſition* preceding, the Focal Length $d\,s$ of the Convex Glaſs $f\,f$ being given, and the Diſtance of an Object $d\,a$ being given leſs than the Focal Length $d\,s$, to find the Diſtance $d\,k$ of the *Imaginary* Focus; the Rule is, As the Difference between the Diſtance of the Object $d\,a$, and the Focal Length $d\,s$, that is, $d\,s - d\,a = a\,s$: To the Focal Length $d\,s$ :: So the Diſtance of the Object from the Glaſs $a\,d$: To the Diſtance of the Imaginary Focus from the Glaſs $d\,k$.

Now whereas in the 8th. *Propoſition* $d\,a$ is given, and $d\,k$ ſought; in this Problem $d\,a$ is ſought, and $d\,k$ is given; for $d\,k$ is equal to the Difference between the Focal Length of the longer Glaſs $k\,x$, and the Glaſs's Diſtance $x\,d$.

Wherefore by the foreſaid 8th. *Propoſition*, and its Corollary, it being $d\,s - d\,a = a\,s : d\,s :: a\,d : d\,k$. It ſhall be alſo *alternando* $d\,s : d\,k :: d\,s - d\,a : a\,d$. And Compounding $d\,s + d\,k : d\,k :: d\,s : a\,d$. But in this Problem the Three firſt Terms of this laſt Analogy are given, and the laſt $a\,d$ required.

Where-

Wherefore by this Analogy, the laſt Term $a\,d$ is found. And according to this Analogy we frame this Rule for ſolving this Problem.

As the Focal Length of the ſhorter Glaſs + the Difference between the Focal Length of the longer, and the Glaſſes Diſtance :
To the Difference between the Focal Length of the longer and the Glaſſes Diſtance ::
So the Focal Length of the ſhorter Glaſs :
To the Diſtance of the Diſtinct Baſe from the Glaſs.

Example.

Let the longer Focal Length be $x\,k = 20$, the ſhorter Focal Length $d\,s = 8$. The Diſtance of the Glaſſes $d\,x = 15$.

Then $x\,k - d\,x\ (= d\,k) + d\,s : d\,k :: d\,s : a\,d$

In Numb. $20 - 15\ (= 5) + 8 = 13 : 5 :: 8 : 3{,}07692$.

Schol. 1. The ſame Solution holds, if the ſhorter Glaſs be placed before the longer, and the longer Glaſs at any Diſtance from the ſhorter leſs than the Focal Length of the ſhorter.

Let the ſhorter Focal Length be $= 8$, the longer $= 20$, and the Glaſſes Diſtance $= 6$; then $8 - 6 = 2$; and the Proportion is $8 - 6 + 20 = 22 : 2 :: 20 : 1{,}8182 =$ to the Diſtance of the *Compound* Diſtinct Baſe (for ſo I'll call it) of theſe two Glaſſes from the laſt and longer Glaſs.

Corollary.

From hence is manifeſt the Rule which is before laid down after our 3d *Propoſition* for Determining the Focus of a double Convex-Glaſs. The Rule is this, *As the Sum of the Radii of both Convexities : To the Radius of either Convexity :: So the double Radius of t'other Convexity : To the Diſtance of the Focus.*

For

For Example, Let the Radius of one Convexity be 20, and of t'other 8. Then 28 : 8 :: 40 : 11,4286. Or, 28 : 20 :: 16 : 11,4286 = Focus. Let us then conceive this double Convex-Glaſs to be divided into two Plano-Convex Glaſſes, formed on the ſame Spheres as the two Sides of the double Convex, *viz.* One on a Sphere whoſe Radius is 20; and t'other on a Sphere whoſe Radius is 8; and theſe two Glaſſes placed touching each other on their plain Sides, the larger Focal Length would be 40, and the ſhorter Focal Length would be 16, and the Diſtance of the Glaſſes would be *Nothing*. Then by the Rule of the XVI. *Prop.* 16+40=56 : 40 :: 16 : 11,4286 = To the Compound Focus.

Scholium 2.

By this Propoſition, and by the Corollaries of *Prop.* XXVI. hereafter, the Eight firſt Propoſitions of *Dechales Second Book of Dioptricks* are manifeſtly ſolved.

Another Solution of the ſame Problem.

My eſteemed Friend, the Learned and Ingenious Mr. JOHN FLAMSTEED, *Reg. Aſtron.* has very neatly ſolved this Problem, and communicated the Solution thereof to me, as follows.

Tab. 18. *Fig.* 2. Let *b p b* repreſent a Convex-Glaſs, whoſe Focus is at *f*; let *a b* be two Rays falling upon it parallel to the Axis *c f*, which ſhall therefore be collected in *f*; let *n f k* be another Convex-Glaſs of any different Sphere, whoſe Focus is *h*, its Diſtinct Baſe *i h i*. 'Tis required to find, at what diſtance from the Point *f*, the Diſtinct Baſe ſhall be formed, the Glaſs *n f k* being placed any where nearer the Glaſs *b b*, as ſuppoſe in X, Y, or *p*. T.18. F.2.

Through

Through the Points X, Y, p, draw the Lines E X e, D Y d, T p, at Right Angles to the Axis. And from the Points E, D, T, wherein they cut the refracted Ray $b\,f$, draw the Lines E o, D m, T n, parallel to the Axis. And from the Points o, m, n, where they fall on the Glaſs $n\,f\,k$, draw the Lines $o\,q$, $m\,r$, $n\,s$, parallel to the refracted Ray $b\,f$; theſe repreſent this Ray $f\,b$ falling upon the Glaſs $n\,f\,k$ placed at X, Y, p.

Produce $b\,f$ through the Glaſs to the Diſtinct Baſe, which it cuts in i, we may now conſider $q\,o$, $r\,m$, $s\,n$, as three parallel Rays proceeding from the ſame Point of an Object infinitely diſtant, which by the known Properties of Glaſſes, ſhall be collected into the ſame Point i of the Diſtinct Baſe with the Ray $b\,f$. Draw therefore the Lines $o\,i$, $m\,i$, $n\,i$, interſecting the Axis in the Points x, y, g. Now, becauſe the Focus is that Point, where the Ray falling upon the outward Glaſs is by the ſecond Glaſs collected or made to converge with the Axis, the Points x, y, g, ſhall be the Foci ſought; that is, $f\,x$ ſhall be the Compound Focal Length for the Glaſs $n\,f\,k$ placed in X, $f\,y$ when 'tis placed in Y; and $f\,g$ when 'tis placed in p, or when both the Glaſſes touch.

Now that we may find theſe Focal Lengths $f\,x$, $f\,y$, $f\,g$, draw $m\,k$ parallel to the Diſtinct Baſe $i\,h\,i$, and $i\,k$ parallel to the Axis $f\,h$. We ſuppoſe the Thickneſs of the Glaſſes to be as little as poſſible, and therefore the Lines repreſenting the Rays, paſſing *through* them, are *ſtrait*, and not at all bent.

Wherefore the Triangles D Y f, $h\,i\,f$ are ſimilar. Alſo the Triangles $m\,k\,i$, $m\,f\,y$ are ſimilar.

It ſhall therefore be — — —	1	$f\,Y : f\,h :: Y\,D\;(=m\,f) : h\,i$
Converting and Compound.	2	$f\,Y + f\,h : f\,Y :: m\,f + h\,i : m\,f$
But by the Scheme — — —	3	$m\,f + h\,i = m\,k$
Therefore the 2d is thus —	4	$f\,Y + f\,h : f\,Y :: m\,k : m\,f$
And by the ſim. $\triangle^s$ *mki*, *mfy* --	5	$m\,k : m\,f :: k\,i : f\,y$
From the 4th and 5th it foll.	6	$f\,Y + f\,h : f\,Y :: k\,i : f\,y$

Now

Now by Poſition $bf=pf$ is the longer Focal Length, and $fh=ki$ is the ſhorter Focal Length, and Yp the Diſtance of the Glaſſes; and therefore $fY=pf-pY$ is the longer Focus, — the Diſtance of the Glaſſes; and fy is the Diſtance of the Diſtinct Baſe from the inward Glaſs. From all which, the foregoing 6th Analogy, being reſolved into Words, gives his Rule for ſolving the Problem,

As the longer Focus — the Glaſſes Diſtance + the ſhorter Focus :
To the longer Focus — the Glaſſes Diſtance ::
So the ſhorter Focus :
To the Diſtance of the Diſtinct Baſe from the inward Glaſs.

Or thus,

As the Aggregate of the ſhorter Focus and the Difference between the longer and the Glaſſes Diſtance :
To the Difference between the longer Focus and Glaſſes Diſtance ::
So the ſhorter Focal Length :
To the Diſtance of the Diſtinct Baſe from the inward Glaſs.

Which is the very ſame Canon with that I have given, deduced from the 8th Propoſition hereof.

This Problem is of conſiderable Uſe in Dioptricks, being the Foundation of an excellent ſort of Teleſcope much uſed in *England* for the Night. But it ſeems no Optick Writer has ſolved it. *Honoratus Faber* has clearly omitted it. And *Dechales*, though he offers at our foreſaid VIII. Propoſition, on which our Solution of this Problem is founded; yet miſſing ſo enormouſly as he has done in that, he could not pretend to the Solution of this. And had he rightly ſolved our VIII. Propſition, I queſtion whether he would have puſhed it on ſo far as to ſolve thereby this Problem; for in the two next ſucceeding Problems, he has rightly the Propoſitions on which their Solutions depend, but leaves the Problems themſelves untouch'd.

PROP. XVII. PROBL.

A Convex-Glaſs being given, with a Concave of a larger Sphere, the Concave being placed behind the Convex, at any Diſtance leſs than the Focal Length of the Convex; 'Tis required to find the Place of the Compound Focus, or Diſtinct Baſe of theſe two Glaſſes.

T.18. F.3. This Problem is ſolved by the XV. *Propoſition* hereof. *Tab.* 18. *F.* 3. The Convex-Glaſs *x* is placed ſo before the Concave-Glaſs *e*, that this Convex would unite the Parallel Rays *b f*, *b f*, in its Focus at *d*, and the Rays *q d*, *q d*, would croſs the Axis in *d*, unleſs the Concave *e* were interpoſed. Which Concave is ſuppoſed to have its Virtual Focus at *i* more diſtant than *d* the Diſtance of the Focus of the Convex *x*.

Wherefore here is a Ray *q* incident on the Concave *e*, and converges towards a Point *d*, nigher the Glaſs *e* than the Virtual Focus of parallel Rays *i*; and 'tis required to determine the Point *h*, where it croſſes the Axis. The Rule by the XV. *Propoſition* is this,

As the Exceſs of the Focus of the Concave more than this Point of Convergency i d:
To the Focus of the Concave i e ::
So the Diſtance of this Point of Convergency from the Glaſs d e :
To the Diſtance of the Point where this Ray croſſes the Axis h e :

Now in our preſent Problem, the three firſt Terms of this Analogy are given, and the laſt required. For the Problem gives *d x* the Focal Diſtance of the Convex, and *i e* the Focus of the Concave, and *e x* the Diſtance of the Glaſſes. Hence we have *d e* $(=d x - e x)$ the Diſtance from the Concave of the

the Point d, to which the Ray converges. And we have id $(=ie-de)$ the Excess of the Virtual Focus of the Concave beyond the Point of Convergency d.

Wherefore by the foresaid Analogy it being, As $id:ie::de:he$. The Rule for Solving this Problem is this,

From the Focal Length of the Convex subtract the Glasses distance. Mark the difference, then say,

As the Focal Length of the Concave — this difference :
To the Focal Length of the Concave ::
So this difference :
To the distance of the Distinct Base from the Concave.

1. *Example.*

Focus Convex $= dx = 20$
Focus Concave $= ie = 60$
Glasses Distance $= ex = 10$
Hence — — — $= de = 10$ Diff.
And — — — — $= id = 50$
Then $id = 50 : ie = 60 :: de = 10 : he = 12$.

2. *Example.*

Focus Conv. $= 12$
Focus Conc. $= 15$
Glasses Dist. $= 2$
Hence $de = 10$ Difference
And $id = 5$
Then $5 : 15 :: 10 : 30 = he$.

Prop. XVIII. Probl.

The Focal Lengths of a Convex-Glass and a Concave-Glass being given, and the Concave placed towards the Object, distant from the Convex, so that the Sum of the Concaves Focal Length and Glasses distance may be greater than the Focal Length of the Convex, and this distance of the Glasses being given also; 'Tis required to determin the distance of the Distinct Base from the Convex.

This is performed by the V. *Proposition* hereof. *Tab.* 18. T.18.F.4. *Fig.* 4. Let *bzc* be a Concave, on which there fall the parallel Rays *nb*, *mc*, which are ſo refracted by this Glaſs, that after paſſing it, they proceed on in *be*, *cf*, as if they came directly from the Point *a* the Virtual Focus of the Concave, (by *Prop.* X. XI. XII. hereof) but in *e* and *f* meeting with the Convex-Glaſs, whoſe Focus is on each ſide it at *s*, *s*, the Rays diverge not to *i* and *g*, but are refracted, ſo that they croſs the Axis in *k*.

We may therefore conceive theſe Rays, not as paſſing through the Concave *bc*, but as proceeding directly from the nigh Point *a*; and then to determine the Point *k*, where they croſs the Axis, the Rule is in the *Prop.* V. thus,

As s a *the difference between the Focus* s d *of the Convex-Glaſs* e f, *and the diſtance* d a *of the Object from the Glaſs* :
To the Focus s d *of the Convex-Glaſs* : :
So the diſtance d a *of the Object from the Glaſs* :
To the diſtance of the reſpective Focus d k.

Now in this Problem, the three firſt Terms of this Analogy are given, and the fourth *dk* is required. For the Sum of the Concaves Focal Length and the diſtance of the Glaſſes, that is, $za + dz$ is equal to *da*; and *da*—the Convex's Focal Length that is $da - sd$ is equal to *sa*.

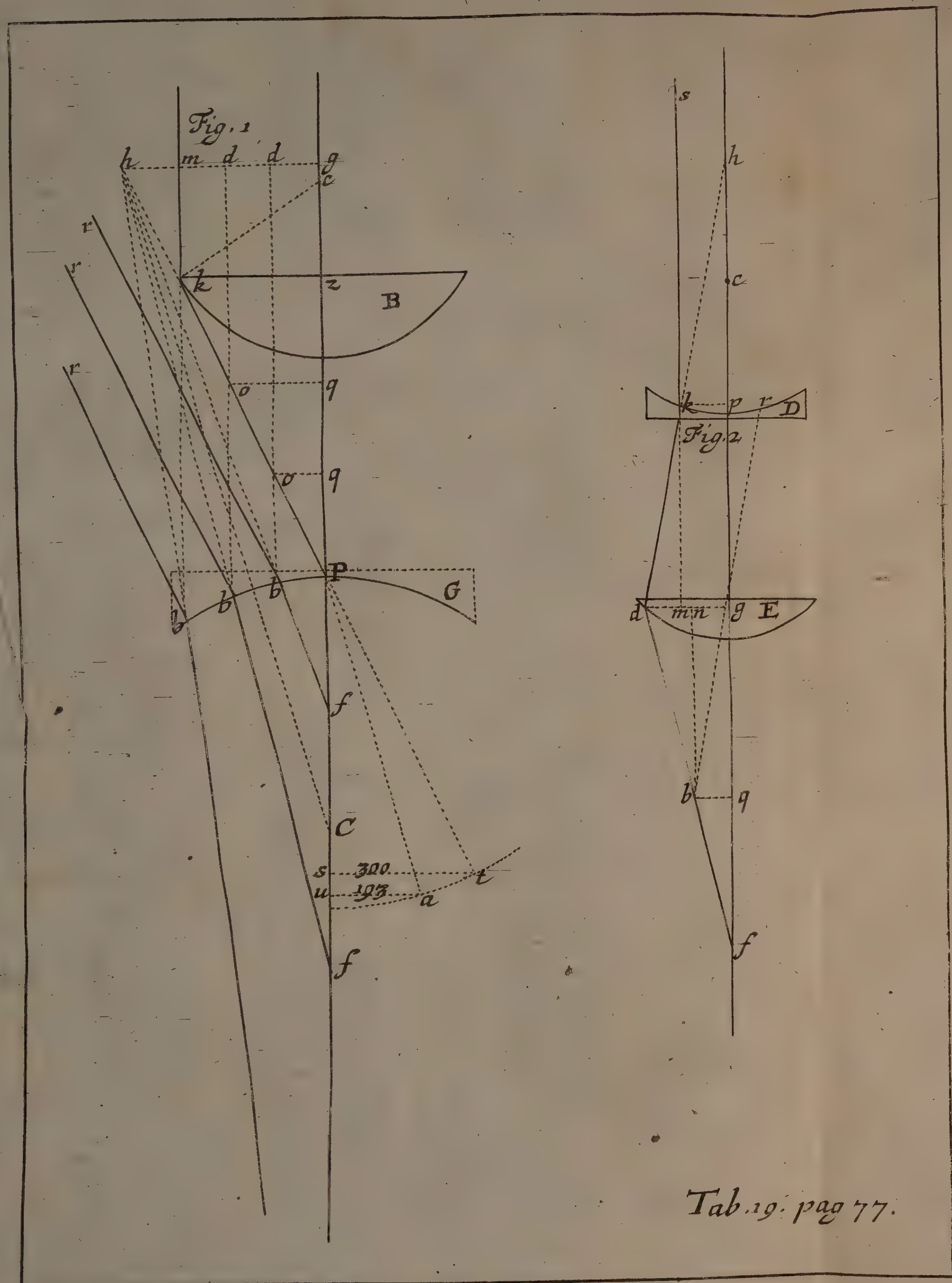
Wherefore we ſolve the Problem by this Canon.

From the Sum of the Focus of the Concave and diſtance of the Glaſſes ſubtract the Focus *of the Convex, that is,* $za + dz - sd =$ sa. *Keep this difference, then ſay,*

As this difference s a :
To the Focus of the Convex s d : :
So the Sum of the Concave's Focus and Glaſſes diſtance $za + dz = ad$:
To the diſtance of the Focus from the Convex d k.

Which is required by the Problem, and is the very Analogy of our *Prop.* V.

An



Tab. 19. pag 77.

Another Solution of the Two laſt Problems in Prop. XVII. *and* XVIII.

Theſe Two laſt Problems were likewiſe communicated to me, moſt ingeniouſly ſolved by my forementioned Honoured Friend Mr. J. Flamsteed. The Solutions I ſhall give in his own Words at large, as I have them in a Letter to me Dated from the *Greenwich-Obſervatory*, *Jan.* 17. 168$\frac{6}{7}$. as follows.

The Firſt Problem is this, *A Convex-Glaſs being given, with a Concave of a larger Sphere; the Concave being placed at any diſtance leſs than the Focal Length of the Convex from it, and remoter from the Object; To find the Place of the Focus or Diſtinct Baſe.*

The Second Problem is, *From the ſame Data; the Concave being placed nigher the Object; To find the Diſtinct Baſe.*

For the Firſt Problem, *Tab.* 19. *Fig.* 1. B is a Convex-Glaſs, T.19. F.1. P its Focus, *m k* a Ray of Light falling upon it parallel to its Axis, which will therefore converge into its Focus at P. G is a Concave-Glaſs of a larger Sphere, placed upon the Focus of the Convex; C P the Radius of the Sphere whereon the Concave is formed. And we are always to conceive the Thickneſs of both theſe Glaſſes where the Rays of Light pierce them as ſmall as poſſible.

Draw the Line *k* P, this ſhews the way of the Ray *m k* after its Emerſion from the Convex B. Produce it both ways at pleaſure to *t* and *h*. Then conceive the Concave G placed any where betwixt P and B, as in *q*, *q*; From which Places draw the Lines *q o*, *q o*, at Right Angles to the Axis. And from the Points *o*, *o*, *k*, wherein they interſect the refracted Ray *k* P; draw the Lines *o b*, *o b*, *k b*, falling on the Concave G in *b*, *b*, *b*. From theſe Points again draw the Lines *b r*, *b r*, *b r*, parallel to the refracted Ray *k* P; theſe ſhall repreſent

present the Ray k P falling on the Concave G placed at q, q; and also when it touches the Convex B. Let these then be consider'd as so many parallel Rays, proceeding from the same distant Point. Wherever they shall intersect the Axis after they have passed the Concave, there shall be the true Focus for both Glasses, which I shall call the *Compound Focus.* To find which we are to remember;

That, as the Parallel Rays of Light falling on a Convex-Glass, are all collected very nearly in the same Point of the distinct Base; so also the Rays of Light falling Parallel to each other on a Concave, after they emerge from it, diverge and separate, as if they had all proceeded from the same Point, which I therefore call the *Point of Divergency* (*This, in my foregoing Propositions, I term with most Optick Writers, the* Virtual Focus) Which Point in a Concave-Glass is just as far distant from it, as the Point of *Convergency*, or Focus of a Convex of the same Sphere or Spheres is distant from it. We know the Focal Point is some little nigher the Glass in the Parallel Rays that fall *widest* from the Axis, than in those that fall near it; so is the Point of *Divergency* for Concaves. But the difference is so small in Glasses of a large Sphere (such as in these Problems we suppose) that for all Rays falling on the Glass Parallel to each other, it may be taken as a *simple Point*. Such therefore we are to conceive it; and then to find this Point of Divergency, The Rule is (*Corol. Prop.* XI.) $300-193=107 : 193 ::$ So CP the Radius of the Plano-Concave-Glass: To Pb: Which in Glasses of a large Sphere, and where the Inclination of Pb is small, will be insensibly different from Pg; bg being perpendicular to the Axis. Or we may determine this Point of Divergence Geometrically thus, On the Centre P with any Radius Pt, on the void side of the Figure, strike the Arch at; then make it $300 : 193 :: ts : au$. Draw Pa, and from C the Centre of the Concave draw Cb Parallel to aP. The Point b, wherein it cuts the Line Pk produced

duced upwards, ſhall be the Point of Divergency ſought.

Lay a Ruler over the Point h, and the Points b, b, b; this, where it cuts the Axis gp produced, as in f,f, ſhall give all the poſſible Compound Foci.

Then hg being perpendicular to the Axis, produce the Lines bo, till they cut it in d, d.

Then the Triangles hPg, oPq are ſimilar: likewiſe the Triangles hfg, bfP are ſimilar alſo.

It will then hold *ob ſimilia Triangula* hPg, oPq	1	$hP : oP :: hg : dg$
Dividing the Firſt — —	2	$hP - oP : oP :: hg - dg : dg$
That is, — — — — — —	3	$ho : oP :: hd : dg = oq$
Moreover by *Simil. Triang.* hfg, bfP — — — — — —	4	$hd : dg :: hb : bf$
From the 3d and 4th it follows — — — — — — —	5	$ho : oP :: hb : bf$.
And Compound the 5th---	6	$ho + oP : oP :: hb + bf : bf$
That is, — — — — —	7	$hP : oP :: hf : bf$
And Dividing the 7th —	8	$hP - oP : oP :: hf - bf : bf$
That is, — — — — —	9	$hP - oP : oP :: hb : bf$:
But $hP = gP = hb$, and ſo likewiſe $oP = qP$, (the Inclination of kP, as is ſaid before, being ſuppoſed very ſmall; and the Glaſſes of ſmall Segments of large Spheres.) Wherefore the 9th is thus — — — — —	10	$gP - qP : qP :: gP : bf$.

But by Poſition	Focus of the Concave	11	$= gP = hP$.
	Focus of the Convex	12	$= zP$.
	Diſtance of the Glaſſes	13	$= zq$
	Fo. Convex — Gl. diſt.		$= qP = zP - zq$.

Wherefore

Wherefore the 10th. Analogy, by the 11th, 12th, and 13th Steps, resolved into Words gives this Rule,

As the Focus of the Concave — Focus of the Convex + distance of the Glasses:
To the Focus of the Convex — distance of the Glasses::
So the Focus of the Concave:
To b f = *to the distance of the Distinct Base from the Concave.*

And this Canon of Mr. Flamsteed *is the very same with ours in the preceding* 17th *Proposition deduced from the* 15th *hereof.*

But if the Concave be placed next the Object: Which is his Problem II, and our XVIIIth Proposition, my forementioned Learned Friend solves it thus.

T.19. F.2. *Tab.*19. *F.*2. D is a Concave, E a Convex, hp the Focal Length of the Concave, $gp = gq$ the Focal Length of the Convex. Let sk be a Ray of Light falling into the Concave at k. From the Point of Divergency h, through k draw the Line kd; this shall shew the refracted Path of the Ray sk after it has passed the Glass D. From q the Focus of the Convex E erect qb perpendicular to the Axis hf; and through g draw the Line rgb Parallel to kd, intersecting the distinct Base of the Convex E in b, through which draw db, continuing it till it intersect the Axis in f: *There* shall be the compound Focus or distinct Base for these Glasses thus posited. Draw now the Line dg at Right Angles to the Axis; produce sk till it cut dg in m; and draw bn Parallel to qg.

Then the Triangles hdg, hkp, bgq, are similar. Also the Triangles bdn, fbq, are similar. And $hp + pg = hg$.

It shall therefore be — — —	1	$hg : gq :: dg : ng = bq$
And converting the First —	2	$hg : hg - gq :: dg : dg - ng$
Inverting the second — —	3	$hg - gq : hg :: dg - ng : dg$.
That is, by the Scheme —	4	$hg - gq : hg :: dn : dg$.

Then

Then by the ſimilar Triangles dnb, dgf — — — —	5	$dn : dg :: nb = gq : gf$
From 4 and 5 it follows —	6	$hg - gq : hg :: gq : gf$
Then hg being = to $hp +$ pg, the 6th is thus	7	$hp + pg - gq : hp + pg :: gq : gf.$
But by Poſition { Focus of the Concave —	8	$= hp$
Focus of the Convex —	9	$= gq$
Diſtance of the Glaſſes —	10	$= pg$
Focus of Concave + Glaſſes diſt. — — — — —	11	$= hg = hp + pg$
Diſt. of the diſtinct Baſe from Convex — — —	12	$= gf$

From all which the 7th Analogy may be thus expreſſed in words,

As the Focus of the Concave + the diſtance of the Glaſſes — the Focus of the Convex :
To the Focus of the Concave + the diſtance of the Glaſſes : :
So the Focus of the Convex :
To the Diſtance of the diſtinct Baſe from the Convex :

Which is the ſame Rule with that which I have before given in *Prop.* XVIII, deduced from the 5th hereof.

And theſe two laſt Propoſitions do naturally lead us to the Conſideration of *Meniſcus*-Glaſſes.

MENISCUS-GLASSES.

That is called a *Meniſcus-Glaſs*, which is Convex on one ſide and Concave on t'other. 'Tis ſo named from its Reſemblance to the New Moon or *Lunula*, in Greek μηνίσκος.

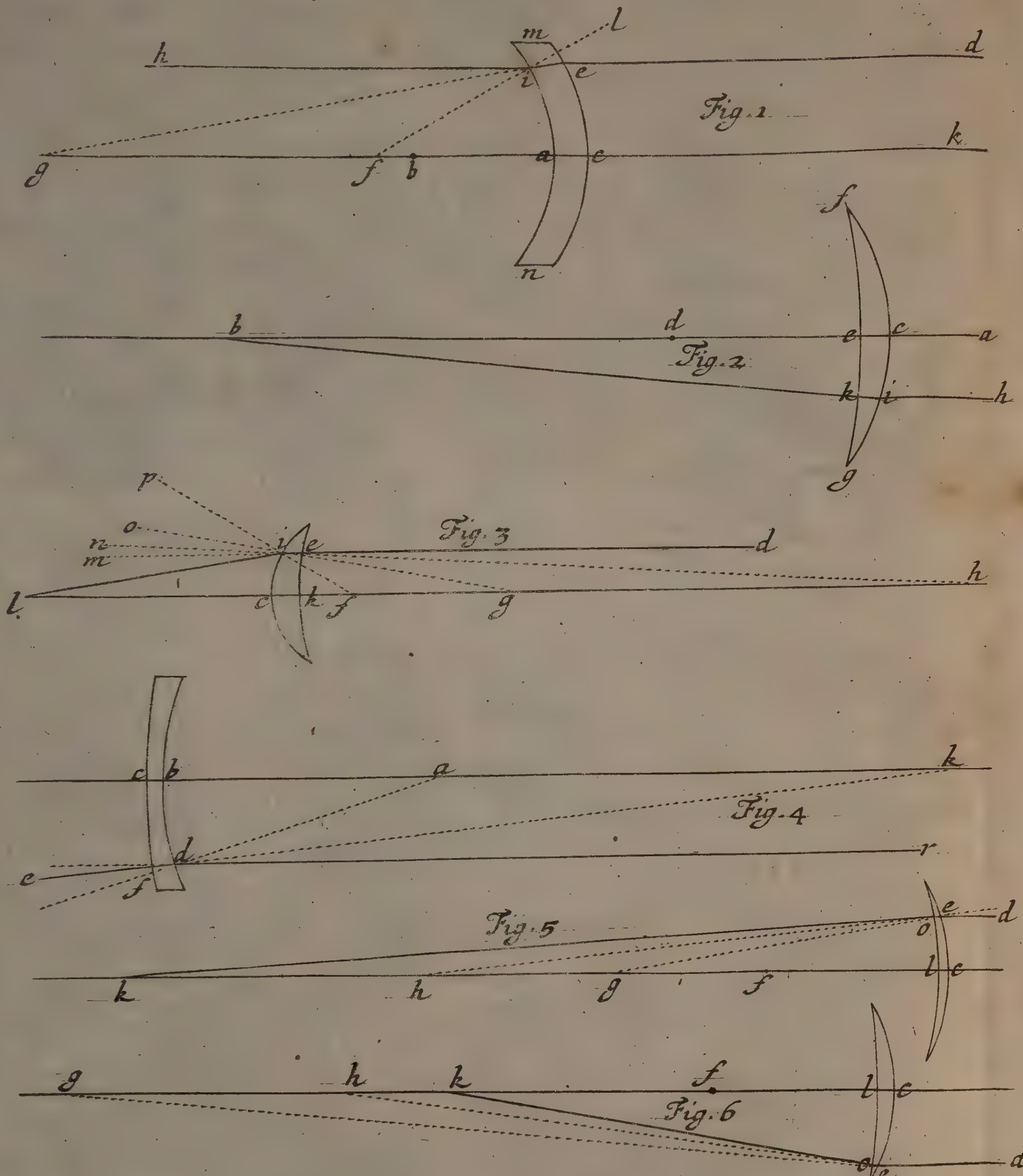
In *Meniſcus-Glaſſes* either the Semidiameter of the Convexity and Concavity are equal or unequal: If they are equal the Ray that falls thereon Parallel to the Axis, after Refraction pro-

ceeds again Parallel: If they are unequal, then either the Semidiameter of the Convexity is leſs than the Semidiameter of the Cavity, and then the Convexity prevails, and the Glaſs has a *Real Focus*: Or elſe the Semidiameter of the Cavity is leſs than the Semidiameter of the Convexity, and then the Cavity prevails, and the Glaſs has only a *Virtual Focus*. But of these more fully in the following Propoſitions; This *Obſervation* being premiſed, That the Focus of a *Meniſcus*, the Semidiameter of whoſe Convexity is leſs than the Semidiameter of the Concavity, and theſe two Semidiameters being given, is eaſily determined by *Prop.* XVII preceding. For in that Propoſition, two Glaſſes are given, one a Convex, and t'other a Concave, and the Focal Length of the Convex is ſhorter than that of the Concave. And whereas in that *Prop.* there is given a diſtance between the Glaſſes, let us conceive this Diſtance to be *nothing*, that is, let us imagine theſe Glaſſes to *touch*; and their Reſults a *Meniſcus* whoſe Focus is determined by the ſame Rule in that *Prop.* Which is this,

Obſerv.

As the Focal Length of the Concave — the Focal Length of the Convex + the Glaſſes Diſtance:
To the Focal Length of the Concave::
So the Focal Length of the Convex — the Glaſſes Diſtance:
To the Diſtance of the Diſtinct Baſe from the Concave.

But becauſe we ſuppoſe, in the Caſe of a *Meniſcus*, the two ſides of the Glaſs to touch, that is, the diſtance of the two Glaſſes to be *nothing* (for every *Meniſcus* may be conceived divided into two Glaſſes, a Plano-Convex, and a Plano-Concave) or the Thickneſs of the Glaſs to be inconſiderable in Compariſon of the Focal Length. The foreſaide Rule applyed to a *Meniſcus* ſhall be this,



Tab. 20. pag. 83.

As the Focal Length of the Concave — the Focal Length of the Convex:
To the Focal Length of the Concave ::
So the Focal Length of the Convex:
To the Diſtance of the Focus of the Meniſcus.

Now if we have given the Semidiameters of the Convexity and Concavity of a *Meniſcus*, we have given the Focal Lengths of the Plano-Convex and Plano-Concave of which this *Meniſcus* is compounded. For theſe Focal Lengths may be aſſumed the Doubles of the Semidiameters, wherefore Halfs being as their Wholes, the foreſaid Rule for finding the Focus of a *Meniſcus* may be thus expreſſed,

As the Difference of the Semidiameters of the Convexity and Concavity:
To the Semidiameter of the Concavity::
So the Diameter of the Convexity:
To the Focal Length.

Which is the Common Rule aſſigned by Optick Writers. But of this more fully hereafter *Prop.* XXIII. And tho this Rule in General were ſufficient for determining the Foci of *Meniſcus-Glaſſes*, yet I ſhall inlarge more fully thereon in the following Propoſitions.

Prop. XIX.

In a Meniſcus, if both ſpherical Superficies have the ſame Diameter, the Ray that falls thereon Parallel to the Axis, after its ſecond Refraction proceeds again Parallel.

Tab. 20. *F.* 1. *m n* is a *Meniſcus*, *b c* the Semidiameter of the Convexity *e c*, *f a* the Semidiameter of the Concavity *i a*, *d e* a Ray of Light Parallel to the Axis *k g*. I ſay this Ray after a double Refraction, one at *e*, and t'other at *i*, proceeds in *i h* Parallel to *k g*.

We here ſuppoſe the Thickneſs of the Glaſs inconſiderable in reſpect of the Semidiameter of the Sphere on which 'tis form'd; and therefore we wholly neglect it.

We ſuppoſe alſo that Sines and Angles and Sides in theſe ſmall Angles are Proportional.

Let $gc=ga$ be made Triple $bc=fa$, the Ray de by its firſt Refraction at its Entrance into the Glaſs at e is refracted towards g, eig being a Right Line; at i the Ray emerges from the Glaſs on the Concave Surface ia. Draw fil. Now the Angle of Inclination within the Glaſs is $lie=fig$, and if the Ray be refracted into ih, the Angle of Refraction is $hig=igf$, which ought therefore to be $\frac{1}{2}$ the Inclination fig, and, that it is ſo, I thus prove.

In the Triangle gif, As gf : To if :: So the Angle gif : To the Angle igf. But gf is double if, therefore $\angle gif$ is double $\angle igf$. *Which was to be Demonſtrated.*

Prop. XX.

In a Meniſcus, if the Semidiameter of the Concavity be triple the Semidiameter of the Convexity, the Focal Length is equal to the Semidiameter of the Concavity.

This is moſt evident, if the Convex-ſide be turned to the T.20. F.2. Ray, as *Tab.* 20. *Fig.* 2. hi is a Ray parallel to the Axis ab, dc the Semidiameter of the Convexity $fcig$, $be=3\,dc$ the Semidiameter of the Concavity $fekg$. The Ray, by the firſt Refraction it ſuffers on the Convex Surface at i, is directed to concur with the Axis at a Diameter and an half of the Convexity, that is, in b. But b is the Centre of the Concavity; wherefore the Ray ik within the Glaſs falls perpendicularly on the Concave Surface fkg; and therefore is not at all refracted by its Emerſion from the Glaſs, but proceeds onwards directly in ikb. *Which was to be Demonſtrated.*

But

But let the Concave-ſide be towards the Ray, as *Tab.*20. *Fig.* 3. fc is the Semidiameter of the Convexity ic, $gk=3fc$ T.20. F.3. the Semidiameter of the Concavity ek, de a Ray parallel to the Axis, produced directly to m. Let lc be made equal to gk, I ſay the Focus ſhall be at l, lc being equal to the Semidiameter of the Concavity. Draw geo. The Angle of the Rays Inclination on the Concave Surface is $deg=oem$ (Ang. *ad Verticem*) and the Ray by the firſt Refraction it receives on the Concave Surface at e is ſo refracted as if it came from the Point h diſtant a Diameter and half of the Concavity then draw $hein$ directly. Here $men=deh=ehg$ is the Angle of Refraction, which is therefore $\frac{1}{3}$ of the Inclination $deg=egk=oem$. But we ſuppoſe, as the Angles ſo are the Sines, and ſo are the Sides. Wherefore in the Triangle egh, $s\angle egh=s\angle egk : s\angle ehk :: eh=kh : eg$, that is, $\angle egk : \angle ehk :: kh : eg$. But the Angle egk is equal to $3\angle ehk$, and therefore kh is equal to $3\,eg$, and conſequently gh is double eg or gk. From f the Centre of the Convexity draw fip, this is perpendicular to the Convex Surface ic. Wherefore ſeeing gh is double gk, and gk is triple fc or fi, hf ſhall be octuple fi (by the Scheme). And the Second Angle of Inclination hif on the Convex Surface ſhall be octuple of the Angle at h, becauſe their Correſpondent ſubtending Sides are as 8 to 1. But hif is equal to pin (*Ang. ad Vert.*) And becauſe at the Rays Egreſs from Glaſs to Air, the Angle of Refraction nil ought to be half the Inclination pin, ſo let it be made; and then ſeeing the Angle pin is octuple the Angle at h, its half nil ſhall be quadruple the Angle at h; from the Angle nil ſubtract the Angle $nim=dih=\angle h$, there remains the Angle $mil=ilf=3\angle h$; wherefore the Angle at l and $egk=3\,ehk$ are equal. And then in the Triangle elg (neglecting the Thickneſs of the Glaſs) el and eg, or cl and kg the Semidiameter of the Concavity are equal. *Which was to be Demonſtrated.*

Obſervation.

Observation.

This is the 28th. *Prop.* of *Dechales* First Book of *Dioptricks*, which he there huddles over in a most confused manner, omitting several Steps in the Demonstration. *Zahn* has the same in *Fund.* 2. *Syntag.* 1. *Cap.* 7. *Prop.* 32. and transcribes from *Dechales* not omitting his very Errors.

PROP. XXI.

In a Meniscus, the Semidiameter of whose Convexity is triple the Semidiameter of the Concavity, The Virtual Focus is distant the Semidiameter of the Convexity.

T.20. F.4. *Tab.* 20. *F.* 4. ab is the Semidiameter of the Concavity bd, $kc = 3\,ab$ is the Semidiameter of the Convexity cf, rd a Ray parallel to the Axis kc. This Ray by the first Refraction it receives on the Concave Surface at d, is refracted into de, as if it came directly from the Point k (by *Prop.* X.), wherefore de falls perpendicular on the Convex Surface (by *Def.* 10.) and consequently is not refracted at its egress from the Glass (by *Exp.* 3.) wherefore the Virtual Focus is at k. *Which was to be Demonstrated.*

PROP. XXII.

In whatever Meniscus wherein the Semidiameters of the Convexity and Concavity are unequal; The General Rule that assigns the Focus *either* Real *or* Virtual *is this,*

As the difference of the Semidiameters :
To either of the Semidiameters, whether of the Convexity or Concavity ::
So is the Diameter of the other Surface :
To the Focus Real or Virtual.

(*Case*

(*Case* 1.) I shall demonstrate this *Proposition* in Three several Cases, and the First shall be, Where the Semidiameter of the Concave is greater than the Semidiameter of the Convex, but less than triple the Semidiameter of the Convex.

(*Case* 2.) Wherein the Semidiameter of the Concave is greater than triple the Semidiameter of the Convex.

And in these Two Cases the Glass has a *Real Focus.*

(*Case* 3.) Wherein the Semidiameter of the Concave is less than the Semidiameter of the Convex, and then the Glass has only a *Virtual Focus.*

'Tis here supposed, that the Breadth of the Glass is so little, and the Angles so small, that Sines and Angles and Sides may be proportional. For an Angle, we often assume its Complement to 180°, as having the same Sine.

'Tis supposed likewise that the Thickness of the Glass is inconsiderable, and therefore 'tis wholly neglected.

First therefore for the First Case. *Tab.* 20. *Fig.* 5. Let de be a Ray parallel to the Axis ck, fc is the Semidiameter of the Convexity ec; gl is the Semidiameter of the Concavity ol. Make hc triple fc. Then by the first Refraction the Ray suffers at e, 'tis directed towards h (by *Prop.* I.). Wherefore the Angle of Inclination upon its Emersion from the Glass is goh. Let the Angle of Refraction hok be made half goh. Then k is the Focus, and it being so made, I say, the difference of the Semidiameters $= fg$: Is to fc the one Semidiameter :: As $2gl$ the other Diameter : To ck the Focal Length. T.20. F.5.

Demonstration.

In the Triangle hog — —	1	$\angle hog : \angle ohg :: hg : go = gl$
And therefore — — — —	2	$\frac{1}{2} hog = hok : ohg :: hg : 2gl$
Moreover in the Triang. hok	3	$hok : ohg :: hk : ok = kl$
From 2 and 3 it follows —	4	$hg : 2gl :: hk : kl$

By

By permuting the 4th. — —	5	$2gl : kl :: hg : hk$
Compounding the 5th. —	6	$2gl+kl : kl :: hg+hk=gk : hk$
But kl is equal to $kg+$ $+gl$, therefore ——	7	$2gl+kl=2gl+kg+gl=$ $3gl+gk$
Wherefore the 6th. runs thus	8	$3gl+gk : kl :: gk : hk$
By permuting the 8th. — —	9	$3gl+gk : gk :: kl : hk$
Dividing the 9th. ——	10	$3gl : gk :: kl-hk=hl : hk$
And permuting the 10th. —	11	$3gl : hl=3fl :: gk : hk$
From 11 and 6 it follows —	12	$3gl : 3fl :: 2gl+kl : kl$
And consequently from 12	13	$gl : fl :: 2gl+kl : kl$
Dividing the 13th. ——	14	$gl-fl=fg : fl=fc :: 2gl : kl=kc$

For the Thickneſs of the Glaſs is neglected, and therefore we aſſume $kl=kc$.

But this 14th. Analogy $fg : fc :: 2gl : kc$. Is the *Propoſition* which was to be Demonſtrated.

Note, *Zahn* does again in this *Propoſition* tranſcribe from *Dechales*, and copies the very Miſtakes of the Preſs. But in our foregoing Demonſtration all is rectified.

Corollary.

Becauſe —— $fg : fc :: 2gl : kc$
It ſhall alſo be — $fg : gl :: 2fc : kc$
For if it be —— $fg : fc :: 2gl : kc$
Then Permuting $fg : 2gl :: fc : kc$

Then halve the firſt Conſequent and double the laſt Antecedent, and it ſhall be $fg : gl :: 2fc : kc$. *Which was to be Demonſtrated.*

T.20. F.6. In the 2. *Caſe*, Let fc, *Tab.* 20. *Fig.* 6. be the Semidiameter of the Convexity ec. Make ch equal to $3fc$. Let gl be the Semidiameter of the Concave lo, de a Ray parallel to

to the Axis, which by the first Refraction tends towards h (by *Prop.* I.) Now the Angle of Inclination on the Concave Surface is hog; for go is the Perpendicular. Wherefore the Ray by the second Refraction from Glass to Air is bent from the Perpendicular by half the Angle of Inclination; Let koh be this Angle of Refraction $= \frac{1}{2}\, hog$. I say then, that As gf: To lf or fc (neglecting the Thickness of the Glass) : : So $2gl$: To ck or lk.

Demonstration.

In the Triangle goh — —	1	$\angle ohg$ or $ohl : \angle goh :: og = gl : gh$.
And therefore — — — —	2	$ohl : \frac{1}{2} goh = hok :: 2gl : gh$
Moreover in the Triang. hok	3	$ohl : hok :: ok = lk : kh$.
From 2 and 3 it follows —	4	$2gl : gh :: lk : kh$.
Permuting 4 — — —	5	$2gl : lk :: gh : kh$.
Compound the 4th. — —	6	$2gl + gh : gh :: lk + kh : kh$.
Permute the 6th. — — —	7	$2gl + gh : lk + kh :: gh : kh$.
From 5 and 7 it follows —	8	$2gl + gh : lk + kh = lh :: 2gl : lk$.
Compound the 8th.	9	$2gl + (gh + lh =) gl = 3gl : lh = 3lf :: 2gl + lk : lk$.
And from the 9th. it follows	10	$gl : lf :: 2gl + lk : lk$.
And dividing the 10	11	$gl - lf = gf : lf :: 2gl : lk$.

Which was to be Demonstrated.

Corollary.

Because — — — $gf : lf :: 2gl : lk$.
It shall be also — — — $gf : gl :: 2lf : lk$, by the foregoing *Corollary*.

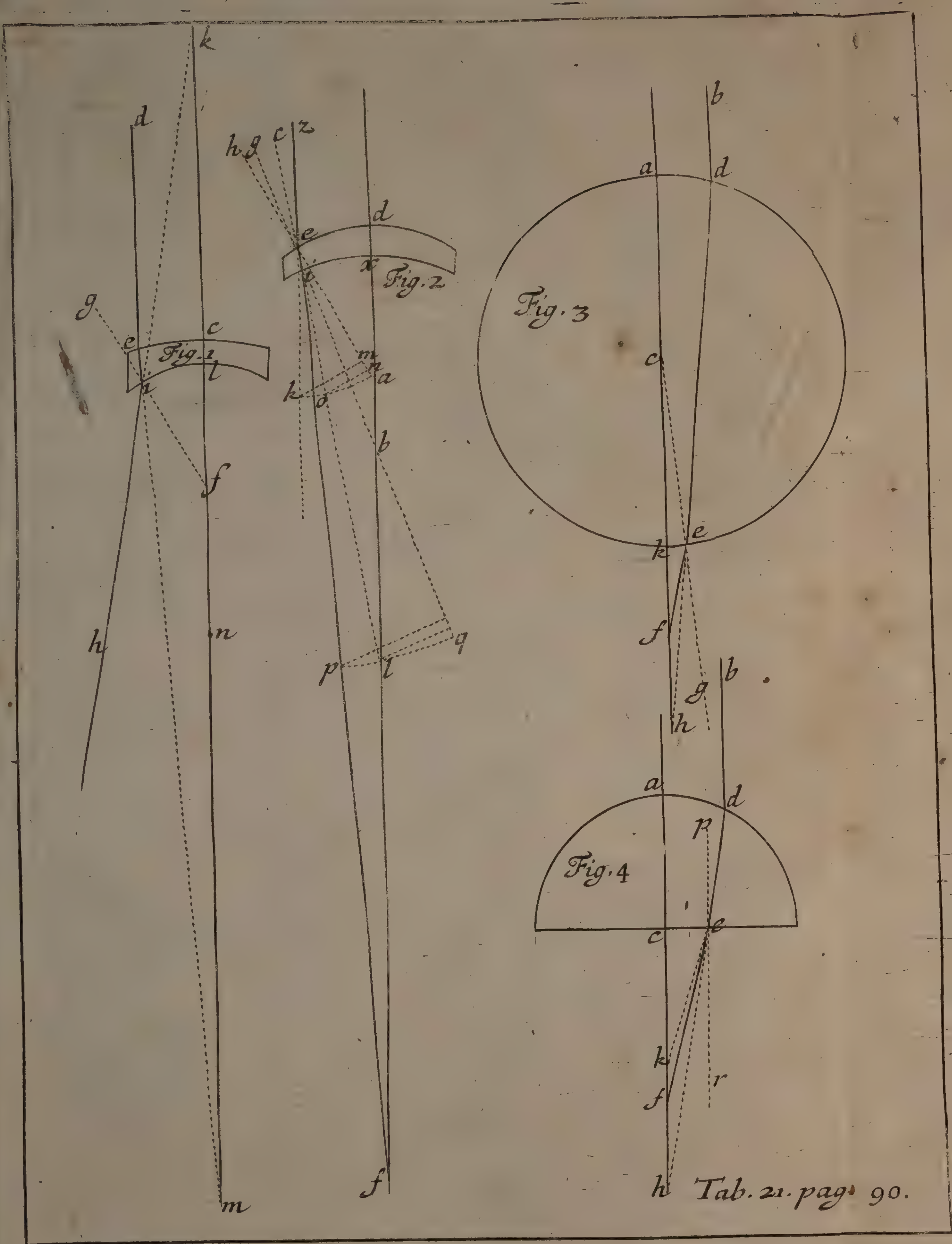
T.21. F.1. Laſtly in the third Caſe, *Tab.* 21. *f.* 1. fl is the Semidiameter of the Concavity il, nc the Semidiameter of the Convexity ec. Let cm be made Triple nc. de is a Ray Parallel to the Axis km. this Ray by its Firſt Refraction proceeds in eim; at i it emerges into Air on the Concave Surface. Draw the Perpendicular fig. The Angle of Inclination on this Concave Surface is $eig = fim$, *ad Vert.* Let the Angle of Refraction him be made equal to $\frac{1}{2} fim$. as it ought to be. Then, I ſay, the Ray Proceeds after its Second Refraction in ih, as if it came directly from the Virtual Focus k, and that then it ſhall be, as $nf : nc :: 2fl : kc$.

Demonſtration.

In the Trian. mif — —	1	$\angle mif : \angle m :: fm : fi = fl$.
Therefore — — — — —	2	$\frac{1}{2} mif = him : \angle m :: fm : 2fl$.
Alſo in Trian. mik — —	3	$\angle mik$ or its Compl. $\angle him : \angle m :: mk : ik = kl$.
From 2 and 3 — — —	4	$fm : 2fl :: mk : kl = kc$ neglecting the Thickneſs.
Dividing 4 — — — —	5	$fm - 2fl : 2fl :: mk - kl = ml = mc : kc$.
But by Poſition in the Scheme	6	$fm - 2fl = ml - 3fl$.
Wherefore the 5th Runs thus	7	$ml\ (=mc) - 3fl : 2fl :: ml = mc : kc$.
And Permuting 7 — — —	8	$mc\ (=3nc) - 3fl : mc :: 2fl : kc$.
And therefore — — — —	9	$nc - fl = nf : \frac{1}{3} mc = nc :: 2fl : kc$.

Which was to be Demonſtrated.

Corol-



Tab. 21. pag. 90.

Corollary.

Seeing it is — — — $nf : nc :: 2fl : kc$.
It ſhall be alſo — — $nf : fl :: 2nc : kc$.

Vid. Preceding Corollaries.

Prop. XXIII. Probl.

The Semidiameter of the Convexity being given, 'tis required to find the Semidiameter of the Concavity, that a Meniſcus formed with this Convexity and Concavity may unite the Parallel Rays at a given diſtance. Vid. Barrow Lect. Opt. 14. pag. 102. *&c.* Edit. Lond. 1672. 4to.

Tab. 21 F. 2. $ae = ad$ is the given Semidiameter of the Convexity, df is the given Diſtance, at which the Parallel Rays are to be united, ze produced to k is a Ray Parallel to the Axis df. Draw aeb on the Point of Incidence e as a Centre at any Interval, ſtrike the Arch koa, and make the ſine of the Angle of Incidence km, to the Sine of the firſt refracted Angle on, as 300 to 193, or as 14 to 9. Draw eol produced the other way to c. The Ray by the firſt Refraction is directed to l; in the Line le take any Point at Pleaſure i, ſo that ei may be the Thickneſs of the Glaſs which is to be formed; which Thickneſs is here repreſented conſiderable, for Illuſtration ſake, but it being really of no Moment in Glaſſes of ſmall Segments of large Spheres, 'tis neglected altogether in Demonſtration. Draw fi which we ſuppoſe equal to fd. And make the Angle liq: To the Angle lif :: As 193 : To 107. Draw qi which cuts the Azis in b. On b as a Centre ſtrike the Arch of the Concavity ix. I ſay $bi = bx$ is the Semidiameter of the Concavity, which added on t'other ſide the Glaſs to the former given Convexity, ſhall unite the Parallel Rays at the given diſtance df. T.21. F.2.

Demonstration.

By the first Refraction the Course of the Ray is through eil, but at i it meets the Concave Surface, and emerges from thence into Air. Now the second Angle of Inclination is bil, for bi is perpendicular to the Concave Surface. But the Angle lif is (by Construction) To the Angle liq :: As 107 : To 193. Wherefore lif is the second Angle of Refraction agreable to the second Angle of Inclination bil (by *Exp.* 6.) Wherefore ipf is the refracted Ray. *Which was to be Demonstrated.*

Calculation of an Example.

Let the Semidiameter of the Convexity $ae=ad$ be given 10000. and the Focal Distance $df=fi$ 63000.

Let us suppose the first Angle of Inclination kea to be 5°. 0′. 0″.

Then as 300 : to 107 :: $s.\angle kea : s.\angle ila$, which is the first Angle of Refraction = 1°. 46′. 50″. Wherefore $ilf=$ 178°. 13′. 10″.

Also (by *Prop.* 1.) As 107 : 300 :: $ae=10000$: To $el=il=ld=28037$.

Then in the Triangle ifl, we have the three sides and the Angle ilf, to find the Angle, $ifl=0°.\ 47'.30''$ and $fil=0°.59'.20''$.

Say then, As 107 : To 193 :: $s.\angle fil : s.\angle liq$ which we shall find to be 1°. 47′. 0″.

And $fil+liq=fib=2°.\ 46'.\ 20''$. Also $180° - fib - ifl = ibf = 176°.\ 26'.\ 10''$.

Lastly in the Triangle fib

As	$s.\angle ibf =$	176°. 26′. 20″ *Log. comp. ar.*	1.2064785633
To	$s.\angle ifb =$	0 47 30 — — — —	8.1404059077
So	$if =$	63000 — — — — — — —	4.7993405495
To	$bi =$	14000 — — — — — — —	4.1462250205

Where-

Wherefore I ſay that a *Meniſcus*, whoſe Convexity hath the Semidiameter 10000, and the Concavity the Semidiameter 14000, has its Focal Length 63000.

Further Proof.

To Prove this by ſome of the foregoing Rules for finding the Focus of a *Meniſcus*. Let us conceive ſuch a *Meniſcus* to conſiſt of a Plano-Convex Glaſs, and a Plano-Concave Glaſs. And the Semidiameter of the Plano-Convex to be 10000, and the Semidiameter of the Plano-Concave 14000. Then to find the Focal Lengths of theſe Glaſſes ſeparately, ſay,

By *Corol. Prop.* II. 107 : 193 :: 10000 : 18037 = *Fo. Conv.*
And by *Corol. Pr.* XI. 107 : 193 :: 14000 : 25252 = *Fo. Conc.*
Diff. 7215

Then by the Obſervation Preliminary to *Prop.* XIX. 7215 : 25252 :: 18037 : 63128 = to the Focal length of ſuch a *Meniſcus*. Which comes ſufficiently nigh 63000, conſidering the foregoing Angles are Calculated only in Round Numbers to every 10″ ſeconds.

What is here performed by ſuppoſing the Firſt Angle of Inclination $kea = 5^\circ.\ 0'.\ 0''$, may be likewiſe done ſuppoſing it of an other Inclination, and as the Angle is ſmaller it will be the more exact.

Prop. XXIV.

An intire Glaſs-Sphere Unites the Parallel Rays at the Diſtance almoſt of half its Semidiameter behind it.

'Tis here ſuppoſed that the Incident Rays do not fall diſtant from the Axis more than 20 or 30 Degrees.

Tab.

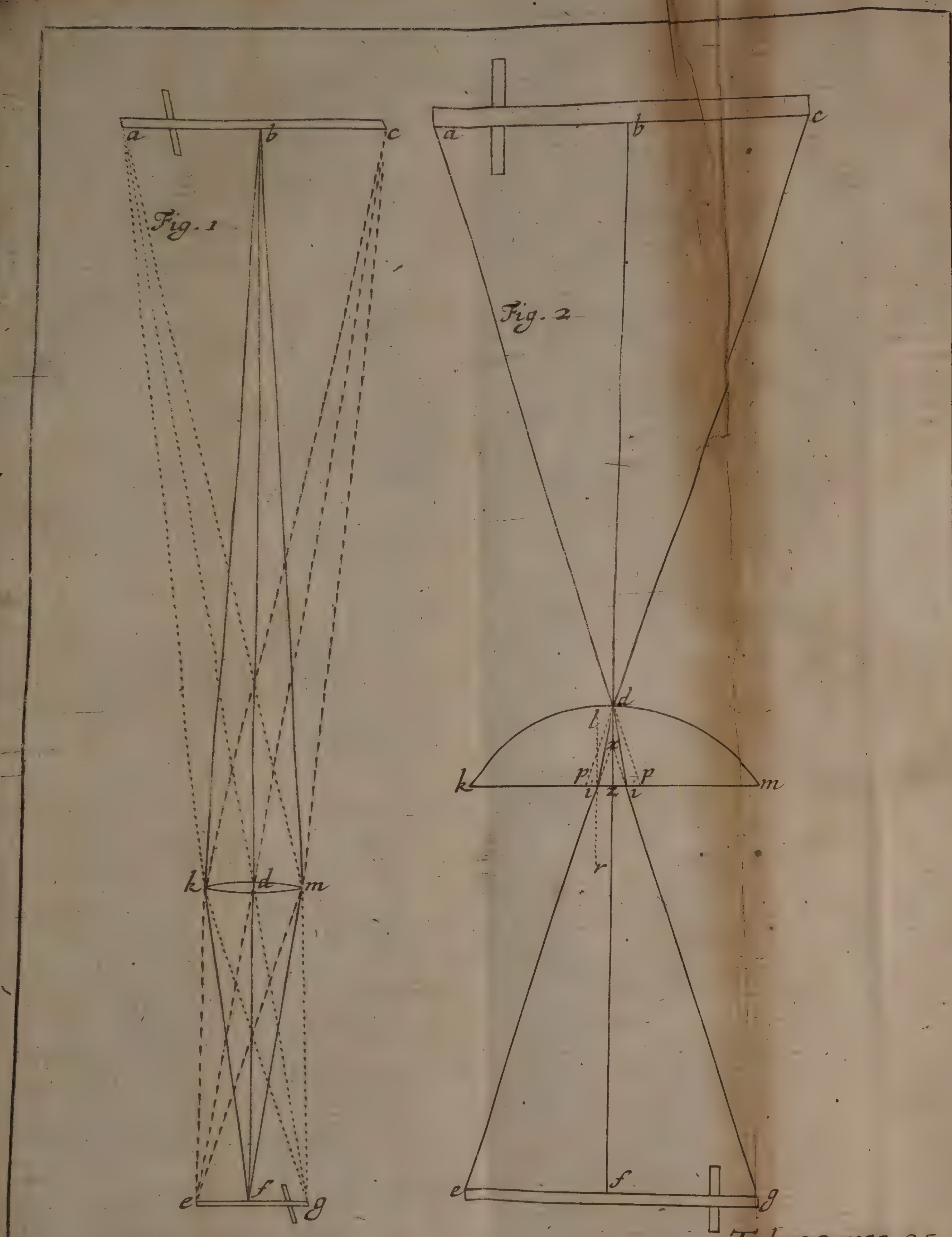
T.21. F.3. *Tab.* 21. *f.* 3. *a k* is a Glaſs-Sphere. On the Point *d* there falls the Ray *b d* Parallel to the Axis *a k*. I ſay this Ray after a Double Refraction concurs with the Axis in the Point *f*, ſo that *f k* is almoſt half the Semidiameter of the Sphere.

c is the Centre of the Sphere, make *k h* equal to the Semidiameter *c k*, then *a h* is a Diameter and Half. And the Ray *b d* by the Firſt Refraction proceeds in *d e* directly towards the Point *h*. But at *e* it emerges from Glaſs to Air. Draw the Perpendicular *c e g*, the Angle of this ſecond Inclination is $ced = geh$. Now the Ray inſtead of proceeding directly to *h* is refracted from the Perpendicular *e g* by the Angle *f e h*, which is therefore to be half *c e d*. And that it is ſo, I thus prove. *c e d* is equal to $ech + ehc$ (32. 1. *Eucl.*) but *e c h* and *e h c* are to ſenſe equal, their Subtenſes *e h* and *e c* being very near equal: wherefore *e h c* is half *c e d*. But *e h c* is equal to *f e h* (their Subtenſes *f e* and *f h* being as to ſenſe equal; for *f k* by ſuppoſition is equal to *f h*, and *f e* is very near equal to *f k*, therefore *f e* is almoſt equal to *f h*) and conſequently the Angle *f e h* is half *c e d* or *g e h*. *Which was to be Demonſtrated.*

Prop. XXV.

A Glaſs Hemiſphere Unites the Parallel Rays at the Diſtance of a Diameter and one third of a Semidiameter from the Pole of the Glaſs.

T.21. F.4. *Tab.* 21. *f.* 4. *a d c* is a Glaſs Hemiſphere, *b d* a Ray of Light Parallel to the Axis *a h*. This Ray after a Double Refraction Croſſes the Axis in the Point *f*, ſo that *f c* is a Semidiameter and one third of a Semidiameter. Make *c k* and



Tab. 22. pag. 95.

and *k h* each equal to *c a*. By the firſt Refraction at *d*, the Ray is directed towards *h* in *d e h*. At *e* the Point where it emerges from the Glaſs, draw *p e r* Perpendicular to the Surface *c e*. Now the ſecond Angle of Inclination is $dep = her$ (15. 1.) $= khe$ (29. 1.) $= keh$. (For *k e* being nighly equal to *k c*, 'tis alſo nighly equal to *k h*; Then in the Equilateral Triangle *k e h*, the Angle *k e h* ſhall be nighly equal to the Angle *k h e*) And the Angle of Refraction is *f e h*. And this we are to prove equal to $\frac{1}{2}$ the Inclination *h e r* or *e h f*. Seeing *k h* is equal to *c k*, and *k f* is $\frac{1}{3}$ of *k h* by Conſtruction, it follows that *c f* is double *f h* (for $ck + kf = cf = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$, and $fh = \frac{2}{3}$, but $\frac{4}{3}$ is = to double $\frac{2}{3}$) But *e f* is nighly equal to *c f*; therefore *e f* is nighly double *f h*. Then in the Triangle *f e h*, as the Sides, ſo the Angles (by ſuppoſition) therefore the Angle *e h f* is nighly double the Angle *f e h*. *Which was to be Demonſtrated.*

Definition.

The Projection of an Object in the Diſtinct Baſe of a Convex Glaſs, I call the *Image*.

Prop. XXVI.

As the Diſtance of the Object from the Glaſs :
To the Diſtance of the Image from the Glaſs : :
So the Diameter of the Objects Magnitude :
To the Diameter of the Image.

Tab. 22. *f.* 1. *a b c* is an Object, *k d m* a Convex Glaſs in the Hole of a Dark Chamber, *e f g* the Image formed by this Glaſs on a White Paper in the Diſtinct Baſe. I ſay, T. 22. F. 1.

As

As *c d* the Diſtance of the Object from the Glaſs: To *e d* the Diſtance of the Image from the Glaſs:: So *a c* the Objects Magnitude: To *g e* the Magnitude of the Image.

To ſhew this, we are to remember the Premiſes to the IV Propoſition. For by them we ſhall find, that the Axes *a d*, and *c d* of the Luminous Cones *k a m*, *k c m*, may be conſider'd as paſſing the Glaſs unrefracted. Becauſe that after they have paſſed the Glaſs and become *d g*, *d e*, they proceed Parallel to their Courſe before they entred the Glaſs. So that in Glaſſes of large Spheres, and ſmall Segments, the thickneſs of the Glaſs being inconſiderable in reſpect of the Focal Diſtance, we may neglect it. And then we have here two Similar Triangles *a d c*, *g d e*; For ∠*a d c* is equal to ∠*e d g* (15. 1. *Eucl.*) and the Side *e g* is Parallel to the Side *a c* (by ſuppoſition) and conſequently the Angle *c a d* is equal to the Angle *e g d* (29. 1.) and *a c d* is equal to the Angle *g e d*. Wherefore it ſhall be *c d* : *e d* :: *a c* : *g e*. *Which was to be Demonſtrated.*

Note. From hence appears the Miſtake committed by Monſieur *Comiers* in *DeBlegny's Zodiacus Medico-Gallicus. An. 3tio. pag* 117. Who ſays, *Diameter Imaginis Solis in Diſtinctâ Baſi obtinebit ad minimum in ſua Diametro Funem ſeu Chordam Dimidii Gradus magni Circuli Sphæræ, cujus Vitrum eſt ſegmentum.* Whereas it ſhould be, *Magni Circuli Sphæræ, cujus Radius eſt Diſtantia Focalis Vitri.*

What is here Demonſtrated concerning the Real Image of a Convex Glaſs may be accommodated to the Virtual Image of a Concave.

Scholium.

But if we are yet more ſcrupulous, and will conſider alſo the thickneſs of the Glaſs; Then the Point within the Glaſs

Glaſs from whence the Diſtance of the Diſtinct Baſe is to be counted, may be thus determin'd in a Plano-Convex Glaſs. *Tab.* 22. *f.* 2, *k d m* is a Plano-Convex Glaſs with its Convex Side towards the Object *a b c*; *a d*, *c d*, the Axes of Radious Cones falling Obliquely on the Glaſs in its Pole *d*; Which (if the Glaſs were not interpoſed) would proceed directly on to *p*, *p*, but by the Glaſs are now refracted into *d i*, *d i*; But at *i i* meeting with the Second ſurface of the Glaſs, inſtead of going ſtrait onwards, they are again Refracted into *i e*, *i g*, Parallel to *a d*, *c d*. Let *e i*, *g i* be produced backwards till they Interſect the Axis *b f* in *x*, I ſay *x* is the Point in the Glaſſes Axis, from whence the Diſtance of the Diſtinct Baſe is to be reckon'd. *And as the Thickneſs of the Glaſs* d z *: To the Diſtance of this Point* x *from the Inner ſurface* z x *: : So is the Co-tangent of the Angle of Incidence from Glaſs to Air : To the Co-tangent of the Refracted Angle.* T.22. F.2.

For to the Point of Incidence *i* draw *l i r* Perpendicular to *k m*. Here the Angle of Incidence from Glaſs to Air is $lid = idz$ and the Complement of idz is diz, whoſe Tangent is dz (making iz Radius) and the Refracted Angle is $rie = ixz$ whoſe Complement is xiz, and the Tangent hereof is xz.

In Double Convexes the Caſe is ſomething different, but 'tis needleſs to inlarge any farther thereon.

Wherefore we ſee, that if we uſe a Plano-Convex Glaſs with its Convex-ſide towards the Object, and if we allow for its Thickneſs, we are to compute the Diſtance of the Object from the Pole of the Glaſs *d*, and the Diſtance of the Diſtinct Baſe from the Point *x*. But ſuppoſing the Plane Side towards the Object *e f g*, then we are to reckon the Diſtance of the Object *e f g* from the Point *x*, and the Diſtance of the Diſtinct Baſe *a b c* from *d* the Pole of the Glaſs. *Vid. Prop.* XXVII.

Corollary 1.

It follows from this *Prop.*XXVI. That the Diameter of the Sun ſubtending an Arch of 32 Minutes in a Great Circle of the Heavens, the Diameter of the Sun's Image, Repreſented in the Diſtinct Baſe of a Convex-glaſs, ſubtends an Arch of 32 Minutes in a Circle, whoſe Radius is the Diſtance of the Diſtinct Baſe from the Glaſs.

T.23. F.1. But *Tab.* 23. *Fig.* 1. b is a Convex-Glaſs, $a\,b$ a Ray of Light proceeding from the Sun's Eaſtern Limb, $d\,b$ a Ray from its Centre, $c\,b$ a Ray from its Weſtern Limb. Let the ſingle Glaſs b Repreſent the Image of the Sun in the Diſtinct Baſe $h\,q\,i$, and let us ſuppoſe the Sun's Diameter to be 32′ Minutes. Draw $b\,h$, $b\,q$, $b\,i$. Let $e\,x\,z\,f$ be an other Plano-Convex-Glaſs, ſo placed behind the Glaſs b that both theſe Glaſſes together (according to *Prop.* XVI.) may Repreſent the Sun's Image in the Diſtinct Baſe $k\,p\,o$: 'Tis required to find the Breadth or Diameter $k\,o$ of this Image, from theſe Data, $h\,b\,q$ the Angle of the Sun's Semidiameter 16′ Minutes, $b\,q$ the Focal Length of the Glaſs b, $b\,f$ the Diſtance of the Glaſſes, $p\,f$ the Diſtance of the Diſtinct Baſe from the Glaſs $f\,x$, $e\,g$ the Radius of the Convexity $e\,x\,z$, or inſtead thereof, the Focal Length of the Plano-Convex Glaſs $e\,x\,z\,f$, for having one we may eaſily obtain the other.

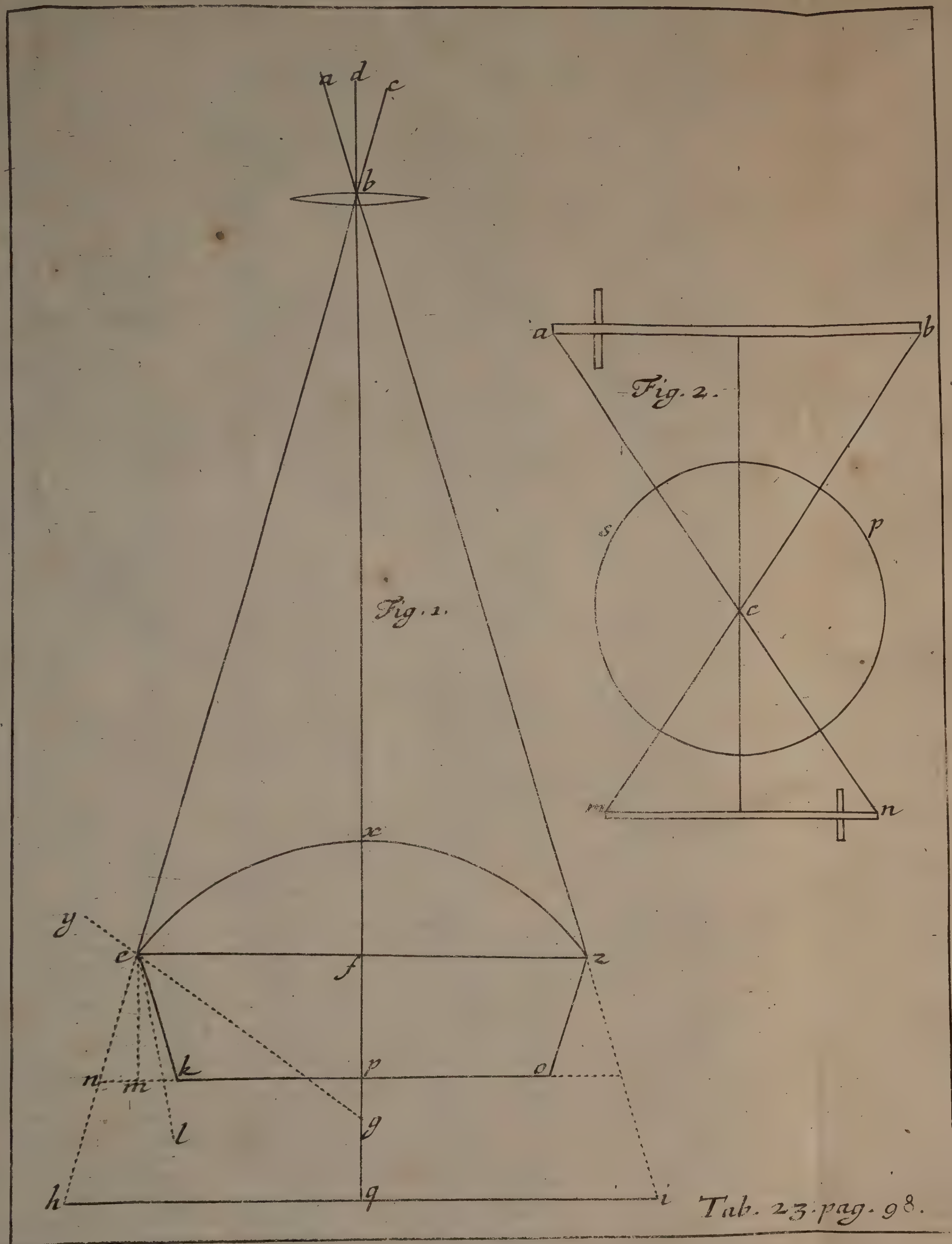
We ſuppoſe that the Central Ray $d\,b\,x\,q$ is Co-incident with the Axis of the Glaſſes.

Firſt therefore in the Right Angled Triangle $h\,b\,q$; we have the Angle $h\,b\,q$ (and conſequently $b\,h\,q = b\,e\,f$) and $b\,q$ to find hq.

(2) Then $b\,f + f\,p = b\,p$. And $b\,q : h\,q :: b\,p : p\,n :: b\,f : e\,f$.

(3) In the Right-Angled Triangle $e\,f\,g$, $e\,f$ and $e\,g$ are known to find the Angle $f\,e\,g$.

(4) 180



Tab. 23. pag. 98.

(4) $180 - bef - feg = yeb = geb$, Which is the Angle of the Inclination of the Ray be on the Convex ſurface ex.

(5) $300 : 193 :: s.\angle heg : s.\angle hel$, Wherefore the Ray by its firſt Refraction would proceed in el.

(6) Draw me Perpendicular to the Plane Surface ef, then $ef = mp$. And $me = pf$. And $np - mp = nm$; And the Angle nem is = to qbh, And $hel - nem$ is = to mel, Which is the Inclination of the Ray on the Plane Surface ef.

(7) Then $193 : 300 :: s.\angle mel : s.\angle mek$.

(8) In the Right-angled Triangle mek, me and the Angles are known to find mk.

(9) Laſtly $nm + mk = nk$, and $pn - nk = pk = \frac{1}{2} ko$; Which was required to be found.

Corollary 2.

The foregoing Trigonometrical Calculation in the firſt Corollary I had from my Eſteem'd Friend Mr. *John Flamſteed Aſtr. Reg.* But Inſtead thereof, or as an Additament thereto, I ſhall ſubſtitute this Problem.

To Determine the Breadth of the Diſtinct Baſe Reſulting from the Combinations of Glaſſes expreſſed in Prop. XVI, XVII, XVIII. *There being given (together with the Data in thoſe Propoſitions) the Breadth of the Object, or the Angle it ſubtends before the outermoſt Glaſs, And if it be a nigh Object, its Diſtance from the outermoſt Glaſs.*

We here ſuppoſe our Glaſſes of the leaſt Thickneſs Imaginable, or that all the Refractions are performed in the Right Line, that paſſes through the Glaſſes Breadth at Right Angles to the Glaſſes Axis, for avoiding Confuſion in the Schemes.

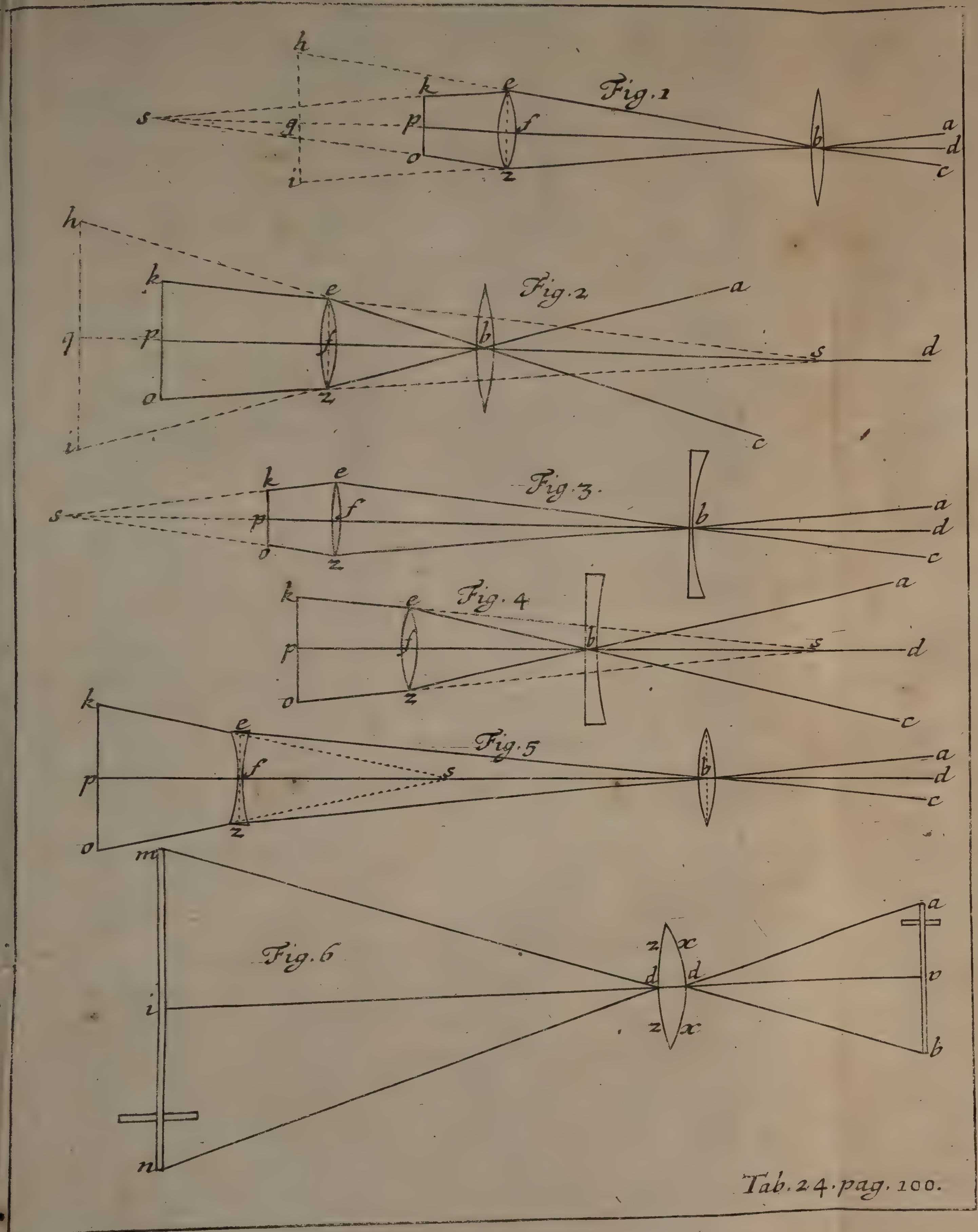
T.24. F.1, 2,3,4,5. Wherefore in *Tab.* 24. *Fig.* 1, 2, 3, 4, 5. *b* is the Glaſs next the Object, whether Convex (as in *Prop.* 16, 17.) or Concave (as in *Prop.* 18.) We ſhall take the Sun for our Object, and ſuppoſe *a b* a Ray proceeding from its Eaſtern Limb, *d b* a Ray from its Centre, *c b* a Ray from its Weſtern Limb. Let the ſingle Glaſs *b*, in the firſt and ſecond *Figures*, Repreſent the Image of the Sun in the Diſtinct Baſe *h q i*. The Breadth of the Baſe *h q i* is eaſily obtain'd from the Data. Let *e f z* be another Glaſs, ſo placed behind the Glaſs *b*, that both theſe Glaſſes together may Repreſent the Suns Image in the Diſtinct Baſe *k p o*, 'Tis required to find the Breadth or Diameter *k o* of this Image from theſe Data; *b q* the Focal Length of the Glaſs *b*; The Breadth of the Image *h i*, or the Angle $abc = ebz$; The Focal Length of the Glaſs *e z*; The Diſtance of the Glaſſes *f b* and *p f* the Diſtance of the Diſtinct Baſe from the Glaſs *e z*.

Through *p* at Right-angles to the Axis *d s b f p* of the Glaſſes draw *k p o* infinitely. Firſt from theſe Data, let us obtain the Breadth of the Glaſs *e z*, where the Rays *b e*, *b z*, meet it, or its half breadth *e f*, which is eaſily had in the Right-angled Triangle *e f b*, the Angle *e b f*, and the Side *f b* being given.

Let us then conſider *b* as a radiating Point, either farther from or nigher to the Glaſs *e z* than its Focus; And by *Prop.* V. and VIII. for Convexes, and by *Corol.* XV. for Concaves, let us determine the Reſpective, Imaginary, or Virtual Focus *s*, whereby we obtain *f s*. From *s* draw *s e*, *s z*, directly; Where theſe Lines *s e*, *s z*, meet the Diſtinct Baſe in *k* and *o*, there the Breadth of the Diſtinct Baſe is determin'd; To obtain the Meaſure whereof, ſay as $sf : fe :: sp : pk = po = \frac{1}{2} ko$.

And note that in *Fig.* 1 and 3. $sp = sf - pf$. But in *Fig.* 2, 4, 5. $sp = sf + fp$.

Corol-



Tab. 24. pag. 100.

Corollary 3.

If the Glaſs be a Compleat Sphere; Then, as the Diſtance of the Object from the Centre of the Glaſs : To the Breadth of the Object : : So the Diſtance of the Image from the ſame Centre of the Sphere : To the Breadth of the Image.

This is Manifeſt from *Tab.* 23. *Fig.* 2. Whereby it appears, that a Glaſs Sphere *s p c* expoſed to an Object *a b*; From each point *a*, *b*, of the Object there proceeds one certain Ray *a c n*, *b c m*, which paſſes Unrefracted; And theſe Rays Croſs in the Centre *c* of the Sphere, by which means they are Perpendicular to both Sides of the Sphere, at their Ingreſs and Egreſs, and ſo paſs Unrefracted. T.23. F.2.

Corollary 4. PROBL.

The Diſtance of an Object, and its Diameter being given, 'tis required to Repreſent its Image in the Diſtinct Baſe under a given Meaſure; The Thickneſs of the Glaſs being given alſo.

This is the 52 Problematical Propoſition of *Gregorii Opt. Promot.* And is thus ſolved by our Doctrine.

Tab. 24. *Fig.* 6. *a v b* is an Object, whoſe Diameter *a b* is given, and its Diſtance from the Glaſs *v d* is given alſo : 'Tis required to Project this Object under a Given Meaſure *m n*; The Thickneſs of the Glaſs *d d* being alſo given. Let it be made, As *a b* : *v d* : : *m n* : *d i*. Then ſtrike the Arch *x d x* with ſuch a Radius, that the Rays flowing from the Point *v* may thereby be tranſmitted Parallel within the Body of the Glaſs (which ſhall be by the foregoing Doctrine, by making $\frac{1}{2}$ *v d* the Radius) Afterwards ſtrike the Arch *z d z* with ſuch T.24. F.6.

a Radius, that it may Unite the Parallel Rays at *i*, which ſhall be by making ⅓ *d i* the Radius: I ſay the Glaſs *z x* ſhall project the Image of *a b* under the given Meaſure *m n*. This is ſo manifeſt from *Prop.* IX. together with the foregoing Doctrine, that it wants no farther Explication.

Note. I have here conſider'd the whole Thickneſs *d d* of the Glaſs in Conformity to *Gregory's* Propoſition. But what foregoes concerning the Conſideration of the Glaſſes Thickneſs may be apply'd here.

Prop. XXVII.

*The Object and its Image in the Diſtinct Baſe are Reciprocal. vid. Prop.*VI.

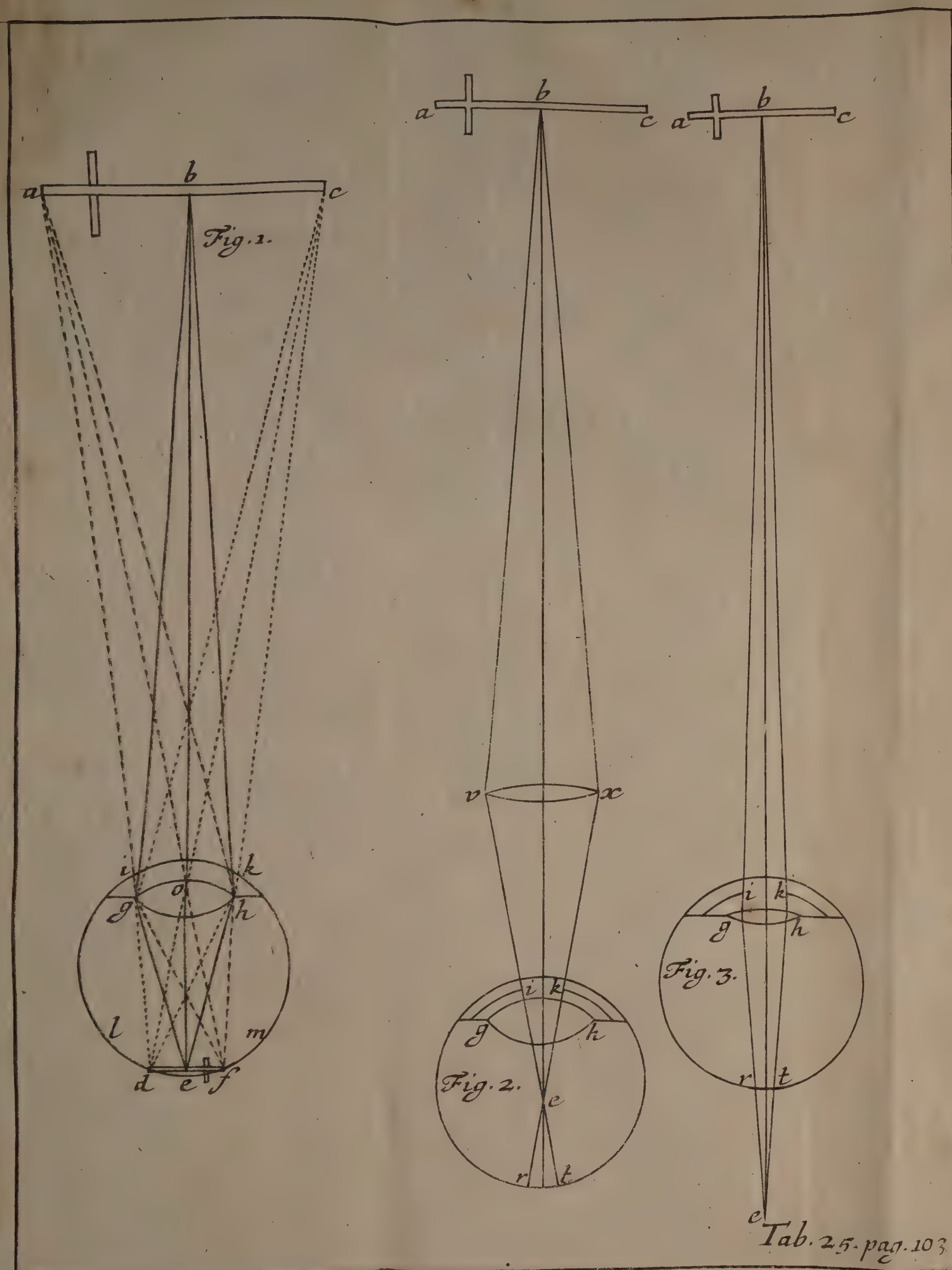
T.22. F.1. *Tab.* 22. *Fig.* 1. *a b c* being an Object, and the Convex Glaſs *k m* Repreſenting the Image thereof in the Diſtinct Baſe *e f g*. Let us now conceive *e f g* an Object, I ſay the ſame Glaſs *k m* the ſame way poſited ſhall project the Image thereof in the Diſtinct Baſe at *a b c*.

This does neceſſarily follow from the precedent Propoſition, and from *Exp.* 8. And therefore needs no farther Demonſtration.

Corollary.

Hence it follows that the Image in the Diſtinct Baſe may be ſometimes larger than the Object. On which depends the Doctrine of the double *Microſcope*, as hereafter ſhall appear more fully.

Let us imagine the Image of the Sun projected in the Focus of a Convex-glaſs, And let us now conceive the *Converſe*, *viz.* That this Image were now the Real Sun, the Diſtinct Baſe of



Tab. 25. pag. 103

of this would be as Large and as Remote as is now the Real Sun. So that every Convex-glaſs may be conceived to have two Foci, or Diſtinct Baſes; one at the Object, t'other at the Image.

Prop. XXVIII.

The manner of Plain Viſion with the naked Eye is expounded.

Tab. 25. *F.* 1. *a b c* is an Object, *i k l e m* is the Globe of the Eye, furniſh'd with all its Coats and Humors; But in this Figure we have only expreſſed the Cryſtalline Humour *g o h*, as being Principally concern'd in Forming the Image on the Fund of the Eye. T.25. F.1.

(1) From each Point in the Object we may Conceive Rays flowing on the Pupil of the Eye *i k*; as here from the middle Point *b*, there proceed the Rays *b g*, *b o*, *b h*; Theſe by means of the Coats and Humours of the Eye, and eſpecially by the Chryſtalline Humour *g h*, are refracted and brought together on the *Retina* or Fund of the Eye in the Point *e*, and there the Point *b* is repreſented. For we may conceive the Cryſtalline Humour *g h* as it were a Convex-glaſs, in the Hole of a Dark Chamber *i l m k*, and that *d e f* is the Diſtinct Baſe of this Glaſs. What is here ſaid of the Point *b*, and its Repreſentation at *e*, may be underſtood of all the other Points in the Object, as of *a* and *c* and their Repreſentations at *f* and *d*.

(2) And as in a dark Chamber, that has a Hole furniſh'd with a Convex-glaſs, if the Paper, that is to receive the Image in the Diſtinct Baſe, be either nigher to, or farther from the Glaſs, than its due diſtance, the Repreſentation thereon is confuſed; For then the Radious Pencils do not exactly determine with their Apices on the Paper; But thoſe from one Point are mixt and confuſed with thoſe from the Adjacent Points: ſo in the

the Case of Plain Vision, 'tis requisite that the Pencils should exactly determine their Apices at d, e, f, on the Retina, or else Vision is not Distinct.

'Tis therefore contrived by the *Most Wise and Omnipotent Framer of the Eye*, That it should have a Power of adapting it self in some Measure to *Nigh* and *Distant* Objects. For they require different Conformations of the Eye; Because the Rays proceeding from the Luminous Points of *Nigh* Objects do more Diverge, than those from more Remote Objects.

But whether this variety of Conformation consist in the Crystallines approaching nigher to, or removing farther from the *Retina*; Or in the Crystallines assuming a different Convexity, sometimes greater, sometimes less, according as is requisite, I leave to the scrutiny of others, and particularly of the curious Anatomist. This only I can say, that either of these Methods will serve to explain the various Phænomena of the Eye; And I am apt to believe, that both these may attend each other, *viz.* a Less Convex Crystalline requires an Elongation of the Eye, and a more Convex Crystalline requires a shortning thereof; As a more Flat Convex-Object-glass or of a Larger Sphere requires a Longer Tube, and one more Protuberant, bulging or of a smaller Sphere requires a shorter Tube.

(3) By the forementiond Scheme we perceive, the Rays from each Point of the Object are all confused together on the Pupil in $g\ h$, so that the Eye is placed in the Point of the Greatest Confusion: But by means of the Humors and Coats thereof each Cone of Rays is separated, and brought by it self to determine in its proper Point on the *Retina*, there Painting distinctly the Vivid Representation of the Object; Which Representation is there perceived by the *sensitive Soul* (whatever it be) the manner of whose Actions and Passions, He only knows who Created and Preserves it, *Whose Ways are*

Past

Past finding out, and by us unsearchable. But of this Moral truth we may be assured, *That He that made the Eye shall see.*

(4) We are likewise to observe, that the Representation of the Object *a b c* on the Fund of the Eye *f e d* is *Inverted.* For so likewise it is on the Paper in a dark Room; there being no other way for the Radious Cones to enter the Eye or the dark Chamber, but by their Axes *a o, b o, c o,* crossing in the Pole *o* of the Crystalline or Glass. And here it may be enquired; How then comes it to pass that the Eye sees the Object *Erect?* But this Quæry seems to encroach too nigh the enquiry into the manner of the Visive Faculties *Perception*; For 'tis not properly the Eye that *sees,* it is only the Organ or Instrument, 'tis the *Soul* that *sees* by means of the Eye. To enquire then, how it comes to pass, that the Soul perceives the Object *Erect* by means of an *Inverted* Image, is to enquire into the Souls Faculties; which is not the proper subject of this Discourse. But yet that in this Matter we may offer at somthing, I say, *Erect* and *Inverted* are only Terms of *Relation* to *Up* and *Down*, or *Farther from* and *Nigher to* the Centre of the Earth, in parts of the same thing: And that that is an *Erect* Object, makes an *Inverted* Image in the Eye, and an *Inverted* Object makes an *Erect* Image; That is, that part of the Object which is *farthest from* the Centre of the Earth is Painted on a Part of the Eye *Nigher* the Centre of the Earth, than the other parts of the Image. But the Eye or Visive Faculty takes no Notice of the Internal Posture of its own Parts, but uses them as an Instrument only, contrived by Nature for the Exercise of such a Faculty.

But to come yet a little nigher this difficulty; This enquiry results briefly to no more than this, How comes it to pass, that the Eye receiving the Representation of a Part of an Object on that part of its Fund which is *Lowermost* or nighest the Centre of the Earth, perceives that part of the Object as *Upper-*

most or fartheſt from the Centre of the Earth? And in anſwer to this, let us imagine, that the Eye in the Point *f* receives an Impulſe or Stroke by the Protruſion forwards of the Luminous Axis *a o f*, from the Point of the Object *a*; Muſt not the Viſive Faculty be neceſſarily directed hereby to conſider this ſtroke, as coming from the Top *a*, rather than from the bottom *c*, and conſequently ſhould be directed to conclude *f* the Repreſentation of the Top?

Hereof we may be ſatisfy'd by ſuppoſing a Man ſtanding on his Head: For here, tho the Upper Parts of Objects are painted on the Upper Parts of the Eye, yet the Objects are judged to be *Erect*. And from this Poſture of a Man, the Reaſon appears, why we have uſed the Words *Fartheſt from*, and *Nigheſt to the Centre of the Earth*, rather than *Upper* and *Lower*. For in this Poſture, becauſe the *Upper* Parts of the Object are painted on that part of the Eye nigheſt the Earth, (though really the upper Part of the Eye) they are judged to be fartheſt removed from the Earth.

What is ſaid of *Erect* and *Reverſe* may be underſtood of *Siniſter* and *Dexter*: But of theſe Phyſical Conjectures enough.

(5) The Image of an *Erect* Object being Repreſented on the Fund of the Eye *Inverted*, and yet the ſenſitive Faculty judging the Object *Erect*; it follows that when the Image of an *Erect* Object is Painted on the Fund of the Eye *Erect*, the ſenſe Judges that Object to be *Inverted*.

This is a neceſſary Concluſion, and is of conſequence for explaining ſome Particulars that follow.

(6) The *Magnitude* of an Object is Eſtimated by the Angle the Object ſubtends before the Eye: Thus in the ſame forementiond *Tab.* 25. *Fig.* 1. the Length of the Object *a c*
T.25.F.1. is eſtimated by the Angle *a o c*=*f o d*, this we call *The Optick Angle*.

From

From hence,& from *Prop.*XXVI,XXVII.it follows,that if the Eye were placed instead of the Glass at *d* (*T.*22.*F.*1.)and *a b c* or *e f g* were Objects,the Eye would perceive them of Equal Bigness. T.22. F.1

I know very well the Point *o*, which is the Vertex of the Optick Angle,is variously assigned by various Authors; some placing it in the Centre of the Eye; Others in the Vertex of the Crystalline; Others in the Vertex of the outward Coat or Cornea of the Eye: But 'tis a Matter of no great consequence,wherever we place it; for according to the Bigness of this Angle *a o c*, the Image on the Fund of the Eye is Bigger or Less. *Vid. Tacquet Optica. Lib.* 1. *Def.* 4. and *Prop.* 3.

I know likewise, that by a curious Experiment in Opticks discover'd by an Ingenious *French-Man* Monsieur *Mariotte*, 'tis controverted, whether the *Retina* or *Choroide* be the Seat of Vision, or the Place on which the Pictures of outward Objects are expressed (*vid. Philosoph. Transact.* Num. 35. and 59.) But to our business it matters not which of them we pitch on; and therefore I chuse to speak as commonly 'tis presumed; and mention the *Retina*, or rather the *Fund of the Eye*, as the Place that receives this Picture.

(7) In the same (*Tab.* 25. *Fig.* 1.) We perceive the Rays that flow from the Point *b* do proceed to the Eye *Diverging*, as *b g*, *b o*, *b h*; And if the Object *a c* were infinitely distant from the Eye, or so distant from the Eye, that the Breadth of the Pupil *i k* were insensible in Comparison to this Distance, then the Rays *b g*, *b o*, *b h*, would proceed as it were Parallel, and so fall on the Eye: In both which Cases, by means of the Refractions in the Eye, they are brought together, and paint the Image of the Point *b* on the Fund of the Eye at *e*. *Vid. Gregorii Opt. Prom. Pr.* 30. T.25. F.1.

But in *Tab* 25. *Fig.* 2. If the Diverging Rays *b v*, *b x*, that flow from the Point *b*, meet the Convex Glass *v x*, and are thereby made to converge as *v i*, *x k*, and so fall on the Eye, T.25. F.2.

and there passing through the Crystalline *g h*, are made to Converge yet more as *i e*, *k e*; Here they cross in the Point *e*, before they reach the *Retina r t*, and consequently do paint thereon the Image of the Point *b* confusedly, for 'tis Painted on the space *r t*; whereas to cause distinct Vision, it should only be painted on a correspondent *Point* on the *Retina*.

And this is the Fault of their Eyes, who are called *Myopes*, *Purblind*, or *Short-sighted*. For in them the Crystalline is too Convex (as in this *Tab.* 25. *Fig.* 2. both the Convex Glass and Crystalline joyn'd together make too great a Convexity) uniting the Rays before they arrive at the *Retina*. And therefore they are helped by Concave-glasses, which take off from the too great Convexity of their Crystalline some part of its Refractive Power: Or rather these Concaves make the Rays *Diverge* so, that their Crystalline shall be sufficient only to bring them again together, so that they be not united, till they arrive at the Fund of the Eye.

Myopes are also helped by holding the Object very near; for then the Rays that fall on their Eye from any single Point do more Diverge, than when the Eye is farther from the Point, and consequently their too Convex Crystalline does but suffice to bring them together on the *Retina*.

(8) On the contrary, the Eyes of Old Men have their Cry-
T.25. F.3. stalline too Flat (as *Tab.* 25. *Fig.* 3.) and cannot correct the Divergence of the Rays *b i*, *b k*, to make them meet on the *Retina r t*, but beyond the Eye at *e*. Wherefore for their Help 'tis requisite they add the Adventitious Convexity of a Glass; that both it and the Crystalline together, may be sufficient to unite the Rays just at the *Retina*: And from hence it appears, that Spectacles help Old Men, not by magnifying an Object, but by making its Appearance Distinct; for Old Men cannot read the largest Print without Spectacles, and yet with Spectacles, they read the smallest, though these with Specta-

cles

cles do not appear ſo large, as thoſe without Spectacles.

(9) What is ſaid of the *Confuſed* or *Diſtinct* Repreſentation of a *Point* in the Object, may be underſtood of the *Confuſed* or *Diſtinct* Repreſentation of the whole Object; at leaſt for thoſe Parts that lye pretty nigh adjacent to that Point that is looked at. For here we do not take a *Point* in the ſtrict ſenſe of the Mathematicians, but in a Phyſical Senſe, for the ſmalleſt Part imaginable; or as we have aſſumed it in the firſt ſuppoſition. And the whole Object conſiſting of ſuch Points, what is ſhewn of one Point may be underſtood of every Point in the Object, that is, of the whole Object.

(10) 'Tis Requiſite alſo (before I proceed farther) to explain, what I mean by *Clear* Viſion, and *Faint* Viſion; *Diſtinct* Viſion, and *Confuſed* Viſion.

By what foregoes, I ſuppoſe theſe two latter Terms are pretty well underſtood, *viz.* *Diſtinct* Viſion is then cauſed, when the Pencils of Rays from each Point of an Object do accurately determine in Correſpondent Points of the Image on the *Retina*: *Confuſed* Viſion on the contrary, when theſe Pencils do intermix one with another.

But *Clear* Viſion is only cauſed by a Great Quantity of Rays in the ſame Pencil, illuminating the Correſpondent Points of the Image *ſtrongly* and *vigorouſly*.

Faint Viſion is then when a Few Rays make up one Pencil; And tho this may be *Diſtinct*, yet 'tis *Dark* and *Obſcure*, at leaſt not ſo *Bright* and *Strong*, as if more Rays concurr'd.

Of Single Glaſſes apply'd to the Eye.

Hitherto I have ſpoken of Glaſſes by themſelves; And of the Eye by it ſelf: I come now to conſider them both together.

But firſt 'tis to be Noted, that when we ſpeak of the *Diſtinct* or *Confuſed* Appearance of an Object, 'tis needful only to explain my ſelf concerning ſome one ſingle Point in the Object; and for this we chuſe the middle Point; for if we ſhew that ſingle Point in the Object to be *Diſtinctly* or *Confuſedly* Repreſented on the *Retina* (according to the XXVIII. *Prop.*) the ſame may be underſtood of the Adjacent Points in the Object.

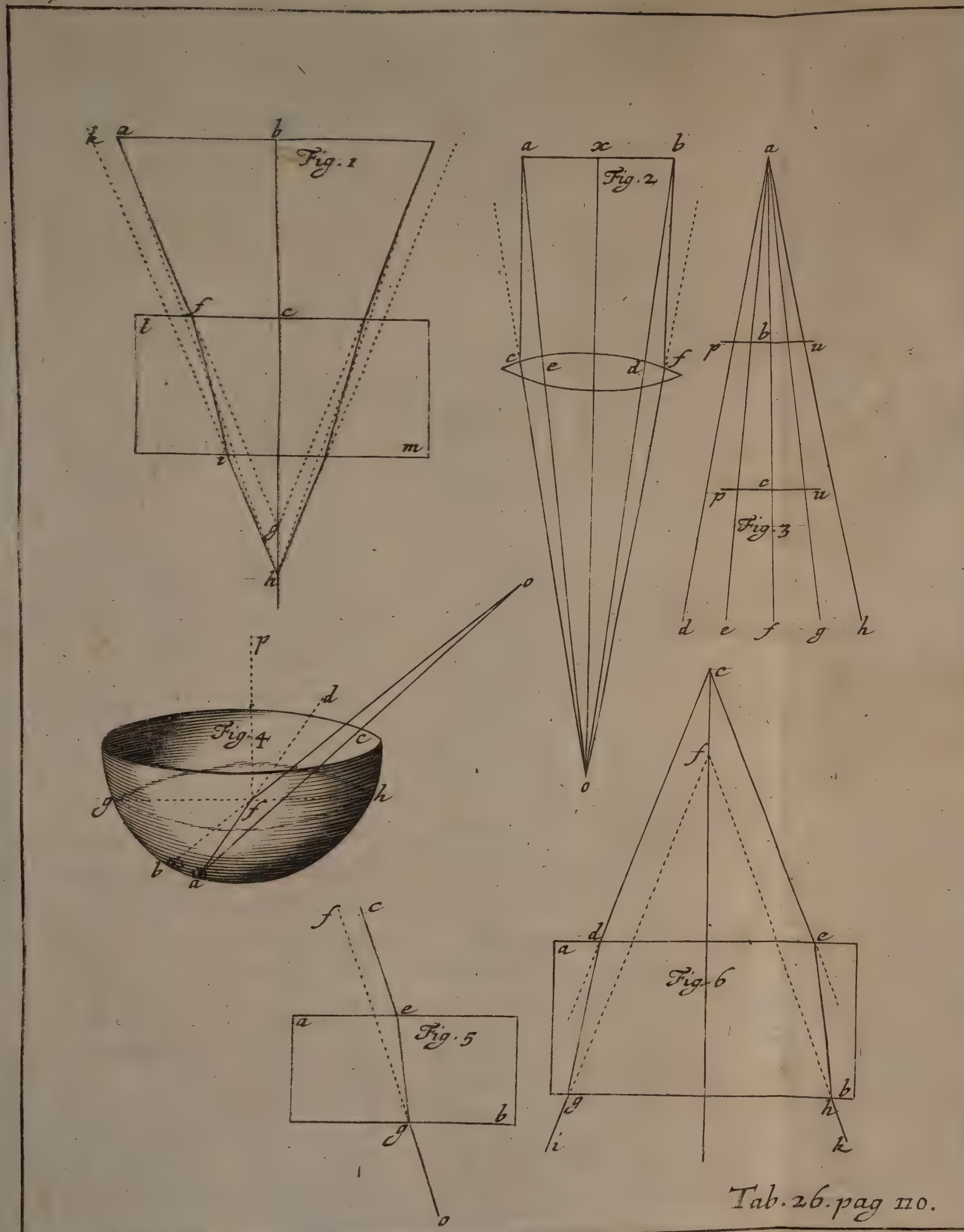
But when we diſcourſe of the *Erect* or *Inverted* Appearance of an Object: Or of the *Magnify'd* or *Diminiſh'd* Appearance of an Object; 'tis requiſite we conſider the whole Object. For a ſingle Point, though Phyſical, cannot properly be conſider'd as *Erect* or *Inverted*, or *Magnify'd* or *Diminiſh'd*.

So that we perceive *Diſtinct* or *Confuſed* Viſion to depend on the Formation of Rays proceeding from each ſingle Point. But *Erect* and *Inverted*, or *Magnify'd* and *Diminiſh'd* Appearances depend on the Conſideration of Rays proceeding from different Points of the Object. I ſhall always conſider thoſe, that proceed from the extremities of the Object.

PROP. XXIX.

An Object ſeen through a Plain Glaſs whoſe Surfaces are Parallel is Magnify'd thereby.

This Propoſition being directly contradictory to what HONORATUS FABER aſſerts in the XLIII. *Prop. Sec.* 2. T.26. F.1. of his *Synopſis Optica*, I ſhall mark my *Tab.* 26. *Fig.* 1. with the ſame Letters wherewith he marks his 89th Fig. He acknowledges that the *Apparent Place* of an Object is changed by a Plain Glaſs; but not that the Viſual Angle is alter'd thereby. But I ſhall ſhew that the Optick Angle is Magnify'd thus; Let *m l* be a Plain Glaſs, *a b* an Object, *a f* a Ray produced



Tab. 26. pag 110.

duced directly to g, but Refracted into $f\,i$; And at its Emersion from the Glaſs at i, Refracted again into $i\,h$, parallel to $a\,f\,g$. (by *Experim.* 1, 2.) Produce $i\,h$ directly towards k; let the Eye be at h, draw $h\,a$. The Angle,under which the Object $a\,b$ appears through the Glaſs $m\,l$ to the Eye at h, is $i\,h\,b$, or $k\,h\,b$, which is certainly greater than $a\,h\,b$, which is the natural Optick Angle. Wherefore the Eye at h through the Glaſs ſees the Object $a\,b$, under a greater Angle than it would do without the Glaſs, and conſequently the Object is magnify'd by the Glaſs. *Which was to be Demonſtrated.*

But that which deceived Faber is, that becauſe the Angle $a\,g\,b$ is equal to the Angle $k\,h\,b$, therefore (ſays he) the Optick Angle *with* and *without* the Glaſs are the ſame; or (as he has it) the Eye at g without the Glaſs would ſee the Object $a\,b$ under the Angle $a\,g\,b$, and through the Glaſs the Object is ſeen under the Angle $k\,h\,b$ equal to $a\,g\,b$: Which is granted him. But this does only prove, that the naked Eye at g, and the Eye through the Glaſs at h, ſees the Object under the ſame Angle. Whereas he ſhould have proved (if it were poſſible) that the naked Eye at h, and the Eye through the Glaſs at h, ſees the Objects under the ſame Angle; And then indeed he had rightly proved what he propoſed, *viz.* that the Optick Angle is not magnify'd by the Interpoſition of the Glaſs: For 'tis not Mathematical, firſt to place the Eye at g *without* the Glaſs, and then at h *with* the Glaſs, and proving the Optick Angles the ſame, to conclude that therefore the Plain Glaſs does not magnifie. For at that rate one may ſhew that even a Convex Glaſs does not magnifie; For at *one* Station an Object ſhall appear under as great an Angle *without* it, as at *another* Station *through* the Glaſs.

The Reaſon, why the Common Window Glaſs,&c. through which we look, cauſes no ſenſible Alteration of Objects, is becauſe 'tis too Thin. *Vid. Barrow Lect. Opt.* 15.

Con-

Concerning the *Apparent Place* of an Object through a Plain Glaſs, more hereafter.

Prop. XXX.

All Objects ſeen Erect through Convex Glaſſes are Magnify'd thereby.

T.26. F.2. *Tab.* 26. *Fig.* 2. Let *a x b* be an Object, *o* the Eye, draw *a e o*, *b d o*, directly ſtrait from the extremities of the Object to the Eye. The Angle compriſed by the Direct Rays *a e o*, *b d o*, that is the Angle *a o b*, is the *Natural* Optick Angle. Let now the Glaſs *c f* be interpoſed; here becauſe the Rays *a e*, *b d*, would *naturally* and by their *Direct* courſe concur at *o*, now the Glaſs is interpoſed, their Concourſe ſhall be accelerated before they arrive at *o*. Wherefore the Eye at *o* ſhall not perceive the extremities of the Object through the Glaſs by the Rays *a e*, *b d*, but by ſome *other Rays*. And theſe *other Rays* muſt fall either *without* (that is, farther from the Axis *o x*, than) *a e*, *b d*, or they muſt fall *within a e*, *b d*: But they cannot fall *within a e*, *b d*; for if *a e*, *b d* themſelves be made by the Glaſs to concur *before* they arrive at *o*, much more ſhall any other Rays that fall *within a e*, *b d*, be made to concur *before* they arrive there: And conſequently they cannot convey the Appearance of the Extreme Points *a*, *b*, to the Eye at *o*. Wherefore it remains, that the Rays that do this muſt fall *without a e*, *b d*: Let theſe Rays be *a c*, *b f*, which by the Refractive Power of the Glaſs are bent from their Direct Courſe and are made to proceed in *c o*, *f o*, croſſing at the Eye in *o*; Here the Optick Angle through the Glaſs is *c o f*, which is greater than the Natural Optick Angle *a o b*, and conſequently the Object is *Magnify'd*. *Which was to be Demonſtrated.*

Prop.

Prop. XXXI.

Concerning the Apparent Place of Objects ſeen through Convex-glaſſes.

(1) In Plain Viſion the Eſtimate we make of the *Diſtance* of Objects (eſpecially when ſo far removed, that the Interval between our two Eyes, bears no ſenſible Proportion thereto; or when look'd upon with one Eye only) is rather the Act of our *Judgment*, than of *Senſe*; and acquired by *Exerciſe* and a Faculty of *comparing*, rather than *Natural*. For *Diſtance* of it ſelf, is not to be perceived; for 'tis a Line (or a Length) preſented to our Eye with its End towards us, which muſt therefore be only a *Point*, and that is *Inviſible*. Wherefore Diſtance is *chiefly* perceived by means of Interjacent Bodies, as by the Earth, Mountains, Hills, Fields, Trees, Houſes, &c. Or by the *Eſtimate* we make of the *Comparative Magnitude* of Bodies, or of their *Faint Colours*, &c. Theſe I ſay are the *Chief* Means of apprehending the Diſtance of Objects, that are conſiderably *Remote*. But as to *nigh* Objects, to whoſe Diſtance the Interval of the Eyes bears a ſenſible Proportion, their Diſtance is perceived by the turn of the Eyes, or by the Angle of the Optick Axes. (*Gregorii Opt. Promot. Prop.* XXVIII.) This was the Opinion of the Antients, *Alhazen*, *Vitellio*, &c. And tho the Ingenious Jeſuit *Tacquet* (*Opt. Lib.*I. *Prop.*II.) diſapprove thereof, and Objects againſt it a New Notion of *Gaſſendus* (of a Man's ſeeing only with one Eye at a Time one and the ſame Object) yet this Notion of *Gaſſendus* being abſolutely Falſe (as I could Demonſtrate, were it not beſide my Preſent Purpoſe, but I refer to the 7th Chap. of the 2d Part.) it makes nothing againſt this Opinion. —p'290

(2) Wherefore Diſtance being only a *Line*, and not of it ſelf perceivable; if an Object were convey'd to the Eye by one

one ſingle Ray only, there were no other means of judging of its Diſtance, but by ſome of thoſe hinted before. Therefore when we eſtimate the Diſtance of *nigh* Objects, either we take the help of both Eyes, or elſe we conſider the Pupil of one Eye as having *Breadth*, and receiving a *Parcel* of Rays from each Radiating Point. And according to the various Inclination of the Rays from one Point, on the various Parts of the Pupil, we make our Eſtimate of the Diſtance of the Object: And therefore (as is ſaid before) by one ſingle Eye we can only Judge of the Diſtance of ſuch Objects, to whoſe Diſtance the Breadth of the Pupil has a ſenſible Proportion. To illuſtrate all this by *Tab.* 26. *Fig.* 3. Let *a* be a Radiating Point, ſending forth the *Rays a d*, *a e*, *a f*, *a g*, *a h*, with all the intermediate Rays. Let *p u* be the Breadth of the Pupil, and firſt placed at *c*, there receiving only *a e*, *a f*, *a g*: Let *p u* be tranſlated to *b*, where it receives all the Rays; and 'tis manifeſt that the Rays from *a* are differently inclined on the Pupil in one and t'other Poſture, for the Rays, which fall on the Pupil when placed at *b*, diverge very much more than thoſe that fall upon it, when placed at *c*: And therefore the Eye, or Viſual Faculty will apprehend the Diſtance of *a* from *b* and *c* to be *Different*. For it is obſerved before (*Prop.* 29. *Sect.* 2. ſee alſo, *Gregorii Opt. Promot. Prop.* XXIX.) that for viewing Objects *Remote* and *Nigh*, there are Requiſite Various Conformations of the Eye: The Rays from *Nigh* Objects, that fall on the Eye, Diverging more than thoſe from more *Remote* Objects.

T.26. F.3.

(4) If therefore by Refraction through Glaſſes, that parcel of Rays which falls on the Pupil from each Point in *Nigh* Objects be made to flow as cloſe together as thoſe from *Diſtant* Objects; or the Rays from *Diſtant* Objects be made to *Diverge*, as much as if they flow'd from *Nigh* Objects, the Eye through ſuch Glaſſes ſhall perceive the *Place of the Object changed*.

(5) But

(5) But firſt for a ſenſible and common Experiment, to ſhew the *Change* of an Objects *Place* by Refraction. *Tab.* 26. *F.* 4. represents a Veſſel, on whoſe Bottom at *a* there is laid a piece of Mony, or any other remarkable Object, ſo that the Eye at *o* may juſt perceive the Mony over the Edge *c* of the Veſſel. The Mony now appears to the Eye *o* by the direct Line *a c o*. Let now the Veſſel be fill d with Water up to *g f h*; let the Ray *a f* proceed from the Mony, and draw *p f* perpendicular to *g h*; the Ray *a f*, inſtead of going onwards directly to *d*, emerging from Water, a denſe Medium to Air, deflects from the perpendicular *p f*; and becomes (ſuppoſe) *f o*. Wherefore the Eye *o*, now the Veſſel has Water in it, ſees the Silver, not by the direct Ray *a c o* (for that is bent from it, and eſcapes it) but by the refracted Ray *a f o*. Produce *o f* directly to *b*, the Mony ſhall appear as if it were at *b*. For the Eye is not ſenſible of the bending of the Ray, but is affected by it, as if it were directly ſtrait. T.26. F.4.

(6) In like manner *Tab.* 26. *F.* 5. if the Point *c* ſend its Ray *c e* obliquely on the plain Glaſs *a b*; and after a double Refraction it arrive at the Eye *o*; This Point *c* is not now ſeen in its own proper Place, but ſomewhere in the Ray *o g* produced, as at *f*. For the Eye is not ſenſible of the outward accidental Refraction, that attends the Ray at its paſſage through the Glaſs; but is directed by the next immediate Ray *o g* that falls upon it, and conſiders it as *ſtrait*, and coming directly from the Point *f*. T.26. F.5.

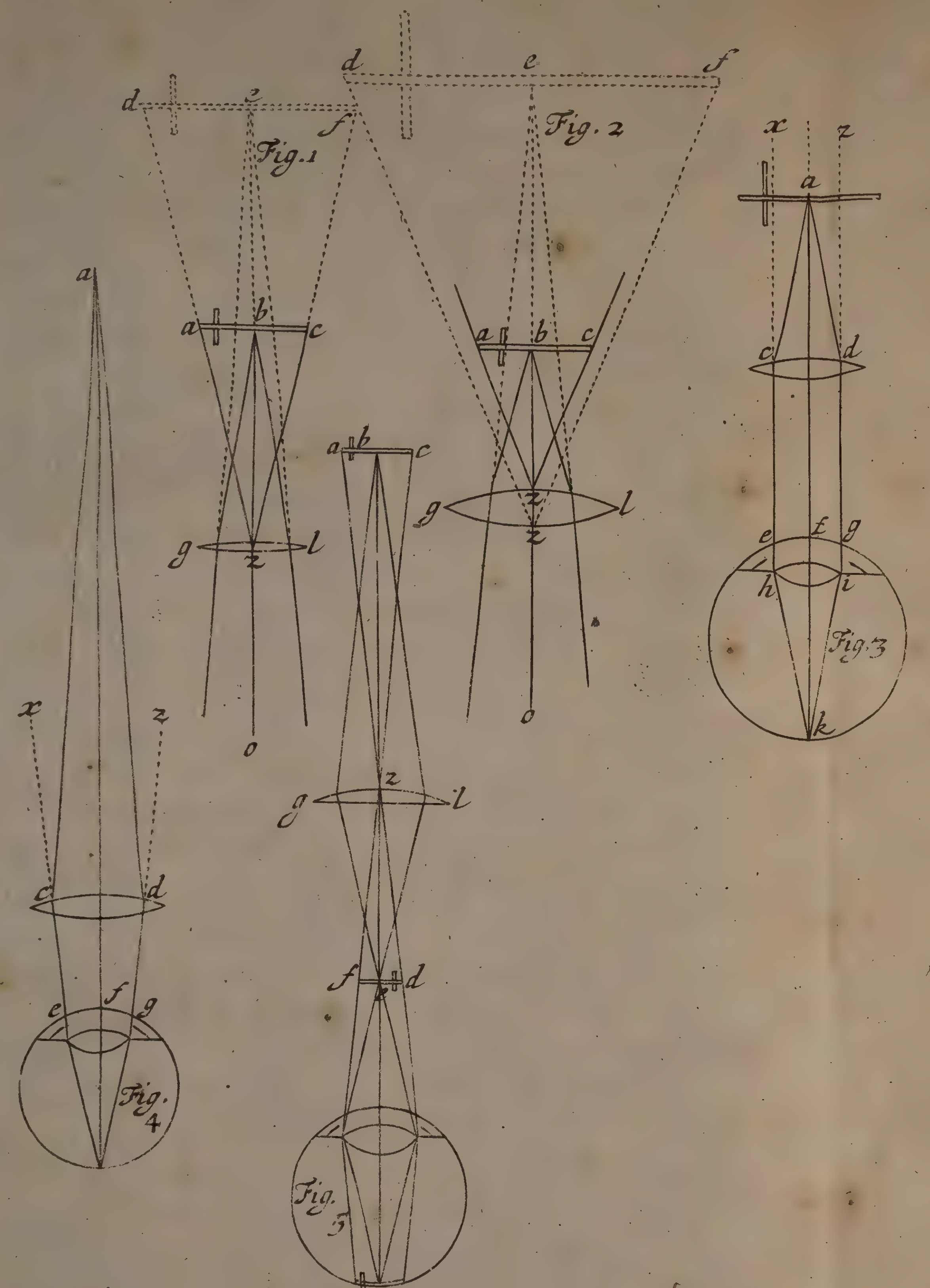
I ſay moreover, that by a plain Glaſs, the place of an Object is changed, and brought nigher the Eye. *Tab.* 26. *F.* 6. *c* is an Object ſending its Rays *c d*, *c e*, upon the plain Glaſs *a b*; theſe after a double Refraction proceed (by ſuppoſition) in *g i*, *h k*. Produce theſe directly towards *f*; and ſuppoſe the Pupil of the Eye large enough to receive the Rays *g i*, *h k*; the Point *c* ſhall appear to the Eye as at *f*. T.26. F.6.

 (7)

(7) *To determine the* Locus Apparens *of an Object placed nigher a Convex-glass than its Focus.*

To perform this, we are to have, the Power of the Glass, the Distance of the Object from the Glass, and the Length of T.27. F.1. the Object given. *Tab.* 27. *Fig.* 1. *a b c* is an Object, whose Distance from the Glass *b z* is given. Let us suppose the Glass of the least thickness imaginable, that we may not be at the trouble of considering the Optick Angle, both at the Immersion and Emersion. Let the middle Point *b* Radiate upon the Glass, and after the Rays have passed the Glass, let them be so Refracted, as if they came directly from the Point *e*. From the forementiond *Data*, this Point *e*, being the *Imaginary Focus*, is easily determin'd by *Prop.* VIII. hereof: Through *e* draw the Infinite Right Line *d e f* Parallel to *a b c*. Wherefore as the *Locus Apparens* of the Point *b* is at *e*, so the *Loci* of the Collateral Points *a*, *c*, shall be somewhere in the Line *d e f* (unless perhaps Convexes on very small Spheres, will Represent the Object Crooked or Bowed, but of this we shall take no notice) to determine which, from the Vertex of the Glass *z* draw the Lines *z a d*, *z c f* (or T.27. F.2. if the Glass have any Thickness, as in *Tab.* 27. *Fig.* 2. from the Vertex of Immersion, or outward Vertex, draw *z a*, *z c*; And from the Vertex of Emersion or Inward Vertex, draw *z d*, *z f*, Parallel to *z a*, *z c*, and then the Thickness of the Glass must be one of the *Data*. *Vid. Prop.* 47. *Gregorii Opt. Promot.*) the Points *d* and *f* are the *Loci Apparentes* of the Points *a* and *c*. For certainly were the Eye behind the Glass just at *z*, the Object *a b c* would appear under the Angle *a z c*, and the Point *b* would appear as at *e*, and therefore the Points *a* and *c* would appear somewhere, so as to make the Object keep the same Angle; But that can only be by the Points *a* and *c* appearing at *d* and *f* (supposing the Object to appear in its own Natural strait shape) Wherefore *d* and *f* are

F.3.



Tab. 27. pag. 116.

are the *Loci* of the Points *a* and *c*, and *d e f* is the *Locus* of the Object *a b c*.

Hence it appears, that a Convex-Glaſs Repreſents Objects as farther off than really they are: And this is the Reaſon, why Pieces of Perſpective (as of Churches and Long Porticoes) appear very Natural and ſtrong through Convex-Glaſſes duly apply'd. For theſe Glaſſes making Objects appear further off than really they are, muſt conſequently make the Parts of the Perſpective ſeem really *Hollow'd* or ſunk in, the *French* term it *Renfoncé*.

Hence alſo 'tis Manifeſt, why Convex-Glaſſes help the Eyes of thoſe, that ſee only *Diſtant* Objects, as *Old Men*, for theſe make *Nigh* Objects appear as *Diſtant*. *Vid. Prop.* XXVIII. *Sec.* 8.

We may perceive alſo that the Diſtance between the Object and Glaſs continuing the ſame, the *Locus Apparens* is never alter'd, though the Eye be removed to and from the Glaſs. *Gregorii Opt. Promot. Corol. Prop.* XLVII. For the Diſtance between the Object and Glaſs continuing the ſame, the Imaginary *Focus e* ſhall always be the ſame.

(8) *The Locus of an Object expos'd to a Convex-Glaſs in its Focus is not to be determin'd.*

When the Radiating Point *a*. *Tab.* 27. *Fig.* 3. is placed in the *Focus* of the Convex-Glaſs *c d*, the Rays *c e*, *d g*, after paſſing the Glaſs, run Parallel, and being produced to *c x*, *d z*, they never Interſect. Wherefore in this Caſe there is no Rule whereby to determine the *Locus* of the Object. And *Barrow* tells us only, *Quod Remotiſſime Poſitum Æſtimatur. Lect.* 18. *ad Finem.* T.27. F.3.

(9) *The Locus of an Object, beyond the Focus of a Convex-Glaſs, to the Eye between the Glaſs and Diſtinct Baſe, cannot be determined.*

Tab.

T. 27. F. 4. *Tab.* 27. *Fig.* 4. Let *a* be a Radiating Point placed more diſtant from the Glaſs *c d* than its Focus; the Rays *c e*, *d g*, after paſſing the Glaſs, do *Converge* towards the Diſtinct Baſe; let the Eye *e f g* be placed between the Glaſs and Diſtinct Baſe, it ſhall then receive the Rays *c e*, *dg*, Converging; and they being produced towards *x* and *z* ſeparate the further.

In this and the laſt Section lies the great Difficulty, which the *Incomparable* and *moſt profoundly Learned* BARROW (*Lect. Opt.* 18. *Sect.* 13.) confeſſedly paſſes over as inſuperable, and not to be explained by whatever Theories we have yet of Viſion. For ſeeing that the Object which applies to the Eye by *Leſs-diverging* Rays, is judged the *more remote*; And that which applies to the Eye by *Parallel Rays*, is reputed *moſt remote*; it ſhould ſeem reaſonably to follow, that what is ſeen by *Converging Rays*, ſhould appear yet *moſt remote* of all: And yet Experience contradicts this, and teſtifies, that the Point *a*, *Tab*, 27. *Fig*. 4. appears variouſly *diſtant*, according to the various Situations of the Eye between the Glaſs and Diſtinct Baſe; and that it does almoſt never (if ever) appear *more diſtant* than the Point *a* it ſelf to the naked Sight, and ſometimes it appears *much nigher*: Or rather, by how much the Rays, which fall through the Glaſs on the Eye, do *more Converge*, by ſo much the *nigher* does the Object appear to approach, inſomuch, that if the Eye approach the Glaſs *very nigh*, the Object *a* appears in its *natural Place*: The Eye being a little farther removed from the Glaſs towards the Diſtinct Baſe, the Point *a* ſeems yet to approach. The Eye being yet farther, the Point ſeems yet *nigher*; and ſo by degrees, till at laſt the Eye being placed at a certain ſtation, as at the *Diſtinct Baſe*, the Point *a* appears *very nigh*, ſo that it begins to vaniſh away in mere *Confuſion*. All which (continues the candid BARROW) ſeems repugnant, or at leaſt not ſo well to agree to what we have laid down. And ſo he

leaves

leaves this Difficulty to the ſolution of others, which I (after ſo great an Example) ſhall do likewiſe; but with the reſolution of the ſame admirable Author, of not quitting the *evident* Doctrine, which we have before laid down, for determining the *Locus Objecti*, on the account of being preſſed by *one Difficulty*, which ſeems inexplicable, till a more intimate Knowledg of the *Viſive Faculty* be obtained by Mortals. In the mean time, I propoſe it to the conſideration of the ingenious, whether the *Locus Apparens* of an Object placed as in this 9th. Section, be not as much *before* the Eye, as the Diſtinct Baſe is *behind* the Eye. *Vid. Corol.* 1. *Prop.* LVII.

(10) *If an Object be more diſtant from a Convex-Glaſs than its Focus, and the Eye beyond the Diſtinct Baſe, the* Locus Apparens *of the Object is in the Diſtinct Baſe.* Vid. *Prop.* XXXIX. *Sect.* 5. item *Schol. Prop.* L.

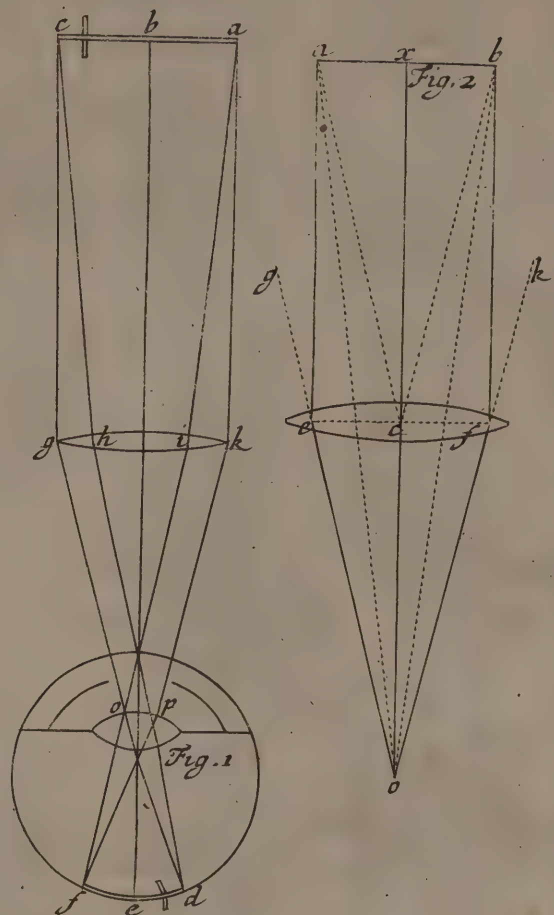
Tab. 27. *Fig.* 5. The Object *a b c* is projected by the Glaſs *g z l* in the Diſtinct Baſe *d e f*. The *Locus* of each Point in the *Object* is in the correſpondent Point of the *Image* in the *Diſtinct Baſe*: Thus *a* appears at *d*, *b* at *e*, *c* at *f*. *Gregorii Opt. Prom. Prop.* XLVI. T.27. F.5.

Dechales (*Dioptr. Lib.* II. *Prop.* XI.) remarks a Matter of ſome moment in this Buſineſs of the *Locus Objecti*. The reaſon (ſays he) that the Appearances through Glaſſes of the Change of the Objects Place, do not ſo ſtrongly ſtrike the Senſe, as the Doctrine here laid down ſeems to intimate, proceeds from hence; that Optick-Glaſſes are ſeldom or never made ſo large, as to be look'd through by both Eyes at once; for if they were, he aſſerts, That the *Locus apparens Objecti* would be much more plainly and ſenſibly determin'd to the ſight: In this particular certainly he is much in the right; for we ſee at all times, that two Eyes make a more exact eſtimate of the Poſition of an Object, than one ſingle Eye. And we have a ſenſible Experiment hereof in *Catoptricks*,

tricks, by the large concave Miroirs, where an Hand and Dagger preſented beyond the Focus, ſeem to ſtrike far without the *Speculum*, at him that preſents it: But not ſo ſtrongly if the *Speculum* be but ſmall, ſo that the Image can be ſeen but by one Eye at once.

Scholium.

To this Property of the Change of the *Apparent Place* of an Object, may we attribute that common Effect of Objects ſeeming to *Dance* and *Move*, being ſeen through Spherick-Glaſſes, whether Convex or Concave, nimbly ſhaken between the Eye and the Object. 'Tis a noted Experiment, that to know, whether a Glaſs be Plain or Spherical (which is not to be found by Inſpection of the Glaſs it ſelf, or by Touch, if the Glaſſes be formed on large Spheres, and be but ſmall portions thereof), the common Tryal is, to ſhake them ſomething nimbly between the Eye and an Object; and if the Object ſeem to move by the motion of the Glaſs, the Glaſs is *not Plain*: The reaſon hereof is this; That toward the *Extremities* of a Glaſs, the Glaſs refracts *more* than towards the middle (for the very middle Ray, that is perpendicular, is not refracted at all) and conſequently the *Apparent Place* of an Object is more changed by the Refraction in the *Extreme* parts of the Glaſs, than in the *middle* of the Glaſs: So that the Object, by the Motion of the Glaſs, appearing ſometimes through the middle of the Glaſs, ſometimes through the extremities of the Glaſs; the *Apparent Place* thereof is *varied* likewiſe, and the Object ſeems therefore to *change its Place*, or to *move*. But in a plain Glaſs the Caſe is otherwiſe, for the Refraction thereof is equally prevalent throughout the whole Glaſs, and neither *ſtronger* nor *weaker* in the *middle*, than in the *extremities*; ſo that the ſhaking of *it* between the Eye and



Tab. 28. pag. 122.

and Object, makes no other difference in the *Apparent Place*, than if the Glaſs were look'd through fixt and immoveable; and therefore the Object ſeems not through it to ſhake and move. And tho the *Locus* be changed by a Spherick Glaſs's being removed *to and from* the Eye, yet in Glaſſes of large Spheres, the *Locus* is not ſo *ſenſibly* changed, as by being ſhaken *before* the Eye. The ſame Reaſon holds for the apparent Dancing of Objects ſeen through riſing Smoaks or Vapors.

Prop. XXXII.

An Object being placed in the Focus of a Convex-Glaſs, and the Eye on t'other ſide the Glaſs, ſees this Object diſtinctly and erect.

That an Object ſo poſited, is ſeen diſtinctly, is evident, becauſe the Rays from each particular Point, after paſſing the Glaſs, become parallel (by *Prop.* VI.) and ſo fall on the Eye. And therefore (by *Prop.* XXVIII. *Sec.* 7.) they are fit to cauſe Diſtinct Viſion. *Tab.* 27. *Fig.* 3. The Point *a* ſendeth forth T.27. F.3.
its Diverging Rays *a c*, *a d*, on the Convex-Glaſs *c d*; theſe are tranſmitted by the Glaſs parallel in *c e*, *d g*, and ſo fall on the Eye; by whoſe Coats and Humors, eſpecially by the Cryſtalline *h i*, they are refracted and brought together in the Point *k* on the *Retina*, there repreſenting diſtinctly the Point *a* of the Object.

Or thus, *Tab.* 28. *Fig.* 1. The Eye *o p*, being in the place T.28. F.1.
where ſome of the Rays from every Point in the Object are mixt together, is in the place of the greateſt *Confuſion*; and therefore by *Prop.* XXVIII. *Sec.* 3. the Viſion is diſtinct.

I ſay alſo the Object is ſeen *erect*. In the ſame Figure *g k* is a Convex-Glaſs, *a b c* an Object; we may imagin that each Point of this Object ſends from it a Cone of Rays, fall-

ing on the whole Surface of the Glaſs; but for avoiding Confuſion in the Scheme, I only expreſs and conſider the Rays *c g*, *c h*, from the Point *c*; and *a k*, *a i* from the Point *a*. *o p* is the Cryſtalline of the Eye. The Rays *c g*, *c h*, after paſſing the Glaſs become parallel as *g o*, *h p*, and fall ſo on the Cryſtalline, by whoſe Refractions they are again brought together in the Point *d* on the *Retina*, there painting the lively Repreſentation of the Point *c*; and the ſame may be ſhewn of the Repreſentation of the Point *a* at *f* on the *Retina*. Wherefore the *ſiniſter* Point *c* is repreſented on the *dexter* part of the Fund of the Eye *d*; and the *dexter* part of the Object *a* is painted on the *ſiniſter* part of the Eye *f*. And conſequently the Image on the *Retina* being *inverted*, the Object is ſeen erect (by *Prop.* XXVIII. *Sec.* 4, 5.). *Which was to be Demonſtrated.*

Prop. XXXIII.

An Object being placed in the Focus of a Convex Glaſs, and the Eye being placed in the Focus, on t'other ſide the Glaſs, ſees this Object under the ſame Angle, as were the naked Eye placed at the ſtation of the Glaſs.

T.28.F.2. *Tab.* 28. *Fig.* 2. *a x b* is an Object placed in the Focus of the Glaſs *e f* (which we ſuppoſe of the leaſt thickneſs imaginable). Let the Rays *a e*, *b f*, run parallel to the Axis *x c o*, theſe Rays are united in the Focus at *o*, at which Point we ſuppoſe the Eye. Produce *o e* and *o f* directly to *g* and *k*, and draw *e c f* (which by ſuppoſition is parallel to *a x b*, and then draw *a c*, *b c*. The Optick-Angle through the Glaſs is *e o f*, which we are to prove equal to *a c b*. Thus, *a x* and *e c* being parallel, and *a e*, *x c*, being ſo likewiſe; *a x* is equal to *e c*, and *a e* to *x c*, being the oppoſite Sides of a Parallelogram. But *x c* is equal to *c o* (the Focal Length on one

ſide

ſide and t'other). Wherefore the Right-angled Triangle $e\,c\,o$ is equal and ſimilar to the Right-angled Triangle $a\,x\,c$. Wherefore the Angle $e\,o\,c$ is equal to the Angle $a\,c\,x$, or $e\,o\,f$ is equal to $a\,c\,b$. And conſequently the Eye at o ſees the Object through the Glaſs under the ſame Angle, as the naked Eye being placed at c, would ſee it. *Which was to be proved.* Vid. *Gregorii Opt. Prom. Prop.* XLIV.

Scholium.

This is the Poſture of both Eye and Object for cauſing the moſt perfect Viſion poſſible through a ſingle Convex Glaſs; unleſs perhaps we ſay, That ſeeing all Objects, which we perceive moſt diſtinctly are pretty nigh our Eyes; therefore we may conceive the Rays flowing from each Point of them, as falling on the Pupil *Diverging*. And conſequently, that to ſee an Object moſt perfectly through a Convex Glaſs, 'tis beſt that the Object be placed a very little nigher the Glaſs than its Focus: For then the Rays, after paſſing the Glaſs, do ſomething *Diverge*, and fall ſo on the Pupil; and this being the moſt natural and uſual occurrence of Objects, it ſuits beſt, and is beſt adapted and moſt agreeable to the Eye; which by its refractive Coats and Humors eaſily collects each Cone of theſe *Diverging* Rays, and brings them together in a Point on the Fund of the Eye.

Corollary 1.

The Eye and Object being ſo placed as above; the Eye ſees this Object *magnify'd* under an Angle almoſt *double* to that under which the Object would appear to the Eye, were the Glaſs removed; that is, the Angle $e\,o\,c$ or $a\,c\,x$ is almoſt double the Angle $a\,o\,c$ ($a\,o$ and $b\,o$ being drawn directly) for the Angle $a\,c\,x$ is equal to $a\,o\,c + c\,a\,o$ (32. 1.) but $c\,a\,o$

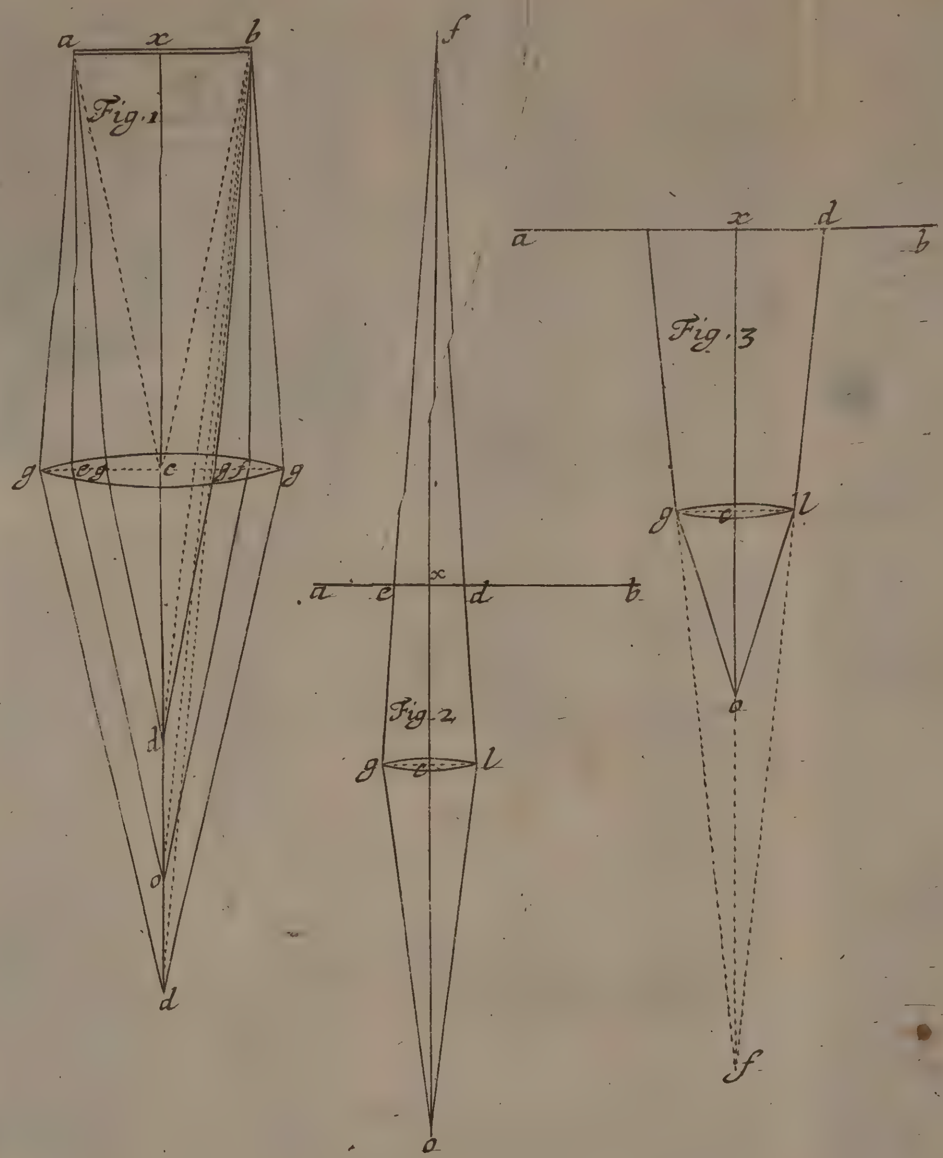
is almoſt equal to $a\,o\,c$. Therefore the Angle $a\,c\,x$ is almoſt double $a\,o\,c$. *Which was to be Demonſtrated.* That $c\,a\,o$ is almoſt equal to $a\,o\,c$, is manifeſt; for the Triangle $o\,c\,a$ is almoſt Iſoſceles, $o\,c$ $(=c\,x)$ being almoſt equal to $c\,a$.

If now we ſuppoſe the Eye to continue in the Focus at o, and the Object $a\,x\,b$ to be brought nigher the Glaſs $e\,f$; ſtill the Optick Angle through the Glaſs ſhall be $e\,o\,f$. By which (tho it continue the ſame, yet) the Object would not be magnifi'd by the Glaſs in this ſecond Poſture, as much as in the former. For the Angle $e\,o\,f$ in this latter Poſture would not ſo much exceed the natural Optick Angle $a\,o\,b$. For at laſt $a\,x\,b$ being brought ſo near $e\,c\,f$, as to be *coincident* therewith, the Angles $e\,o\,f$ and $a\,o\,b$ would be *coincident* alſo.

The ſame may be ſhewn, if we ſuppoſe the Object to continue in the Focus, and the Eye o to approach the Point c. For at laſt the Eye arriving at c, perceives the Object through the Glaſs under its own natural Angle $a\,c\,b$.

Corollary 2.

Hence it appears, that the Eye being in the Focus of a Convex-Glaſs, and the Object in the Focus alſo, the Eye can perceive no greater an *Area* or Space of the Object, than the Breadth of the Glaſs, or the Breadth of that Portion thereof, which the Eye makes uſe of; for $a\,b$ is equal to $e\,f$. And the reaſon is, becauſe thoſe Rays $a\,e$, $b\,f$, that come from the Extremities of the Object a and b, and concurr at the Eye in the Focus o, muſt neceſſarily fall on the Glaſs parallel to the Axis $x\,c$.



Tab. 29. pag. 125.

Prop. XXXIV. Probl.

To determine the Optick Angle, or apparent Magnitude of an Object in the Focus of a Convex-Glaſs, to the Eye at any other ſtation; from theſe Data, *the Glaſſes Focal Length, or the Diſtance of the Object from the Glaſs* x c, Tab. 29. F. 1. *The Diſtance of the Eye from the Glaſs* c d, *and the Breadth of the Object* a b. T.29. F.1.

The Thickneſs of the Glaſs I take no notice of, becauſe I would not perplex the Scheme. And therefore the Glaſs is ſuppoſed of the leaſt Thickneſs imaginable; or all the Refractions are ſuppoſed performed at the Line in the middle of the Glaſs *g e c f g*.

Wherefore let *a x b* be an Object, and let the Rays *a e*, *b f*, fall parallel to the Axis *x c o*; theſe are united in the other Focus at *o*, where if the Eye be placed, it ſees the Object under the Angle *e o f* = *a c b*, by the preceding *Prop.* Wherefore having *b x* the half Object, and *x c*; in the Right-angled Triangle *b x c*, we may find the Semi-optick Angle *b c x*. Draw the prickt Line *b o* directly; the Angle *b o x* is the natural Optick Angle of the Object *b x*, which is eaſily obtained, in the Right-angled Triangle *b o x*, having *b x* and *x o*.

Let now the Eye be out of the Focus at *d*; the Rays *a g*, after paſſing the Glaſs, become parallel to *e o* (*Prop.* VI.) And therefore the Angle *g d x* is equal to *e o x* or *a c x*, which we have found before. Draw the prickt Line *b d* directly; the Angle *b d x* is the natural Optick Angle of the Object *b x*, were the Glaſs removed; and is eaſily had in the Right-angled Triangle *b x d*, having *b x* and *x d* (= *x c* + *c d*). So the Problem is ſatisfied.

Prop.

Prop. XXXV. Probl.

To determine the Visible Area of an Object in the Focus of a Convex Glass; from these Data, *the distance of the Object from the Glass or the Glasses Focal Length, the distance of the Eye from the Glass, and the Breadth of the Glass.*

It is shewn before in the 2d. *Corollary* to *Prop.* XXXIII. That the Visible Area of an Object in the Focus of a Convex Glass, to the Eye placed in t'other Focus, is equal to the Breadth of the Glass. Wherefore I shall only here consider the two other Cases.

T.29. F.2. And first, Let the Eye at o, *Tab.* 29. *Fig.* 2. be placed *further* from the Glass $g\,l$ than its Focus, the Object $a\,x\,b$ is in the Focus, $g\,l$ the Glasses Breadth, $x\,c$ the Glasses Focus or distance of the Object, and $c\,o$ the distance of the Eye from the Glass are given. Draw $g\,o$, $l\,o$, and produce the Axis $o\,c\,f$ infinitely. Let us now imagin the Point o a nigh Radiating Point or Object, sending its Rays $o\,g$, $o\,c$, $o\,l$, on the Glass. By *Prop.* V. Let us determin the Distinct Base or Respective Focus of this Point projected by this Glass, which by the foregoing *Data* is easily performed. Suppose this respective Focus to be $c\,f$. Draw $g\,f$, $l\,f$; where these Lines intersect the Object in e and d, the Visible Area of the Object is determined by $e\,d$.

Demonstration. For by Exper. 8. the Progress of a Ray through different Mediums is *Reciprocal.* Wherefore if the Rays $o\,g$, $o\,l$, be refracted into $g\,e$, $l\,d$; it will necessarily follow, that $e\,g$, $d\,l$, being consider'd as two Rays flowing from the Points e and d, they shall be refracted into $g\,o$, $l\,o$; wherefore this Analogy will hold, $f\,c : c\,l :: f\,x \;(=f\,c - x\,c) : x\,d$. Which $x\,d$ is half the Visible Area to the Eye at o.

Secondly,

Secondly, *Tab.* 29. *F.* 3. Let the Eye at *o* be placed *nigher* the Glaſs than its Focus. Draw *g o*, *l o*. And produce the Axis *o c* infinitely backwards towards *f*. Then (as before) ſuppoſing the Point *o* a Radiating Point or Object; by the *Data*, and by *Prop.* VIII. we may eaſily determin the *Imaginary* Focus thereof, which let be *c f*. From *f* draw *f g* directly to *e*, and *f l* to *d*. *e d*, by the foregoing Demonſtration, is the Viſible Area of the Object *a b*. And $c f : c l :: f x \; (= f c + c x) : x d$. Which is half the Viſible Area to the Eye at *o*. T.29. F.3.

Scholium.

The ſame Rules may be uſed, when the Object is *nigher to* or *further from* the Glaſs than its *Focus*. So that this Propoſition is univerſal to all Caſes in *Erect* Viſion through Convex Glaſſes. But whether this Area be *diſtinctly* viſible or not, depends on other Conſiderations, and will be obvious enough to thoſe that conſider what *has* been before, and what *ſhall* be hereafter laid down concerning the *diſtinct* and *confuſed* appearance of Objects.

Prop. XXXVI.

An Object being placed more diſtant from a Convex Glaſs than its Focus, and the Eye placed on t'other ſide this Glaſs nigher to the Glaſs than the Diſtinct Baſe to the Glaſs, ſees this Object erect and confuſed.

I have ſhewn before (*Prop.* V. XXVI, XXVII.) that if an Object be placed more diſtant from a Glaſs than its Focus, this Object is projected in a diſtinct Baſe ſomewhere on t'other ſide the Glaſs. And that this Diſtinct Baſe, may be ſometimes *Leſs*, ſometimes *equal to*, and ſometimes *Greater than* the Object it ſelf.

Tab.

T.30.F.1. *Tab.* 30. *Fig.* 1. *a b c* is an Object exposed to the Glass *g l*, and projected in the Distinct Base *f e d*. Each Cone of Rays from the Object is made to *converge*, as *b l g* is formed into *g e l*. Let the Eye be placed at *k*. I say, first the Object appears *erect*. For the Rays from *a* the upper part of the Object tend to, and determin in the lower part of the Fund of the Eye. And the Rays from *c* the lower part of the Object tend towards the upper part of the *Retina*; and therefore the Appearance is *erect*, (by *Prop.* XXVIII. *Sec.* 4, 5.)

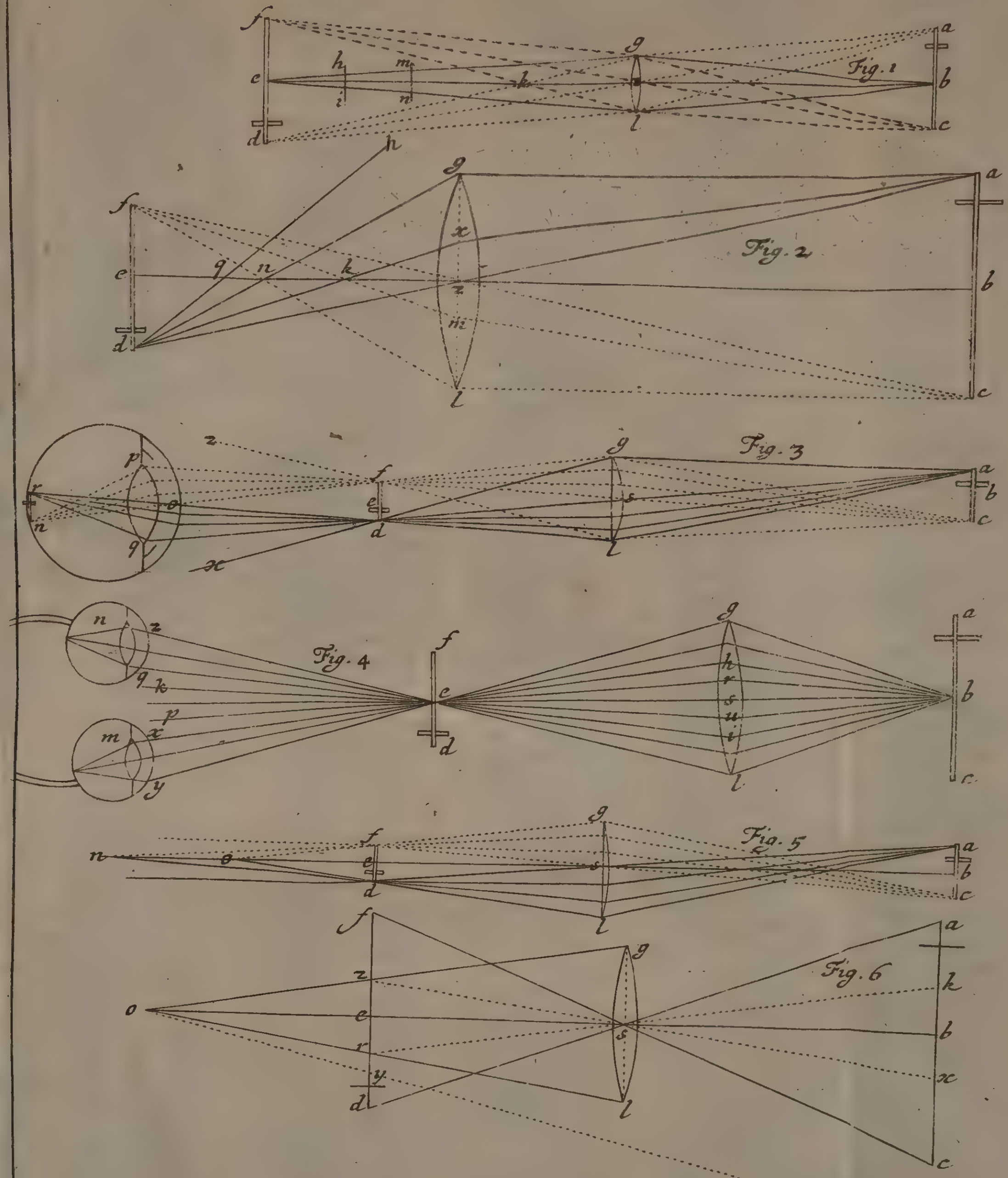
Moreover, the Eye being placed any where between the Glass *g l*, and the Distinct Base *f e d*, does not receive the Species of the Object *inverted*; for that only happens just in the Distinct Base it self. And on this account, 'tis manifest, that the Appearance to the Eye in this posture shall be *Erect*.

I say likewise, that the Apearance of the Object is *confused*. Let us conceive the Pupil of the Eye at *m n*; here it receives the Rays from the Point *b converging*; and therefore by *Prop. XXVIII.* the Representation of the Point *b* on the Fund of the Eye shall be *confused*. And so of the other Points in the Object, that is, of the whole Object. And if the Eye recede from the Glass, as to *h i*, the Vision is *more confused*, because the Rays are *nigher* converged.

Prop. XXXVII. Probl.

To determin the Optick Angle, or Apparent Magnitude, and the Visible Area of an Object placed as in the last; from these T.30.F.2. Data, *The Breadth of the Object* a b c (Tab. 30. Fig. 2.), *Its Distance from the Glass* z b, *The Glasses Power, The Glasses Breadth* g l, *And the Distance of the Eye from the Glass* k z.

(1.) From



Tab. 30. pag. 128.

(1.) From these *Data* by the foregoing Doctrine of Convex Glasses, 'tis easie to determine the Breadth of the Distinct Base. For by *Prop.* XXVI, XXVII. *As the Distance of the Object from a Convex Glass* : *To the Breadth of the Object* :: *So the Distance of the Image in the Distinct Base* : (which is easily obtained from the foregoing *Data*) *To the Breadth of the said Image*. That is (in the foresaid Figure) $bz : ac :: ez : df$. And As $ab : bz :: de : ez$. And consequently the Angle ezd is equal to the Angle azb; the Triangles ezd and azb being similar. And in the Right-angled Triangle abz, we have ab and bz, to find the Angle $azb = ezd$; which is the Angle, under which the Object would appear to the Eye placed just touching the Glass at z; supposing the Glass to be there of the least Thickness imaginable.

(2.) Let us now suppose the Eye at k. I say if the Rays from the single Points a, c, in an Object be converged by the Convex Glass gl to any other Points d, f; the Object shall appear under the Optick Angle xkm, as do the *Apices* of the Pencils from the Extreme Points of the Object, which is $dkf = xkm$. Vid. *Gregorii Opt. Promot. Prop.* XLV.

(3.) Wherefore in the Right-angled Triangle ekd, we have ek and ed, To find the Angle $ekd = zkx$, which is the Semi-Optick Angle to the Eye at k. In like manner may we find the Optick Angle at any other given Station of the Eye, as at n, $end = gnz$ being equal to half fnd.

(4.) As to the *Visible Area* of the Object, 'tis determin'd as before, *Prop.* XXXV. and *Schol.* I shall only add, That if the Eye be at q, and the Line dq being drawn, and produced towards h, it does not fall upon the Glass gl; then the Eye at q shall not see through the Glass the Point a, the *Apex* of whose Pencil is in the Point d. And from hence we may perceive, that at k, we may see through the Glass a *greater* space than the Area of the Object abc. At n we can but *just* see

S the

the *whole* Object *abc*; and at *q* we cannot ſee the extreme Point *ac*. By which the 81 and 82 *Prop.* of *Kepler's Dioptricks* are manifeſt.

(5.) If the Glaſs have any Thickneſs conſiderable, ſee *Prop. XXXI.* concerning the *Locus Objecti*, Sec. 7.

Scholium.

If we firſt determine the *Viſible Area* by *Prop. XXXV.* the Breadth of the Object need not be one of the *Data* for determining the *Optick-Angle.* For the Breadth of the Object is found by obtaining the *Viſible Area.*

Prop. XXXVIII.

An Object being more diſtant from a Glaſs than its Focus, and the Eye placed in the Diſtinct Baſe, the Object appears moſt confuſed.

By *Prop. XXXVI.* 'tis manifeſt, that the nigher the Eye approaches the Diſtinct Baſe, the more *confuſed* the Object appears; becauſe the Rays from each particular Point, do fall on the Eye *cloſer Converged*, and more orderly ſeparated by the Glaſs from the Rays of adjacent Points. But in the Diſtinct Baſe, the Rays are moſt of all Converged, and moſt orderly united in their proper Points, each with the Rays flowing from the ſame Point in the Object. Wherefore the Eye is there in the greateſt Confuſion.

Moreover (by *Prop.* XXVIII. *Sec.* 3.) 'tis requiſite to Diſtinct Viſion, that the Rays from the ſeveral Points in the Object, fall on the Eye altogether *confuſed* thereon (as they do on the Glaſs in the Hole of a dark Room), that the Coats and Humors of the Eye may refract them, and bring them together,

ther, diſtinctly painting the Image on the *Retina*. But when the Eye is in the Diſtinct Baſe, all this is fruſtrated.

Prop. XXXIX.

The Object being more Diſtant from the Glaſs than the Focus; and the Eye further from the Glaſs than the Diſtinct Baſe, begins to perceive the Object inverted; and, at a proper diſtance, Diſtinct.

(1.) This is manifeſt by *Tab.* 30. *Fig.* 3. wherein *a b c* is an Object, *g s l* the Glaſs projecting the Diſtinct Baſe *f e d*, *o* is the Eye. And becauſe half the Pencils of Rays that flow from each of the extreme Points of the Object *a* and *c*, proceeding forwards from the Points *d* and *f* in the Diſtinct Baſe, after they croſs in the aforeſaid Points *d* and *f*, by reaſon of their too great Divergence, do eſcape the Eye. Therefore, for avoiding Perplexity in the Scheme, I have only expreſſed the Half Cones *s a l*, *s d l*, and *s c g*, *s f g*, which incurr the Eye. T.30. F.3.

(2.) Wherefore we may obſerve, that the Diverging Rays *p f o* and *o d q*, falling diverging and confuſed on the Cryſtalline *p q*, are thereby collected, and made to concurr on the *Retina* in the Points *n* and *r*; and there paint the *upper* Point *a* of the Object on the upper part *r* of the Eyes Fund; and the *lower* Point *c* of the Object on the lower part *n* of the *Retina*. Therefore by *Prop.* XXVIII. *Sect.* 4, and 5. the Object appears *inverted*, becauſe the Image repreſented on the *Retina* is *Erect*.

(3.) Or more plainly thus, We may now conceive the Diſtinct Baſe it ſelf *f d* to be as it were an *inverted* Object (as here the Croſs turned with its head downwards). Therefore the appearance thereof to the Eye, ſhall ſeem *inverted*. For if the Eye perceive an *erect* Object, *erect*, it muſt needs perceive the Diſtinct Baſe, being an *inverted* Object, *inverted*.

And this may be laid down as a General Rule, That where an Object, or the Image, or Repreſentation of an Object, which is next the Eye, is *inverted*, there the Object ſhall appear *inverted*. And where the Object, or Image next before the Eye, is *erect*, the Object appears *erect*.

(4.) I ſay alſo, *At a proper diſtance the Inverted Appearance is Diſtinct.* For when the Eye is ſo far removed from the Diſtinct Baſe, as to be able to correct the Divergence of the Rays, that come to it from each Point in the Diſtinct Baſe, and to form them into correſpondent Cones determining their *Apices* on the *Retina*, the Viſion is Diſtinct, otherwiſe not.

Moreover there are two other *Phænomena* of this Poſture of Object, Glaſs, and Eye; which I ſhall here explain as follows.

(5.) If the Eye be moved *upwards* towards *z*, the Object ſhall appear to move *downwards*. If the Eye be moved downwards towards *x*, the Object ſeems to move *upwards*. To explain this, let us conſider the Point *c*; and how the appearance thereof is brought *to* and formed *in* the Eye. And we ſhall find, that in the Poſture of the Scheme, the Point *c* is expreſſed in the Eye, only by that parcel of its Rays that fall on the half Glaſs *s g*, for theſe are brought together in *s f g*, and flowing forward, become *p f o*: But the other parcel of Rays *s c l*, that fall on the half Glaſs *s l*, and are brought together in *s f l*, proceeding forwards, eſcape the Eye; for they go on in *p f z*. Wherefore the Point *c* is now repreſented through the half Glaſs *s g*, or ſeen ſomewhere between *g* and *s*. But the Eye moving upwards towards *z*, ſo that it miſs the Rays *p f o*, and receive *only* the Rays *p f z*; the Point *c* ſhall then be repreſented through the half Glaſs *s l*, or ſeen ſomewhere between *s* and *l*, and conſequently ſhall ſeem to move *downwards*. What is ſhewn of the Point *c*, may be conceived of the other Points in the Object, that is, of the whole Object.

And this *Phænomenon* is the ſtrongeſt Confirmation imaginable, that the *Locus Apparens Objecti*, in this Caſe, is in the *Diſtinct Baſe* (as is ſaid before, *Prop.* XXXI. *Sec.* 10.). For by raiſing the Eye, we depreſs the Point *f*; and by depreſſing the Eye, we raiſe the Point *f*. Wherefore the appearance of the Point *c* is at *f*. For the Point *c* in the Object ſeems to move contrary to the motion of the Eye.

From the foregoing Explication it follows, that if the Glaſs be moved *upwards*, the Object appears to move *upwards*. And if the Glaſs be moved *downwards*, the Object appears to move *downwards*; for the moving of the Glaſs *upwards* is the ſame as moving the Eye *downwards*; but moving the Eye *downwards*, makes the Object appear to move *upwards*; therefore the moving of the Glaſs *upwards* makes the Objects appear to move *upwards*.

Or thus, Becauſe moving the Glaſs *upwards*, the Diſtinct Baſe is moved *upwards* (for it follows the Glaſs), and the Diſtinct Baſe is in this caſe as it were the Object. (*Prop.* XXXI. *Sec.* 10.) Therefore by the Glaſſes motion *upwards*, the Eye ſhall perceive the Object moved *upwards*.

(6.) If with both Eyes we look at the inverted Appearance of an Object through a Convex-Glaſs, we ſhall perceive the Object double. Suppoſing the Eyes not very far removed from the Diſtinct Baſe. *Tab.* 30. *Fig.* 4. *a b c* is an Object projected by the Glaſs *g l* in the Diſtinct Baſe *f e d*; the T.30. F.4. Point *e* being correſpondent to the Point *b*. Let the Cone of Rays *g e l* flow forwards from *e*, and become *z e y*. Let the two Eyes *n*, *m*, meet the Cone of Rays *z e y*. I ſay the Eye *n* perceives the Point *e* (the Repreſentation of the Point *b*) by the Rays *z e q*, which flow forward from *i e l*; and conſequently the Eye *n* perceives the Point *b* or *e*, as it were, ſomewhere between *s* and *l*. In like manner, the Eye *m* perceives the Point *b* or *e* ſomewhere between *s* and *g*: Wherefore the two Eyes ſeeing the Point *b* or *e*, as it were, *in two places*, ſee it *Double*.

Double. What is ſaid of the Point *b*, may be underſtood of any other Point in the Object, and conſequently of the whole Object. And if we ſhut the *Right* Eye *m*, the *Left* Appearance of *b* vaniſhes; if the *Left* Eye *n*, then the *Right* Appearance of the Object vaniſhes.

But if the Eyes be removed very far from the Diſtinct Baſe *f e d*, their Interval continues the ſame; and the Rays *e k*, *e p*, Diverging continually, *they* ſhall at laſt fall on the Eyes; that is, *e k* on the Eye *n*, and *e p* on the Eye *m*. By which the Eye *n* ſhall perceive *e* as it were at *u* (*k e u* being one Right Line), and the Eye *m* ſhall perceive the ſame *e* as it were at *r* (*p e r* being one Right Line), whereby the Difference of the Places *r* and *u* becomes leſs than before, till at laſt the Eye be got at ſuch a Diſtance from the Diſtinct Baſe, that this Difference of the Apparent Places becomes ſo ſmall, that 'tis inſenſible to the ſight.

Quære. How Dr. *Briggs* would explain this *Phænomenon* of *Double Viſion* by his Theory of ſight. *Philoſoph. Collect. Numb.* 6.

Prop. XL.

T.30. F.5. *To determine the Optick-Angle or Apparent Magnitude, and the Viſible Area of an Object, placed as in the laſt, from theſe* Data; *The Breadth of the Object* a b c, (Tab. 30. F. 5.) *Its Diſtance from the Glaſs* s b, *The Glaſſes Power or Focal Length, The Glaſſes Breadth* g l, *And the Diſtance of the Eye from the Glaſs* o s.

(1.) It is ſhewn before (*Prop. XXXVII.*) how we may from theſe *Data* obtain the Breadth of the Diſtinct Baſe *f e d*, and its Diſtance from the Glaſs *e s*. Wherefore $os - es = oe$. Then in the Right-angled Triangle *o e f*, we have *o e* and *e f* to find the Angle *f o e*, which is the Semi-Optick Angle, under

der which the Eye perceives the Inverted Appearance of the Object. In like manner may we find the Optick Angle to any other ſtation as at *n*.

(2.) The Learned GREGORY in his *Opt. Promot. Prop.* 46. has a curious Theorem to this purpoſe, 'tis this, *The Rays from each ſingle Point in an Object* a b c, *being formed by the Glaſs* g s l *into the Diſtinct Baſe* f e d ; *and the Eye* o *being placed behind the Points of Concourſe* f e d; *the Image of each Point in the Object ſhall appear in the* Apex *of its Pencil* (That is, The Image of the Point *a* ſhall appear in the *Apex* of its Pencil at *d*, &c.), *And the Object ſhall appear Inverted; And under that Optick-Angle, as the* Apices *of the Pencils of the Extreme Points of the Object*; That is, under the Angle *f o d*.

(3.) The *Viſible Area* is thus determined, *Tab.* 30. *Fig.* 6. T30. F.6. The Object *a b c* is projected by the Glaſs *g s l* in the Diſtinct Baſe *f e d*. Let the Eye be at *o*; draw *o g*, *o l*; the Eye at *o* ſhall ſee no more of the Object, than what is repreſented between *z* and *r*, where the Lines *o g*, *o l*, interſect the Diſtinct Baſe. To determine the Points in the Object, that anſwer the Points *z*, *r*, in the Diſtinct Baſe. From the Centre of the Glaſs *s* (for I ſuppoſe it of no Thickneſs), draw *z s x*, *r s k*; the Point *k* anſwers *r*, and *x* anſwers *z*. Laſtly, from the foregoing *Data* to determine the Meaſure of *z r* ſay, As $so : sg :: eo : ez = \frac{1}{2} zr$; but the three firſt are known, therefore the fourth is known alſo. *Which was to be found.*

(4.) If the Line *o y* produced do not fall on the Glaſs, then that Point in the Object, whoſe *Apex* of its Pencil is at *y*, or which is projected in the Diſtinct Baſe at *y*, ſhall not be ſeen through the Glaſs by the Eye at *o*. Vid. *Schol. Prop.* XLVI. *Gregorii Opt. Promot.*

Corollary.

Corollary.

Hence it follows, That the *further* the Eye is removed from the Diſtinct Baſe, it perceives the *greater* Area of the Object; but can never ſee more than what is projected in the Breadth of the Glaſs; that is, $z\,r$ can never be *greater* than $g\,l$, nor ever equal thereto, unleſs the Eye o be at an infinite Diſtance.

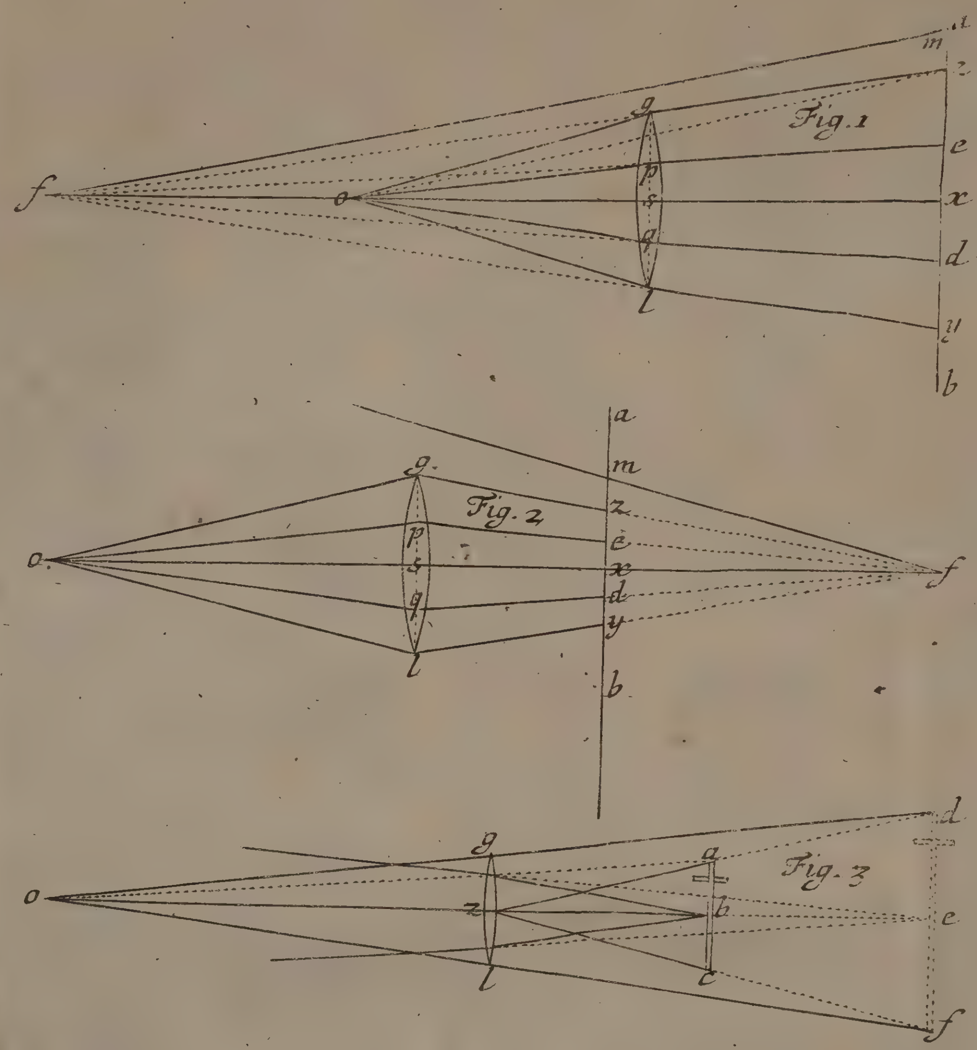
Prop. XLI.

An Object being placed nigher a Convex Glaſs than its Focus, and the Eye on t'other ſide the Glaſs, at any Diſtance within the Eyes power, ſees this Object Diſtinct and Erect.

I ſay, *Within the Eye's Power*; for 'tis ſhewn before in *Prop.* VII. and VIII. That when an Object is placed nigher a Convex Glaſs than its Focus, the Rays from each Point thereof, after paſſing the Glaſs, do Diverge, but not ſo much as if the Glaſs were away. Wherefore, tho the Eye looking at an Object within the Focus of this Glaſs, may be *nigher* the Object, than if the Glaſs were away, and yet ſee it *diſtinctly*; yet there is a *mean* to be obſerved; for even with the Glaſs it ſelf the Eye may be *too nigh* the Object, and not able to correct the Divergence of the Rays it receives.

But that an Object thus poſited appears *Diſtinct* to the Eye at a convenient Diſtance, is manifeſt; for the Rays from each Point proceed *moderately Diverging*, and ſo fall on the Eye. *Prop.* XXVIII. *Sec.* 7.

That an Object thus poſited is ſeen *Erect*, is evident from *Prop.* XXXII. and XXXIX.



Tab. 31. pag. 137.

Prop. XLII.

To determine the Visible Area, and the Optick-Angle or Apparent Magnitude of an Object placed as in the last, from these Data, *The Power or Focal Length of the Glass, The Distance of the Object from the Glass, The Distance of the Eye from the Glass, and the Breadth of the Glass.*

Tab. 31. *Fig.* 1. and 2. *a b* is an Object exposed to the Glass *g l*, *nigher* to it than its Focus. *o* is the Eye, which in *Fig.* 1. is supposed *nigher* the Glass than its Focus; and this is the First Case. But in *Fig.* 2. the Eye *o* is supposed *further* from the Glass than its Focus; and this is the Second Case. 'Tis shewn before (*Prop.* V. VIII.), how the *Respective* and *Imaginary* Foci *f* are determined in one and t'other Case. *Tab.* 32. *Fig.* 1, 2.

First let the given Breadth of the Glass be *g l*. By *Prop.* XXXV. we determine the Visible *Area* *z y*. Let us then suppose the given Breadth of the Glass to be *p q*. By the same *Prop.* XXXV. we determine the Visible *Area* *e d*.

As to the Optick-Angles *g o l* (supposing the Object *z y*) or *p o q* (supposing the Object *e d*) they are determined from the *Data*, by Plain Trigonometry of Right-angled Triangles, as before in *Prop.* XXXIV.

And we shall find by Calculation (as indeed 'tis evident by the very Inspection and Consideration of the Schemes), that the Collateral Parts *z e*, *d y*, of the Object *z y*, are much more magnifi'd (in respect of their Natural Appearance) by Broad Glasses formed on small Spheres, than the Middle Parts *e x*, *d x*; for the Angle *g o p* is the Optick-Angle, through the Glass, of the Part *z e*; and the Angle *p o s* is the Optick-Angle, through the Glass, of the Part *e x*; but the former exceeds the Natural Optick-Angle much more than the latter.

As to the Natural Optick-Angle of the Object zx, were the Glaſs away; draw zo directly (*Fig.* 1.), then in the Right-angled Triangle zxo, we have zx and xo to find the Angle zox, which is the Natural Optick-Angle of the Object zx.

From hence it is, that by Broad Glaſſes formed on ſmall Spheres, the Extreme Parts of *ſtrait* Objects, ſeem to be *incurved* and *bent*; as is manifeſt in the Caſe of the *Micrometer*, or Lattice of fine Hairs, ſtrained before the Eye Glaſs in a Teleſcope, for Meaſuring the Diameters of Objects. As *Pere Cherubin* complains in his *Dioptrique Oculair.* *Part.* III. *Sec.* 7. *Chap.* 1. *pag.* 239. but underſtood not the reaſon. Of this we may make Experiment, by looking with a very Convex Glaſs at two Parallel Lines drawn pretty cloſe on a Paper.

Schol. 1.

If inſtead of the Glaſſes Breadth, we have the Breadth of the Object ed given, we may eaſily determine the Breadth of the Glaſs, through which this Portion of the Object is ſeen: For from the Point f, draw fe, fd, theſe interſect the Glaſs in p, q. I ſay this is the Portion of the Glaſs, through which ed is viſible. And if the Line fm, being produced, do not fall on the Glaſs, the Point m in the Object is not viſible through the Glaſs gl to the Eye at o.

Schol. 2.

This XLII. *Propoſition*, as it relates to the Optick-Angle, may be ſolved an other way by Determining the *Locus Objecti.* But then, to the former *Data* in that *Propoſition*, inſtead of the *Glaſſes Breadth*, we muſt add the *Breadth of the Object.*

T.32. F.1. Wherefore, *Tab.* 31. *Fig.* 3. Let the *Locus* of the Object abc be determin'd def, by *Prop.* XXXI. *Sec.* 7. Let the Eye

Eye be at o, draw do, fo directly; if the Right Line do do not fall on the Glaſs, the Eye at o cannot perceive through the Glaſs the Point of the Object a (Vid. *Prop.* XXXVII. *Sec.* 4. *Prop.* XL. *Sec.* 4. & *Schol. Prop.* XLVI. *Gregorii Opt. Promot.*). But if do, fo, fall on the Glaſs; I ſay the Angle dof is the Angle under which the Object abc appears to the Eye at o through the Glaſs. *For the Object* a b c *is ſeen through the Glaſs under the ſame Angle, as the Image* d e f *would be ſeen without the Glaſs* (by *Prop.* XXXI. *Sec.* 7.). Now becauſe we have ab and bz given, and ez found, we may find alſo ed; for $bz : ba :: ez : ed$. Then in the Right-angled Triangle doe, we have de and eo $(=ez+zo)$ to find the Angle $doe = \frac{1}{2}dof$. Draw then ao directly; the Angle aob is the Natural Optick-Angle, under which the Object ab would appear to the Eye without the Glaſs. Now having ab and bo $(=bz+zo)$ we may find the Angle aob; the Difference between which and the Angle doe (that is doa) ſhews how much the Object *is magnified* by the Glaſs.

I. *Example of the Firſt Caſe computed by the Method in* Prop. XLII. *Tab.* 31. *Fig.* 1. T.31.F.1.

$$\left.\begin{array}{l} \text{Glaſſes Focus} = \text{— — — — — —}\ 4637 \\ \text{Diſt. of the Eye from the Glaſs} = so = 2500 \\ \text{Diſt. of the Obj. from the Glaſs} = sx = 2721 \\ \tfrac{1}{2}\ \text{Breadth of the Glaſs} = \tfrac{1}{2}gl = sg = 145 \end{array}\right\}\ \textit{Data.}$$

$$\left.\begin{array}{l} \tfrac{1}{2}\ \text{Viſible } \textit{Area} = \text{— — — — —}\ zx = ? \\ \tfrac{1}{2}\ \text{Optick-Angle} = \text{— — —}\ \angle gos = ? \end{array}\right\}\ \textit{Quæſita.}$$

Firſt ſuppoſing the Point o a Radiating Point,
By *Prop.* VIII. As Focus Glaſs $-so$: to Foc. Glaſs $:: so : fs$
That is in Numbers — — — $2137 : 4637 :: 2500 : 5425$

Then — — — — $fs : sg :: fx\ (=fs+sx) : zx$

That is $5425 : 145 :: 8146 : 217 = xz$ the Viſible *Area* in this Caſe according to the *Data.*

Secondly, To find the Optick-Angle gos, under which this $zx=217$, appears; ſay,

so : Rad. :: sg : Tang. $\angle gos = 3^{\circ}\ 19'\ 10''$.

The foregoing *Example* calculated according to *Schol.* 2. T. 31. F. 3. *Prop.* XLII. But the Letters refer to *Tab.* 31. *Fig.* 3.

Glaſſes Focus = — — — — — — — — — 4637
Diſt. of the Eye from the Gl. $= zo = 2500$
Diſt. of the Obj. from the Gl. $= zb = 2721$
½ Breadth of the Object = — — $ab =$ 217
} *Data.*

Diſtance of the Imaginary Focus $ze = ?$
Breadth of the Imaginary Focus $= de = ?$
Optick-Angle — — — — — — $\angle doe = ?$
} *Quæſita.*

Firſt, Suppoſing the Point b a Radiating Point, to determine the Imaginary Focus at e, by *Prop.* VIII. we muſt ſay,

As Focus Glaſs — zb	: Focus Glaſs	:: zb	:	ze
That is in Numbers 1916	: 4637	:: 2721	:	6585
Then by *Pr.* 2. 6. *Eucl.* zb	: ba	:: ze	:	ed
That is in Numb. 2721	: 217	:: 6585	:	525

Laſtly, As $eo\ (=ze+zo)$: Rad. :: de : Tang. $\angle doe = 3^{\circ}\ 18'\ 30''$, which wants only 40″ Seconds of the former Calculation, by reaſon of the neglect of the Fractions.

II. *Ex-*

II. *Example of the Second Case calculated according to Prop.* XLII. *Tab.* 31. *Fig.* 2. T31. F.2

Glaſſes Focus = — — — — — — 4637		
Diſt. of the Eye from the Glaſs = so = 5000		
Diſt. of the Obj. from the Glaſs = sx = 2721	Data.	
½ Breadth of the Glaſs = ½ $gl = sg$ = 145		

½ Viſible *Area* = — — — — — zx = ?	Quæſita.
½ Optick-Angle = — — — $\angle gos$ = ?	

Firſt, Suppoſing the point o a Radiating Point, Then by *Prop.* V.

As so — Foc. of the Gl. : Foc. of the Gl. :: so : fs

That is in Numbers 363 : 4637 :: 5000 : 63870

Then — — — — fs : sg :: fx $(=fs-sx)$: zx

That is in Numbers 63870 : 145 :: 61149 : 138, being the Viſible *Area* in this Caſe, according to the *Data*.

Secondly, To find the Optick-Angle gos, under which this $zx = 138$ appears; ſay,

As so : Rad. :: sg : Tang. $\angle gos = 1°. 39'. 40''$

The foregoing II. *Example* calculated according to the Solution in *Schol.* 2. *Pr.* XLII. The Letters relate to *Tab.* 31. *F.* 3. T31. F.3.

Glaſſes Focus = — — — — — — — 4637	
Diſt. of the Eye from the Glaſs = zo = 5000	Data.
Diſt. of the Obj. from the Glaſs = zb = 2721	
½ Breadth of the Object = — — ab = 138	

Diſtance of the Imaginary Focus = ze	
½ Breadth of the Imaginary Focus = de	Quæſita.
Optick-Angle through the Glaſs = $\angle doe$	

Let

Let us ſuppoſe the Point b a Radiating Point. To determine the Imaginary Focus at e, by *Prop.* VIII. ſay,

As Foc. of the Glaſs $- zb$: Foc. of the Gl. :: zb : ze
That is in Numbers 1916 : 4637 :: 2721 : 6585
Then $- - - - - zb : ba :: ze : ed$
That is in Numbers 2721 : 138 :: 6585 : 334

Laſtly, As eo $(= ze + zo)$: Rad. :: de : Tang. $\angle doe =$ $= 1^\circ\ 39'\ 0''$, which by reaſon of the neglect of Fractions wants 40″ Seconds of the foregoing Calculation.

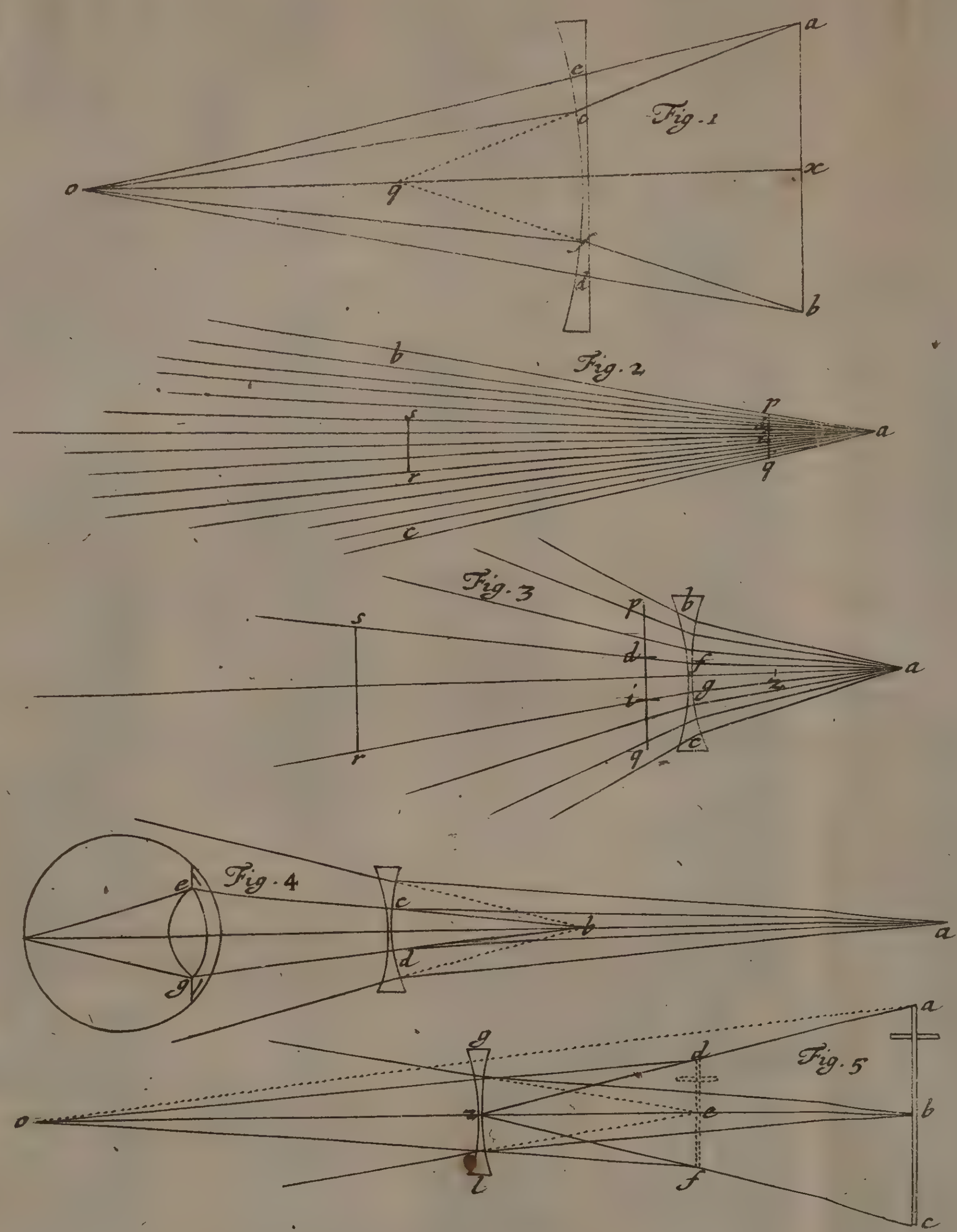
Of CONCAVES.

I Now proceed to the Conſideration of Viſion through *Concave* Glaſſes.

Prop. XLIII.

All Objects ſeen through Concave Glaſſes appear Erect and Diminiſh'd.

That the Object ſhall appear *Erect* is manifeſt; for 'tis before (*Prop.* XXXIX. *Sec.* 3.) laid down as a General Rule, *That where the Object, or Image that is next before the Eye, is Erect, the Eye ſhall perceive the Object Erect; and where Inverted, the Eye ſees it Inverted.* But Concaves have no *Diſtinct Baſe* (as Convexes have) wherein they repreſent the *Image* of the Object *Inverted*, and conſequently, wherever the Eye is placed behind a Concave Glaſs, it ſhall perceive the Object through it, in its Natural Poſture. That Concaves have no *Real Diſtinct Baſe*, is moſt plain from the Doctrine that foregoes relating to them: For a Diſtinct Baſe is cauſed by the Collection



Tab. 32. pag. 143.

lection of the Rays proceeding from a single Point in the Object, into a single Point in the Representation; but Concave-Glasses do not *unite*, but scatter and dissipate the Rays. 'Tis true indeed, a Convex-Glass may cause a Distinct Base, notwithstanding a Concave placed behind it or before it (as we see in *Prop.* XV. XVII, XVIII.); but then this Distinct Base proceeds not from the Concave, but from the Convex: For the Concave exerts its scattering power even in this Case; and it protracts the Uniting of the Rays to a greater Distance from the Convex, as is manifest from the fore-cited *Propositions.*

I say also, *The Appearance of Objects through Concaves is diminish'd.* *Tab.* 32. *Fig.* 1. *a x b* is an Object, *o* the Eye. Draw T32. F.1. *a e o, b d o,* directly strait from the Extremities of the Object to the Eye. The Angle comprised by the direct Rays *a e o, b d o,* that is, the Angle *a o b* is the Natural Optick-Angle. Let now the Concave-Glass *c f* be interposed; here, because the Rays *a e, b d,* would naturally concurr at *o*, now the Concave-Glass is interposed, their Concourse shall be protracted beyond *o*; wherefore the Eye at *o* shall not perceive the Extremities *a, b,* of the Object through the Glass by the Rays *a e, b d,* but by some *other Rays*, and these *other* Rays must either fall *without* (that is farther from the Axis *o x*, than) *a e, b d*; or they must fall *within a e, b d*: But they cannot fall *without a e, b d*; for if *a e, b d,* themselves be made by the Glass to concurr beyond the Eye at *o*; much more shall the Concourse of any other Rays that fall *without a e, b d,* be protracted beyond *o*; and consequently they cannot convey the Appearance of the Extreme Points *a, b,* to the Eye at *o*. Wherefore it remains, that the Rays that do this, must fall *within a e, b d.* Let us suppose these Rays to be *a c, b f,* which in their Natural direct Course, would concur at *q*, but by the Refractive Power of the Concave, are bent and made to proceed in *c o, f o*; their Concourse being protracted till they arrive

at

at the Eye in *o*. Here the Optick-Angle through the Glaſs is *c o f*, which is leſs than the Natural Optick-Angle *a o b*, and conſequently the *Appearance of the Object is diminiſh'd. Which was to be proved.*

Prop. XLIV.

Concerning the Diſtinct and Confuſed Appearance of Objects through Concaves; as alſo of their Faint or Obſcure Appearance.

We are here to remember what before is laid down, concerning the *Virtual Focus* of a Concave expoſed to *Parallel* or to *Diverging* Rays (in *Prop.* XI, XII, XIII. & *Corol. Prop.* XV.). We are likewiſe to take notice, that in Plain Viſion, when the Rays from any ſingle Point in an Object do not Diverge more than what the Refractive Power of the Coats and Humors of the Eye can correct; ſo that theſe Rays may be brought together in a Correſpondent Point on the *Retina*; then the Appearance of that Point is *Diſtinct*. For inſtance, *Tab.* 32. *F.* 2. Let the Point *a* diffuſe the Rays *a b*, *a s*, *a r*, *a c*. Suppoſe the Breadth of the Pupil were *p q*, and the Eye there placed; perhaps the Refractive Power of the Eye is not ſufficient to correct the Divergence of the Rays *a p b*, *a q c*. But if the Pupil (continuing of the ſame Breadth) recede to *r s*, then only the Rays *a s*, *a r*, fall into it; and theſe perhaps may be reduced by the Eye to determine in a Point on the *Retina*; becauſe they do not Diverge *ſo much* as the former. If therefore we ſuppoſe the Pupil in its former ſtation at *p q*, but now only to be of the Breadth *i d*, ſo as to admit only the Rays *a d s*, *a i r*, then the Point *a* may be ſeen as diſtinctly as at *r s*. This is manifeſt even by Experiment; for apply a Minute Object ſo near the Eye, that it appears very *confuſed*; then place before the Eye a very ſmall Hole made with

T.32. F.2.

a

a Pins end in a Paper; The Object ſhall now appear *Diſtinct*. For the Hole in the Paper ſerves to make the Pupil more *Narrow*. Which evidently proves what we have laid down.

The ſame may be ſhewn concerning Viſion through a Concave Glaſs *Tab.* 32. *f.* 3. *a* is a Radiating Point, whoſe Rays *a b*, *a c*, fall on the Concave *b c*. Theſe after paſſing the Glaſs Diverge more than before. Let *pq* be the Breadth of the Pupil, receiving the refracted Rays *a p*, *a q*, Theſe Diverge *ſo much*, that 'tis not in the power of the Eye to reduce them, and form them into an inward Cone, determining its *Apex* on the Fund of the Eye. But let the Pupil continue of the ſame Breadth, and recede to *r s*, where it may only receive the Rays *a d s*, *a i r*; and then perhaps the Eye may prevail to reduce theſe; becauſe they do not Diverge *ſo much* as the other Rays that fell on the Pupil at *p q*. Or otherwiſe, let the Pupil continue at *p q*, But let its Breadth be contracted to *d i*; ſo that it may receive no more Rays in this Poſture, than (continuing of its natural Breadth) at *r s*. Here likewiſe the Appearance of the Point *a* may be *Diſtinct*. Of this likewiſe we may make a moſt convincing Experiment: For take a Concave of a ſmall Sphere, and place it very near the Eye, and the Appearance of diſtant Objects through it is *Confuſed*. Remove it farther from the Eye, and the Appearance ſhall be more *Diſtinct*. But even in a Poſture where the Appearance is *Confuſed*, contract the Pupil by placing before it a ſmall Hole prick'd in a Paper, And the Object ſhall appear *more Diſtinct*, though *Obſcure*, which upon the removal of the Hole, ſhall be again *Confuſed*, though more *Lightſome*. *Vid. Kepleri Dioptr. Prop.* C. T.32. F.3.

Wherefore if we have the Diſtance of a Radiating Point from a Concave, and the virtual *Focus* of the Concave, and the Diſtance of the Eye behind the Glaſs; we may eaſily by the foregoing Rules (*viz. Corol. Prop.* XV.) find the virtual *Focus* of theſe Rays. As ſuppoſe in the ſame *Fig.* the Rays flowing

V from

from *a*, after paſſing the Glaſs, Diverge as if they came directly from *z*. And conſequently, if in Plain Viſion the Eye at *r s* be able to ſee diſtinctly the Point *a*, if it were removed nigher, as at *z*, then it ſhall be able to ſee the Point *a* diſtinctly through the Glaſs. But whether any Particular Eye be able to do this, is impoſſible to be known by Rule; The *Strength* and *Weakneſs* of Mens Eyes being *infinitely various*. And therefore one Man may ſee a Point at a certain Diſtance *diſtinctly* through a Glaſs, which Glaſs to another Man would render it *Confuſed*; As 'tis Plain in the Caſe of *Myopes*, or *Short-ſighted* Perſons.

As to the *Strong* or *Faint* Appearance of Objects through
T.32. F.3. Concaves; Becauſe the Concave (*Tab.* 32. *f.* 3.) ſcatters the Rays flowing from the Point *a*, inſomuch that now the Rays *a b*, *a c*, ſcape the Pupil *p q*, the Point *a* ſhall appear *more Faint* through the Glaſs than naturally. For *Strong* or very *Luminous* Viſion proceeds from a *greater* Quantity of Rays or Light entring the Pupil; And *Faint* or *Obſure* Viſion from a *leſs* Quantity of Rays.

Scholium.

We may here conſider the *different* Effects of Convex and Concave Glaſſes. For when a Point appears *Confuſed* through a Convex, 'tis by Reaſon of the *Too great Convergence* of the Rays that fall on the Eye. And therefore, Becauſe the farther from the Glaſs towards the Point of Concourſe the Rays that fall on the Eye Converge the *more*; the Appearance is through *Them* ſtill the *more Confuſed*.

But in Concaves; Becauſe the *Confuſed* Appearance of a Point proceeds from the *Too great Divergence* of thoſe Rays that fall on the Eye; and becauſe the farther the Eye is from the Glaſs, the Rays that fall on the Eye Diverge *leſs* than thoſe that fell upon it when it was nigher the Glaſs; Therefore through Con-

caves,

caves, the *farther* the Eye is from the Glaſs, the *more Diſtinct* is the Appearance, but ſtill *more Faint*.

PROB. XLV.

Concerning the Apparent Place of Objects ſeen through Concave Glaſſes.

Tab. 32. *f.* 4. If the Point *a* Radiate on the Concave Glaſs *c d*, and the Rays after paſſing the Glaſs Diverge as if they came directly from the Point *b*. And the Eye *e g* receive all or part of the Rays, and thereby perceive the Radiating Point *a*, the Point *b* is the *Locus Apparens* of the Point *a*. That is, the Point *a* is ſeen by the Eye, as if it were at *b*. T.32. F.4.

By which we may Obſerve, that the *Apparent Place* of Objects ſeen through Concaves is brought *nigher* the Eye. And hence 'tis manifeſt, why they help their Eyes, who are ſhort-ſighted, or can only ſee *nigh* Objects. For theſe Glaſſes make *diſtant* Objects ſeem *nigh*. *Vid. Dechales Dioptr. Lib.* 2. *Prop.* XXXIX.

Here alſo by the way we ſhall Note, That ſuppoſe a *Purblind* Perſon, that can Read diſtinctly or ſee Objects at the diſtance of a Foot from his naked Eye: A Concave Glaſs, whoſe virtual *Focus* is a Foot diſtant from it, makes ſuch a Perſon ſee *diſtant* Objects *diſtinctly*. Wherefore knowing the Diſtance at which a *Purblind* Perſon Reads diſtinctly, 'tis eaſie to aſſign him a proper Glaſs for his Eye, to ſee diſtant Objects. *Vid. ſecond Part. C.* 3.

Tab. 32. *f.* 5. Is in all things Correſpondent to the Doctrine laid down in *Prop.* XXXI. *Sec.* 7. and marked with the ſame Letters; ſo that the very Words of that Section may be applied to the Concave, as well as to the Convex. Only the *Imaginary* Focus *e* for the Concave is to be determin'd by *Prop.* XI, XII, XIII. XV. and *Corol. Prop.* XV. &c. I ſhall not therefore Repeat, but refer to that Section. T.32. F.5.

Prop. XLVI. Probl.

To determine the visible Area, and the Optick Angle or Apparent Magnitude of an Object seen through a Concave from these Data; The Power or Focal Length of the Glass; The Distance of the Object from the Glass; The Distance of the Eye from the Glass, and the Breadth of the Glass.

T.33. F.1. *Tab.* 33. *f.* 1. gzl is a Concave Glass, whose virtual *Focus* is given. Likewise gz the half Breadth of the Glass is given also. $a.b\,c$ is an Object, whose Distance from the Glass bz is given. And the Distance of the Eye o from the Glass, oz, is also given.

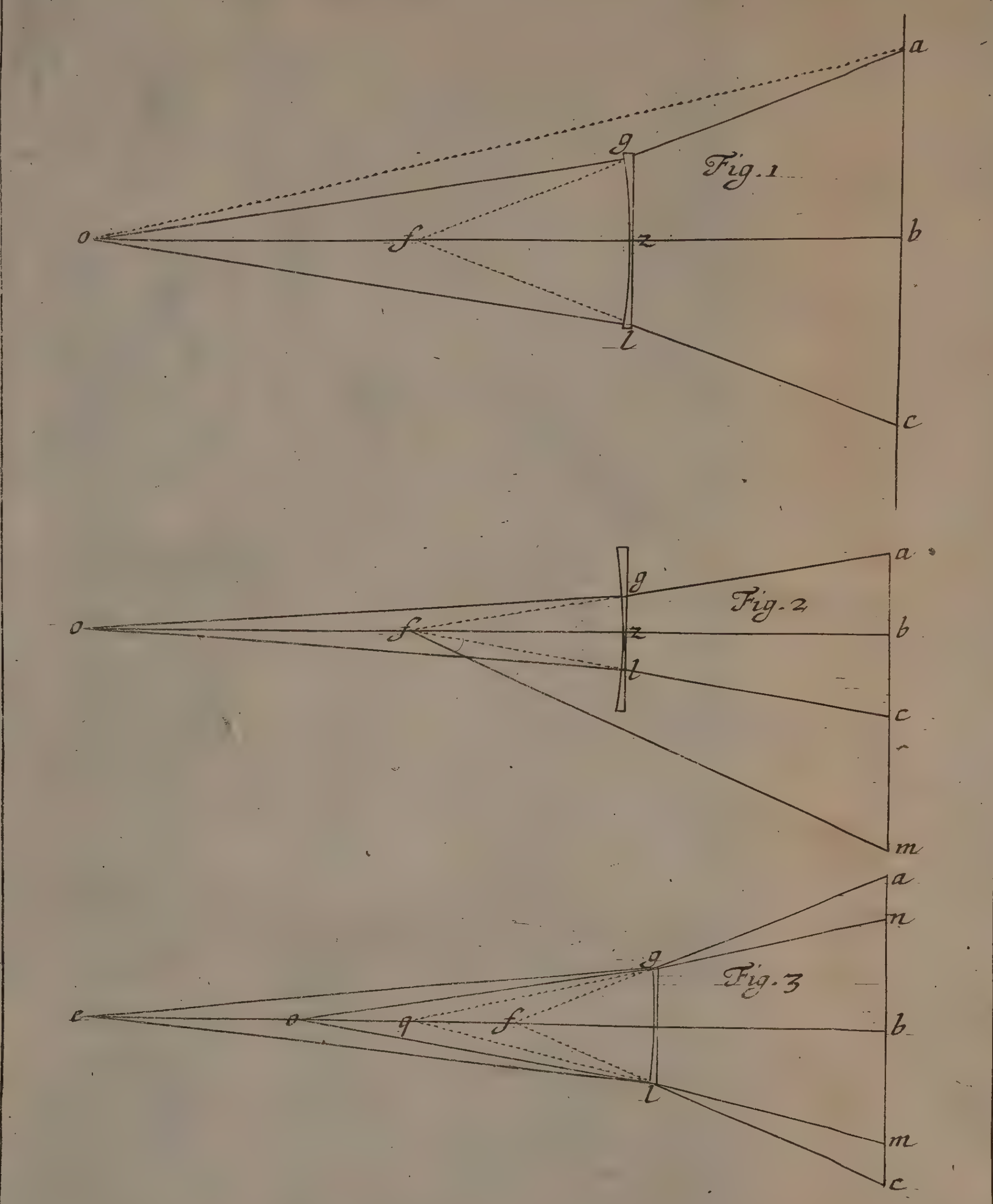
Let us now conceive the Point o an Object or *nigh Radiating* Point, sending its Rays og, oz, ol, on the Concave Glass; These (by the foregoing Doctrine of Concaves) after passing the Glass *Diverge* more than before their Entrance, and proceed in ga, lc, as if they came *directly* from a certain Point f. This Point f or the Line fz, is easily determin'd by *Prop.* XI, XII, XIII. XV. and *Corol.* thereof. Wherefore having fz, we may say, As $fz : zg :: fb\ (= fz + zb)\ ba$. Which is half the *Visible Area.*

As to the Optick Angle through the Glass, goz, 'tis easily obtain'd in the right-angled Triangle goz, having gz and zo.

Draw ao directly, The Angle aob is the natural Optick Angle, under which the Object would appear to the Eye, were the Glass removed; This we obtain in the right-angled Triangle abo by having ab and $bo = bz + zo$.

Scholium 1.

If instead of the Glasses Breadth, we have given the Breadth
T.33. F.2. of the Object abc (*Tab.* 33. *f.* 2.) We may easily determine the



Tab. 33. pag. 148.

the Portion of the Glaſs through which this Object is ſeen. For from the Point f, draw fa, fc, Theſe Interſect the Glaſs in gl; I ſay gl is the Portion of the Glaſs, through which the Object ac is viſible.

And if the Line fm being drawn do not fall on the Glaſs, the Point m in the Object is not viſible through the Glaſs to the Eye at o.

Scholium 2.

The foregoing Problem, as it relates to the Optick Angle, may be ſolved another way by determining the *Locus Objecti.* But then to the former *Data,* inſtead of the *Breadth of the Glaſs,* we cannot add the *Breadth of the Object.*

This is Evident from *Tab.* 32. *f.* 5. To which we may apply the Words of the Second *Scholium, Prop.* XLII. without further Repetition. T.32: F.5.

Example of a Calculation according to *Prop.* XLVI. apply'd to *Tab.* 33. *f.* 1. T.33. F.1.

Focus of the Glaſs $= - - -$ 4573.		
$\frac{1}{2}$ Breadth of the Glaſs $= gz =$ 500		*Data.*
Diſtance of the Object $= bz =$ 10000		
Diſtance of the Eye $= zo =$ 20000		
Half the Viſible Area $= ab = ?$		*Quæſit.*
Half Optick Angle $= goz = ?$		

Firſt by *Corol. Prop.* XV. $zo +$ Fo. Glaſs : zo :: Fo. Gl : zf

That is in Numbers — — 24573 : 20000 :: 4573 : 3722.

Then — — — — — $zf : gz :: bf\ (= bz + zf) : ab$

That is in Numbers — — — 3722 : 500 :: 13722 : 1843.

Laſtly in the right-angled Triangle goz, as zo : Rad : : gz : Tang. $\angle goz = 1^\circ\ 25'\ 50''$.

Example Calculated according to the Method Propos'd in this Second *Schol. Prop.* XLVI. but the Letters refer to *Tab.* 32. *f.* 5. T.32. F.5.

Focus

Focus of the Glaſs = — — — — — 4573.
Diſtance of the Object = bz = — — — 10000
Diſtance of the Eye = zo = — — — 20000 *Data.*
Half Breadth of the Object as Found in the foregoing Example = ab = 1843

Firſt by *Corol. Prop.* XV. bz + Gl. Focus : bz :: Gl. Fo : ze
That is in Numbers 14573 : 10000 :: 4573 : 3138
Then — — — — — — — — — — zb : ba :: ze : ed
That is in Numbers — — 10000 : 1843 :: 3138 : 578.

Laſtly in the right-angled Triangle deo, as oe ($= oz + ze = 23138$) : Rad :: ed : Tang. $\angle doe = 1^{\circ}\ 25'\ 50''$. agreeable to what was found by the immediately preceding Calculation.

I ſhall now ſhew, how by this Method of Demonſtrating the Properties, Effects, and Appearances of Glaſſes; Some of the noted Propoſitions in Dioptrick Writers may be eaſily proved. In which the Authors have been very *Operoſe*, and in ſome very *Obſcure*. And becauſe I will not interrupt the Series of my Propoſitions, I ſhall give them in the following Order.

Prop. XLVII.

The further the Eye is removed from the Concave Glaſs, the Object appears the leſs. Zahn Teleſcop: Fund. 2. Synt. 2. Cap. 5. Prop. *XXIII.* Dechales Dioptr. Lib. 2. Prop. *LII.*

T.32. F.5. This is manifeſt from *Tab.* 32. *f.* 5. For we are to Conſider the Image def, in the virtual Focus, as the Object, and as look'd at without the Glaſs. For the Lines do, fo, which determine the Optick Angle, are drawn directly to the Eye at o. Wherefore if we conceive the Eye o removed farther from the Glaſs; the Angle dof muſt needs decreaſe, and conſequently the Object appear *leſs*. What is here ſaid of the Concave may be apply'd to the Convex in *Tab.* 31. *f.* 3.

T.31. F.3.

Prop. XLVIII.

If a Concave Glaſs be removed from the Eye, ſo large an Area or ſpace of the Object cannot be ſeen through it.

This is the 97*th Prop.* of *Kepler's Dioptricks.* Wherein he ſeems to make no Diſtinction between *removing the Glaſs from the Eye, and removing the Eye from the Glaſs.* Whereas there is a very great Difference between both Motions to be conſider'd in *Dioptricks.* For in *moving the Glaſs from the Eye*, the Glaſs Approaches the Object, and the *Locus Apparens Objecti* is changed. But in *moving the Eye to or from* the Glaſs, the *Locus Apparens Objecti* never alters. *Prop.* XXXI. *Sec.* 7. And from his Proof of this Propoſition 'tis manifeſt, that he ſhould have expreſſed it thus,

If the Eye be removed from a Concave, ſo large a ſpace of the Object cannot be ſeen through it.

Wherefore inſtead of the foregoing 48*th.* Propoſition, I ſubſtitute this; Which is Evident by our foregoing Method from *Tab.* 33. *f.* 3. Wherein *a b c* is an Object, *g l* a Concave Glaſs, T33. F3. *o* the Eye *nigh* the Glaſs, *e* the Eye *more Diſtant* from the Glaſs. Let the Point of Divergence, anſwering to the Station at *o*, be *f* (determin'd by the foregoing Doctrine): draw *f g a*, *f l c*, directly; the ſpace viſible at *o* is *a c*. Now, when the Eye is removed *from* the Glaſs to *e*, the Point of Divergence ſhall alſo be removed from the Glaſs, and determin'd by what foregoes, as ſuppoſe at *q* (till the Eye be at an infinite Diſtance, and then the Point of Divergence is as far from the Glaſs as 'tis poſſible, *viz.* in the virtual Focus): draw *q g n*, *q l m*, directly; The ſpace viſible at *e* is *n m* leſs than *a c*. Which was to be Demonſtrated.

Or

T43. F. 1. Or otherwiſe. *Tab.* 34. *f.* 1. *d e f* is the *Locus Apparens Objecti a b c* through the Glaſs *g l*; the Eye is at *o*; *o g*, *o l* produced directly meet the Image in the Points *d*, *f*. Wherefore exactly the whole Object *a b c*, and no more, is ſeen through the Glaſs *g l* by the Eye at *o*. Let the Eye be removed to *q*; if we draw *q d*, *q f*, theſe fall not on the Glaſs, and conſequently the Extremities of the Object *a* and *c* ſhall not be perceived by the Eye at *q* through the Glaſs. (*Schol.* 1. *Prop.* XLVI.) Draw therefore *q g*, *q l*, and produce them directly to *n* and *m*; the Points in the Object *y x*, anſwerable to the Points *n*, *m*, are the *utmoſt Extremities* viſible through the Glaſs *g l* to the Eye at *q*. Which Points *y*, *x*, are eaſily determinable by what is laid down *Prop.* XL. *Sec.* 3. For from the Center of the Glaſs *z*, draw *z n y*, *z m x*, directly; *y*, *x*, are hereby determin'd.

As to this 48*th*. *Prop.* viz. *That if a Concave Glaſs be removed from the Eye, ſo large an Area or ſpace of the Object cannot be ſeen through it.* 'Tis needleſs to enlarge thereon, after our 46*th*. *Prop.*

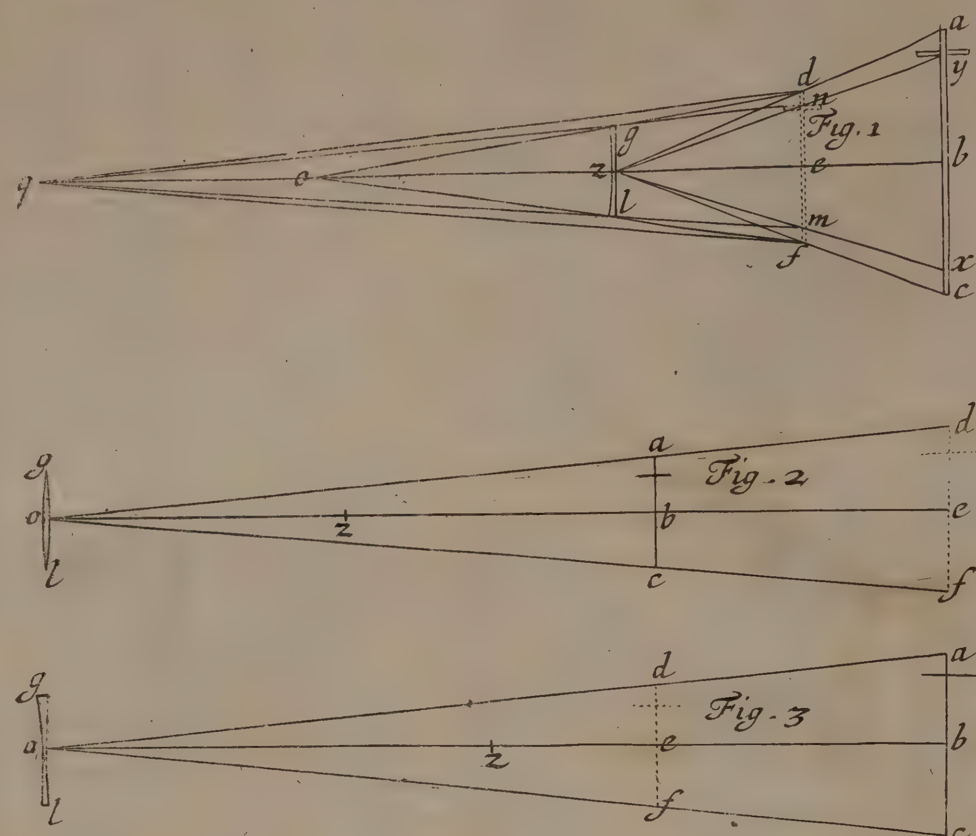
Prop. XLIX.

The farther a Concave Glaſs is removed from the Eye, The Objects are thereby the more diminiſh'd, as long as the Glaſs continues nigher to the Eye than to the Object.

This is the 98*th*. *Prop.* of *Kepler's Dioptricks*. We may find it alſo, In *Cherubin's La Dioptrique Oculaire* pag. 77. *Dechales Dioptrica* Lib. 2. Prop. *XXXVIII. Herigonii Dioptr.* Prop. *XXXI.* But in all of them obſcurely and looſely proved.

The Propoſition is univerſally True, as well in Convex as Concave Glaſſes, only with this Reſtriction in the Convex, *That the Object is to be nigher the Glaſs, than the Glaſſes Focal Length.* And then we may expreſs the Propoſition univerſally thus,

A Con-



Tab. 34. pag. 152

A Convex-Glaſs being equally Diſtant from the Eye and from the Object, renders the Appearance the moſt Magnified; and a Concave the moſt Diminiſhed, that That Diſtance of Eye and Object will allow.

For the Proof hereof I need offer no more than the following Calculations (inſtituted according to the Doctrine Precedent) wherein we ſhall find (*Tab.* 31. *f.* 3. and *Tab.* 32. *f.* 5.) the Angle *d o e*, (which is the Angle under which the Object appears through the Glaſs) in the ſecond Caſe (wherein the Glaſs is equally Diſtant from the Eye and Object) to be *greater* for the *Convex*, and *leſs* for the *Concave*, than the ſame Angle in either of the other Caſes. T.31. F.3. T.32. F.5.

And by the ſame Calculations we may likewiſe obſerve in the firſt and third Caſes, That when the Glaſs is *equally* removed from the *middle* towards the Eye, or towards the Object, the Appearance is *equally* magnified by the Convex, and diminiſh'd by the Concave; for we find in the firſt and third Caſes, the Angles *d o e* equal.

I ſay, this may be ſufficient for the Proof of the foregoing Poſition. But to this I ſhall add alſo this farther Conſideration. That by the preceding Doctrine, the magnify'd Appearances of Objects through Convexes, and their diminiſh'd Appearances through Concaves, being deduced from the *Loci Apparentes* of Objects through thoſe Glaſſes. We may eaſily conceive, That ſuppoſing the Convex-Glaſs *g l* (*Tab.* 34. *f.* 2.) or T.34. F.2.
Concave *g l* (*Tab.* 34. *f.* 3.) to touch the Eye *o*, and that the T.34. F.3.
Apparent Place of the Object *a b c* is *d e f*. The Eye perceives the Object under its own Natural Optick-Angle, neither magnify'd or diminiſh'd by one or t'other Glaſs.

Likewiſe if, in the ſame Figures, we conceive the Glaſſes (which are ſtill ſuppoſed of the leaſt thickneſs imaginable) to touch the Objects, the Object and *Locus Apparens* thereof are the *ſame*; and conſequently, the Optick-Angle in this Poſture can neither be *increaſed* or *diminiſh'd* by either of the Glaſſes.

Wherefore it remains, that seeing the Glasses do *not at all* Exert their Effects in either of the *Extremes*, that is, *either touching the Eye*, or *touching the Object*: And seeing they Exert their Effects *equally* being *equally* removed from the middle *z* between the Eye and the Object; It will follow, that they Exert their Effects *most powerfully* being placed just in the middle *z* between the Eye and Object, that is, the Convex by *Magnifying*, and the Concave by *Diminishing* the Appearance.

And thus much shall suffice concerning *single Glasses apply'd to the Eye*.

Here follow the Calculations.

T.31. F.3.

Calculation for the *CONVEX* Tab. 31. F. 3.

Focus of the Glass = 10,000 } Given the same
Breadth of the Object = *a b* = 1,000 } in all the Cases.

First Case.	*Second Case.*	*Third Case.*
Wherein the Distance between the Eye and the Glass *o z* is given greater than the Distance between the Object and Glass *b z*	Wherein the Glass is equally distant from the Eye and Object, that is, *b z* = *o z*.	Wherein the Distance between the Eye and the Glass *o z* is given less than the Distance between the Object and the Glass *b z*
Data { *b z* = 3,000; *o z* = 9,000	*Data* { *b z* = 6,000; *o z* = 6,000	*Data* { *b z* = 9,000; *o z* = 3,000
z e = 4,286; *e d* = 1,429	*z e* = 15,000; *e d* = 2,500	*z e* = 90,000; *e d* = 10,000
o e = *o z* + *z e* = 13,286; ∠ *d o e* = 6° 8′ 15″.	*o e* = *o z* + *z e* = 21,000; ∠. *d o e* = 6° 47′ 20″.	*o e* = *o z* + *z e* = 93,000; ∠. *d o e* = 6° 8′ 15″.

Calculation

Calculation for the CONCAVE Tab. 32. F. 5. T.32.F.5.

Focus of the Glaſs = 10,000 Breadth of the Object = ab = 1,000	Given the ſame in all the Caſes.

First Caſe.	*Second Caſe.*	*Third Caſe.*
Wherein the Diſtance between the Eye and the Glaſs oz is given greater than the Diſtance between the Object and the Glaſs bz.	Wherein the Glaſs is equally Diſtant from the Eye and Object, that is, $bz = zo$	Wherein the Diſtance between the Eye and the Glaſs oz is given leſs than the Diſtance between the Object and Glaſs bz.
Data $\begin{cases} bz = 3,000 \\ oz = 9,000 \end{cases}$	*Data* $\begin{cases} bz = 6,000 \\ oz = 6,000 \end{cases}$	*Data* $\begin{cases} bz = 9,000 \\ oz = 3,000 \end{cases}$
$ze = 2,308$ $ed = 0,769$	$ze = 3,750$ $ed = 0,625$	$ze = 4,737$ $ed = 0,526$
$oe = oz + ze = 11,308$ $\angle.doe = 3^\circ\, 53'\, 20''$.	$oe = oz + ze = 9,750$ $\angle.doe = 3^\circ\, 40'\, 0''$.	$oe = oz + ze = 7,737$ $\angle.doe = 3^\circ\, 53'\, 20''$.

Hitherto we have Treated of *ſingle* Convexes and Concaves, apply'd to the Eye. I proceed now to the *Combination* of Convexes with Convexes, and Convexes with Concaves; Wherein the Properties, Effects, and Appearances of *Teleſcopes*, and *Microſcopes*, of all kinds ſhall be declared.

Definitions.

The *Object-Glaſs* is the Glaſs next the Object.

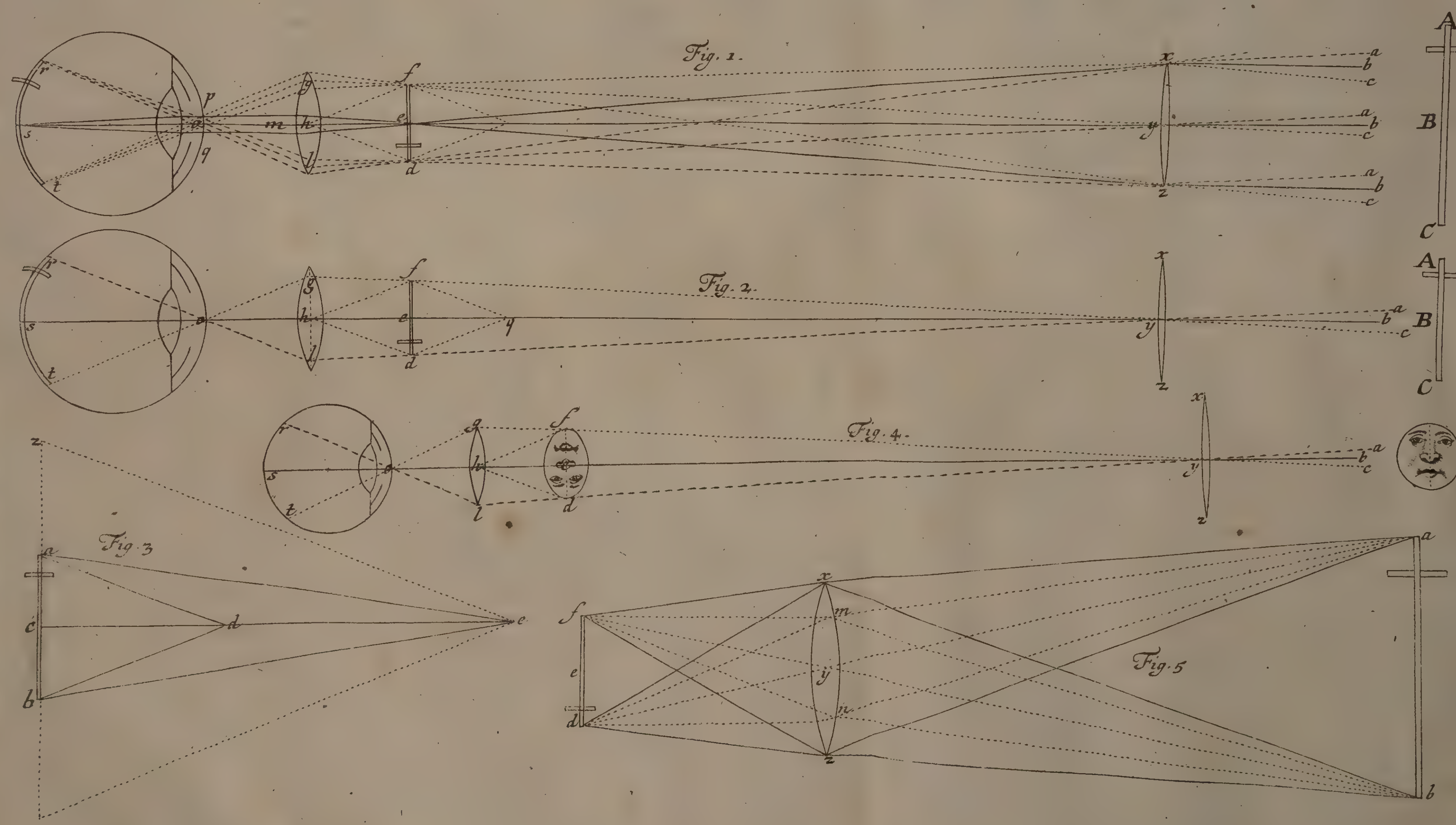
The *Eye-Glaſs*, ſimply ſo called, is the Glaſs *immediately* next the Eye. But if there be more than one, the *firſt Eye-Glaſs* is that *next* the Object-Glaſs; the *ſecond* is that *next* the *firſt*, &c.

Prop. L.

The Teleſcope, conſiſting of a Convex Object-Glaſs and a Convex Eye-Glaſs of a leſs Sphere or greater Convexity, is explained.

T.35. F.1. *Tab.* 35. *f.* 1. Let there be a Diſtant Object ſuch as *ABC*; From whoſe *higheſt* Point *A* let the Rays *a a a*, mark'd by the long Pricks, proceed. And from the *middle* Point *B*, let the Rays expreſſed by the continued Lines *b b b*, proceed. And from its *lower* Point *C*, the Rays mark'd by the round Pricks *c c c*. And ſo Rays from all the other Points in the Object. Theſe falling on the Object-Glaſs *x y z*, are formed thereby into the Diſtinct Baſe *f e d*. Let now the Eye-Glaſs *g h l* be placed as far diſtant from this Diſtinct Baſe *f e d*, as is the Focus of this Eye-Glaſs, that is; Let *e h* be the Focal length of the Eye-Glaſs *g h l* (And conſequently the Diſtance of the Glaſſes *y h* is the Aggregate of both their Focal lengths) And let the Eye *o* be placed as far diſtant (or rather a little more diſtant) from the Eye-Glaſs *g h l* as is the Focal length of the ſame Eye-Glaſs. I ſay the Eye ſhall perceive the Object *ABC*, *Diſtinct*, *Magnified*, and *Inverted*.

Firſt, I ſay the Object is ſeen *diſtinctly*. For the Rays from each Point, being made by the Object-Glaſs to Converge towards the Diſtinct Baſe, proceed forward from the Diſtinct Baſe Diverging, and ſo fall on the Eye-Glaſs. Thus the Rays



Tab. 35. pag. 156

Rays *bx*, *by*, *bz*, from the middle Point of the Object *B*, are made to Converge into *xe*, *ye*, *ze*; And croſſing at *e*, they flow forward, and fall on the Eye-Glaſs about *b* Diverging. Wherefore the Point *e* being now in the Focus of the Eye-Glaſs, the Rays that flow from it upon the Eye-Glaſs, after paſſing the Eye-Glaſs become parallel (*Prop.* VI.) and fall ſo on the Eye at *o*; By whoſe Coats and Humors they are refracted and brought together on the Point *s* on the *Retina* (*Prop.* XXVIII. *Sec.* 7.) there painting the lively Image of the Point *B* in the Object.

Secondly, In like manner may we conceive the Repreſentation of the Collateral Points: thus the Rays *ax*, *ay*, *az*, proceeding from the upper Point A of the Object, are by the Object-Glaſs *xyz* made to Converge towards the Diſtinct Baſe in *d*; From whence flowing forward they fall Diverging on the Eye-Glaſs at *l*; by which they are made to run parallel amongſt themſelves, and are bent towards the Focus at *o*; where falling on the Pupil parallel, they are by the Eye refracted and brought together in the Point *r* on the *Retina*. There painting the Repreſentation of the Point A in the Object.

Thirdly, That the Rays flowing *Diverging* from each particular Point in the Diſtinct Baſe *fed* are brought by the Eye-Glaſs to a *Parallelism* amongſt themſelves, is manifeſt from *Prop.* VI. And that the Rays from *f*, from *e*, and from *d*, are mixt and confuſed by the Eye-Glaſs in its Focus at *o* (or thereabouts) is manifeſt from hence, that we may conceive one Ray in the Cone of Rays *gf*, or in the Cone of Rays *zfx*, that runs parallel to the Axis *yehos*, or at leaſt that would run ſo parallel, if the Breadth of the Object-Glaſs in reſpect of the Breadth of the Eye-Glaſs will permit. This Ray, I ſay, by the known Properties of the Convex Eye-Glaſs, ſhall be refracted into its Focus at *o*; and all the other Rays of the ſame Cone *fg*, after paſſing the Eye-Glaſs, ſhall

be

be refracted and made to proceed parallel to that ſingle Ray, that is bent into the Focus *o*, becauſe the Point *f* is ſuppoſed in t'other Focus of the Eye-Glaſs. Wherefore the Rays from all the Points in the Diſtinct Baſe are confounded together about the Focus of the Eye-Glaſs.

Fourthly, Or otherwiſe I ſhall explain this matter thus; If we conceive *y*, a Radiating Point, ſending forth the Rays *y g*, *y h*, *y l*: And the difference between the Focal length of the Object-Glaſs and Focal length of the Eye-Glaſs to be very great (as ſuppoſe the Object-Glaſs to be twelve Foot, and the Eye-Glaſs three Inches) we ſhall find by *Prop.* V. the Point *o*, where the Rays *g o*, *l o*, croſs the Axis or Perpendicular Ray *h o*, to be very nigh the Focus of the Eye-Glaſs (*viz.* in our Suppoſition *h o* ſhall be 3,063 Inches) and let the Focal lengths of the Object-Glaſs and Eye-Glaſs bear whatever Proportion, the Point *o*, where *g o*, *l o*, ſhall croſs *h o*, may be determin'd by *Prop.* V. But then indeed the Rays that fall on the Eye *without* *g o*, or *within* it, do croſs the Perpendicular Ray *h o*, *farther* from the Eye-Glaſs, or *nigher* to the Eye-Glaſs than *g o* it ſelf; For they do not proceed from the ſame Point *y*; And *g o* is only the Refraction of the Ray *y g*. And therefore, unleſs the difference between the Focal lengths of the Object-Glaſs and Eye-Glaſs be *Conſiderable*, the Eye may move conſiderably *nigher to* the Eye-Glaſs, or *farther from* the Eye-Glaſs than the Point *o*, and not perceive any Alteration in the Appearance of the Object. But if the ſaid Difference be *Conſiderable*, (as it always is in Teleſcopes) the Eye can move but very little either *farther from* or *nigher to* the Eye-Glaſs than *o*, but it ſhall perceive a great Alteration in the Appearance (that is, in the Viſible *Area*) of the Object. As ſhall be manifeſted more plainly hereafter.

From all which it appears, That the Pupil of the Eye at *o*, being in the place of the *greateſt Confuſion*, where the Rays from

from all Points in the Object are mixt together, and the Rays from each single Point fall parallel amongst themselves; The Appearance of the Object must needs be *Distinct*. By *Prop.* XXVIII. *Sec.* 3.

Fifthly, I say likewise the Object is *magnified*. If the naked Eye were in the place of the Object-Glass at *y* (*Tab.* 35. *f.* 2.) T.35. F.2. the Object would appear to it under the Angle $ayc = fyd$. That is, the Object would appear to the Eye at *y*, under the same Angle as the Distinct Base *f e d*, were it an Object, would appear to the same Eye at *y*. Wherefore let us now consider the Distinct Base *f e d* as the Object: This appears to the Eye at *o* through the Eye-Glass under the same Angle *g o l*, as were the naked Eye viewing it at *h*, by *Prop.* XXXIII. Make *e q* equal to *e h*, and draw *h f*, *h d*, and *q f*, *q d*. The Angle *f h d* by *Prop.* XXXIV. is equal to *g o l*, the Optick-Angle through the Telescope, and the same Angle *f h d* is equal to *f q d*. But *f q d* is much greater than $fyd = ayc$ the Natural Optick-Angle to the Eye supposed at *y*. and yet much greater than the Natural Optick-Angle would be to the Eye at *o*. Because *o* is yet further from the outward real Object than *y* by the whole length of the Telescope. Wherefore the Object is *Magnified*.

Lastly, I say the Object appears *Inverted*. This is manifest from the very Scheme (*Tab.* 35. *f.* 1.) without farther Ex- T.35. F.1. plication. For the Object is *Inverted* in the Distinct Base, and the Eye-Glass does not return it again before it arrive at the Eye, but 'tis painted on the *Retina* r s t *Erect*; wherefore by *Prop.* XXVIII. *Sec.* 4, 5. the Object appears Inverted. See also *Prop. XXXIX. Sec.* 4, 5.

Scholium.

The *Locus Apparens* of an Object through this Glass is the Distinct Base *f e d* (*Prop. XXXI.*) as is manifest from this Ex-

Experiment. Stretch an Hair exactly in this Diſtinct Baſe, it ſhall appear as it were fixt to the very Object. *Vid. Chap.* 5. *Sec.* 3. of the Second Part.

Prop. LI. Lemma.

If the Eye directly approach to or recede from an Object; it ſhall be, as the Tangent of the Semioptick-Angle of one Station: to the Tangent of the Semioptick-Angle of t'other Station: : So (reciprocally) the Diſtance of the Eye from the Object in this latter Station: to the Diſtance of the Eye from the Object in the Former Station.

T.32.F.3. *Tab.* 35. *f.* 3. Let *a b c* be an Object, from whoſe middle Point *c* erect *c e* Perpendicular to *a b*. Let the firſt Station of the Eye be at *e*, and its ſecond Station at *d*; and draw *a d*, *a e*; I ſay therefore, As *c e*: to *c d*:: ſo Tangent of the Angle *a d c*: to the Tangent of the Angle *a e c*.

Demonſtration.

Produce *c a* infinitely towards *z*, draw *e z* parallel to *a d*. The Angle *z e c* is equal to the Angle *a d c* (29. 1. *Eucl.*) then *e c* being put Radius, *z c* is the Tangent of the Angle *z e c* = *a d c*; And *a c* is the Tangent of the Angle *a e c*. And it ſhall be, As *c e*: to *c d*:: ſo *z c*: to *a c*: (2. 6. *Eucl.*) which was to be Demonſtrated.

This is the 31 *Prop.* of *Gregorii Opt. Promot.* but there otherwiſe Demonſtrated.

Prop.

Prop. LII. Lemma 2.

If the Eye directly approach to, or recede from an Object, its apparent Bigness increases or diminishes, as the Tangents of the Semioptick-Angles at one and t'other Station.

This is manifest, for the Eye at *d* (*Tab.* 35. *f.* 3.) sees *a c* as big as the Eye at *e* would see *z c*, by *Prop.* XXVIII. *Sec.* 6. Because the Angles *a d c*, *z e c* are equal; and *Quæ sub æquali Apparent Angulo, Æqualia videntur.* So that the Eye advancing from *e* to *d*, sees *a c* as much bigger, as if, continuing at *e*, the Object *a c* had increased to *c z*. T. 35. F. 3.

Corollary.

From this and the last Proposition it follows, that if the Eye directly approach to or recede from an Object, its apparent Magnitude increases or diminisheth, as the Distances of one and t'other Station reciprocally, that is, the apparent Magnitude of *a c* to the Eye at *d*: is to the apparent Magnitude of *a c* to the Eye at *e* : : as *c e* : to *c d*.

Prop. LIII.

The apparent Diametral Magnitude of an Object viewed through the Telescope of Prop. L. *Is to the apparent Diametral Magnitude of the Object viewed by the naked Eye at the Station of the Object-Glass : : As the Focal length of the Object-Glass : to the Focal length of the Eye-Glass.*

This is the *great Proposition asserted* by most Dioptrick Writers, but hitherto *proved* by none (for as much as I know)

they offer indeed Experiments and Methods of Tryal to confirm the Truth thereof, but proceed no farther.

Vid. *Cherubin Diop. Oculaire Part. II. Prop. XXI. LIX. LX. LXII. Kepleri Dioptr. Prop. CXXIV. Galilei Nuncius Sidereus pag.* 12. *Edit. Lond.* 1653. *8vo.*

Honoratus Faber in his *Synopsis Optica Prop. XLIV.* for the Telescope consisting of a Convex Object Glass and Concave Eye-Glass; and in *Prop. XLV.* for the Telescope consisting of a Convex Object-Glass and Convex Eye-Glass, indeavours at something, which he calls a *Demonstration* of this Property. But whether that which he there offers will amount to clear Satisfaction, I leave to their Judgments, who shall Read him.

Dechales in *Prop.* LIV. *Lib. II. Dioptr.* thinks this Proposition so far from *Demonstrable*, that he takes it to be *False*; and says, He never met with any Demonstration thereof, that did not include manifest *Paralogisms*. Perhaps he may be right in this latter part of his Assertion; but the Reason he gives for his concluding it a False Proposition is manifestly Weak and Erroneous: And that on the account of an inartificial kind of Notion and Method, that he takes for Explicating the Magnifying of Telescopes; especially of the *Telescope* furnish'd with a Concave Eye-Glass, which he explains after his manner in *Prop.* LIII.

It were needless to inlarge in this matter, I shall therefore pass it over, and hasten to the *Demonstration* of this *Proposition*.

T.35. F.2. *Tab.* 35. *f.* 2. Let the Object-Glass xyz Project the Image of the Object ABC in the Distinct Base fed, ghl is the Eye-Glass, he the Focal length of the Eye-Glass, to which let eq be made equal. Draw hf, hd, and qf, qd. And let the Eye at o be placed in the Exteriour Focus of the Eye-Glass. The Rays yfg, ydl, are refracted by the Eye-Glass, and cross at o. Wherefore gol is the Angle under which the Object appears through the Telescope. But the Angle gol is equal to the

the Angle $f\,h\,d$ (as well by what foregoes in *Prop.* L. *Sec.* 3. & 5. as by *Prop.* XXXIV.) And the Angle $f\,h\,d$ is equal to the Angle $f\,q\,d$.

Wherefore were the naked Eye at y the Station of the Object-Glaſs, it would perceive the Object under the Angle $f\,y\,d$ $= a\,y\,c$. But now being armed with the Teleſcope, it ſees the Object under the Angle $f\,q\,d$. Let us take their halfs $f\,y\,e$, $f\,q\,e$; and conſider $f\,e$, half the Diſtinct Baſe, as the Object viewed by the naked Eye at the Stations y and q. I ſay (by *Lemma* 2.) The apparent bigneſs of the Object $f\,e$ at the Station q: Is to its apparent bigneſs at the Station y : : As the Tangent of the Angle $f\,q\,e$: To the Tangent of the Angle $f\,y\,e$. But (by *Lemma* 1.) the Tangent of the Angle $f\,q\,e$: Is to the Tangent of the Angle $f\,y\,e$: : As $e\,y$: To $e\,q$. Therefore the apparent bigneſs of the Object $f\,e$ at the Station q : Is to its apparent bigneſs at the Station y : : As $e\,y$: To $e\,q$: that is, As the Focal length of the Object-Glaſs : To the Focal length of the Eye-Glaſs. But the Angle $g\,o\,h$, under which the Eye ſees half the Object through the Teleſcope, is equal to the Angle $f\,h\,e$ or $f\,q\,e$. Therefore the apparent Diametral bigneſs of an Object viewed through a Teleſcope : Is to the apparent Diametral Magnitude of the Object viewed by the naked Eye at the Station of the Object-Glaſs : : As the Focal length of the Object-Glaſs : To the Focal length of the Eye-Glaſs. Which was to be *Demonſtrated*.

The ſame may be declared otherwiſe. Thus; *Tab.* 35. *f.* 2. Let us ſuppoſe the naked Eye at h to view the Object Inverted by means of the Diſtinct Baſe $f\,e\,d$; The Inverted Object ſhall appear under the Angle $f\,h\,d$ (by *Prop.* XL.) But the Eye at o through the Glaſs perceives the Inverted Image of the Object under the Angle $g\,o\,l$ equal to $f\,h\,d$, (by *Prop.* XXXIII.) and $f\,h\,d$ is equal to $f\,q\,d$, and conſequently (as in the foregoing Demonſtration) the Propoſition is manifeſt. T.35.F.2.

 I ſhall

I ſhall now mention the common Method for trying the Truth of this Propoſition by Experiment. Having the Focal length of an Object-Glaſs (for Inſtance) 144 Inches, and the Focal length of an Eye-Glaſs three Inches. A Teleſcope compoſed of theſe, ſhall make the apparent Diametral Magnitude of an Object: To the apparent Magnitude of the ſame Object viewed by the naked Eye :: As 144 : To 3 :: or 48 : To 1. Wherefore, ſuch a Glaſs is ſaid to Magnifie 48 times in the Diameter of the Object, and 2304 (= ſquare of 48) in the Surface of the Object. The Superficies of like Figures being to each other, as the Squares of their Diameters, or Homologous Sides.

Wherefore from a convenient Scale take one part, and therewith deſcribe a Circle; And from the ſame Scale take 48 parts, and deſcribe another Circle. Let theſe two Circles be cut out in Paper, or other Conſpicuous Material, and placed at three or four Foot from each other, on a Wall at ſuch a Diſtance as will require the length between the Glaſſes in the Teleſcope but juſt 147 (= 144 + 3) Inches, to ſhew theſe Objects diſtinctly; Then with one Eye through the Teleſcope obſerve the ſmaller Circle, and at the ſame time with t'other Eye naked look upon the greater Circle; theſe two Circles ſhall appear equal to both Eyes.

Perhaps it may be objected, That the Compariſon is not fair between both Appearances. For the Propoſition ſuppoſes the naked Eye at the Station of the Object-Glaſs; But this Experiment ſets the naked Eye Diſtant from the Object-Glaſs the whole length of the Teleſcope. This would be a material Objection againſt this Method of Tryal, were not the Diſtance of the two Circles from the Eyes vaſtly greater than the length of the Teleſcope; ſo that the Teleſcopes length may not bear any ſenſible Proportion thereto. And ſuch we ſuppoſe it in this Experiment, by advertiſing that this Diſtance is

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to be so great, that the Distance between the Glasses may be no longer than for viewing a Distant Object, *viz.* the just Aggregate of the Focal lengths of the Glasses, that is, 144 + 3 = 147 Inches, that is, $ye + eh = yh$.

I shall now give an Example of a Calculation according to this Proposition. Wherefore in *Tab.* 35. *f.* 4. let us take the Moon A B C for our Distant Object; and let us suppose its Diameter to subtend an Arch of a great Circle of Heaven of 30′ Minutes. Let the Ray ayd proceed from its upper Limb, bye from its Centre, cyf from its lower Limb. These cross in the Vertex y, or middle Point of the Object-Glass xyz; making the Angle $fyd = aye$, equal to 30′ Minutes. Let the Focal length of the Object-Glass ey be given twelve Feet = 144 Inches, or 144,00 Parts: And the Focus of the Eye-Glass he be given three Inches, or 3,00 such Parts. Let the Distinct Base, wherein the Image of the Moon is Projected by the Object-Glass be fed, and draw fh, dh. It is shewn before that the Angle goh is equal to the Angle fhe. T.35. F.4.

Wherefore in the Right-angled Triangle fey, we have $ey = 144{,}00$, and the Angle $fye = 15'$, to find $fe = 0{,}63$.

Then in the Right-angled Triangle feh, we have $he = 3{,}00$, and $fe = 0{,}63$, to find the Angle $fhe = 11^\circ\ 51'\ 40''$.

Wherefore the Semidiameter of the Moon, which by the naked Eye would be seen under the Angle of 15′ Minutes, is seen through this Telescope under an Angle of 11° 51′ 40″.

Let us now enquire, whether the Object appearing under an Angle of 15′ Minutes, and being afterwards made to appear under an Angle of 11° 51′ 40″, doth not thereby appear 48 times bigger than naturally. (for so much, by what foregoes, does this Glass Magnifie).

And for shewing this, let us imagine fe increased 48 times its length 0,63; And then inquire what Angle fye would be. Wherefore fe is now supposed = 30,24 = 48 times 0,63; Then

in

in the Right-angled Triangle fey, we have $fe = 30{,}24$, and $ey = 144{,}00$, to find the Angle $fye = 11^\circ\ 51'\ 40''$. Which ſhews that the Semidiameter of the Moon, being made by the Glaſs to appear under an Angle of $11^\circ\ 51'\ 40''$, is ſeen by the Eye as big, as if the Semidiameter of the Moon it ſelf were really increaſed 48 times, and viewed by the naked Eye. Which is the propoſed Deſign of this Calculation.

Corollary 1.

From hence it follows, That the ſame Object-Glaſs being at one time combined with an Eye-Glaſs whoſe Focus is 1. And at another time with an Eye-Glaſs whoſe Focus is 2. The firſt Teleſcope Magnifies twice as much as the latter.

Corollary 2.

Suppoſing two Teleſcopes of different lengths; If the Focus of the Eye-Glaſs of the ſhorter bears the ſame Proportion to the Focus of its Object-Glaſs, as the Focus of the Eye-Glaſs of the longer bears to its Object-Glaſs: Theſe two Teleſcopes Magnifie *equally*.

And hereupon perhaps it may be enquired, To what end then is all the Pains and Trouble in forming and managing Teleſcopes of 30. 40. 50. 100. 200. 300, &c. Feet; When Objects may be Magnified as much by ſmaller Object-Glaſſes, or Object-Glaſſes of ſhorter Focal lengths, combined with Proportional Eye-Glaſes?

I anſwer Firſt, That Object-Glaſſes of a ſhorter Focus will not bear proportionably Eye-Glaſſes of ſuch ſhort *Foci*, without coloring the Object and rendring it dark, as Object-Glaſſes of longer *Foci*. For Inſtance, let us ſuppoſe that an excellent Object-Glaſs of twelve Foot Focus will receive an Eye-Glaſs

Glaſs of no ſhorter a Focus than three Inches with Clearneſs and Diſtinctneſs. I ſay an Object Glaſs of 24 Foot Focus of the ſame Perfection ſhall receive an Eye-Glaſs of leſs than ſix Inches Focus with equal Clearneſs and Diſtinctneſs. And perhaps it may take an Eye-Glaſs of five or four Inches Focus. And then an Object-Glaſs of twelve Foot with an Eye-Glaſs of three Inches Magnifies but 48 times. But an Object-Glaſs of 24 Foot with an Eye-Glaſs of four Inches Magnifies 72 times, *viz.* $\frac{1}{3}$ more than the former, which is a great Difference, and of vaſt Advantage, when it may be obtained with the ſame Clearneſs and Diſtinctneſs. I confeſs the longeſt Teleſcopes do generally render the Objects more Dark and Obſcure, yet when ſhorter Glaſſes have proportionably as ſhort Eye-Glaſſes, and as cloſe Apertures, they are more Obſcure, than the longer Teleſcopes.

I anſwer Secondly, That the Image of the Moon or other Object in the Diſtinct Baſe of an Object-Glaſs of 24 Foot is twice as long as the Image in the Diſtinct Baſe of an Object-Glaſs of twelve Foot. And conſequently we ſhall not wonder, that the Picture in the former, ſhould be much more Diſtinct and Perfect, than in the latter ; As 'tis much more eaſie to repreſent every Feature and Line of a Face in a large Piece, than in a ſmall Piece of Miniature.

Corollary 3.

And if the Object-Glaſs be formed on a leſs Sphere than the Eye-Glaſs (as ſuppoſe the Object-Glaſs formed on a Sphere of ſix Inches Radius, and the Eye-Glaſs on a Sphere of twelve Inches Radius) hereby the Appearance of the Object ſhall be Diminiſhed. And the Appearance through the Glaſs ſhall be to the naked Appearance as ſix to twelve, or $\frac{1}{2}$ the Natural Appearance.

Scholium.

Scholium.

From hence it is manifeſt, how requiſite it is in relating any *Phænomena* obſerved by the Teleſcope (or even by the Microſcope) to mention not only the length of the Tube in general; But to ſpecifie the particular Focus of the Eye-Glaſs, as well as of the Object-Glaſs; as alſo the Aperture of the Object-Glaſs. For by this means, they that intend to obſerve the ſame *Phænomena*, may underſtand how to adapt their Teleſcopes proper for the Obſervation. This the *Learned and Ingenious Monſieur* Hugens *in his* Syſtema Saturnium *puts down exactly, pag.* 4. Where alſo we find this Paſſage. *Illud in Dioptricis Noſtris Demonſtratum invenietur, Speciei per Tubum viſæ ad eam quæ Nudo Oculo percipitur, hanc ſecundum Diametrum eſſe Rationem, quæ Diſtantiæ Foci in Exteriori vitro (Objectivo Scilicet) ad illam quæ in Inferiori ſive Oculari vitro eſt Foci Diſtantiam.* But hitherto we are ſo unhappy as to want that excellent Perſons Dioptricks. In the mean time, let that which I have given in the foregoing *Prop.* LIII. ſerve till a better be offered.

Prop. LIV. Probl.

To Determine the Angle received by a Teleſcope of the foregoing Combination. The Rule is, as the Diſtance between the Object-Glaſs and Eye-Glaſs : To half the Breadth of the Eye-Glaſs :: So Radius : To the Tangent of half the Angle received.

T.35. F.2. *Tab.* 35. *f.* 2. The Diſtance of the Glaſſes is *hy*. Let half the Breadth of the Eye-Glaſs be *gh*. Then, as *hy* : To *gh* :: So Radius : To the Tangent of the Angle *gyh*, which is half the Angle *gyl*, the Angle received. That is, the Eye at *o* ſhall perceive no more of the Object, than ſubtends this Angle before the Object-Glaſs.

Schol.

Scholium 1.

But if the Eye Approach *nigher to*, or recede *further from* the Eye-Glaſs *g h l* (*Tab.* 35. *f.* 1.) than its Focus at *o*, it shall perceive a leſſer *Area* of the Object, though what it ſees shall be as Diſtinct as at *o*. For let us ſuppoſe the Pupil of the Eye at *m*; The Rays *g o*, *l o*, do not enter the Eye, and conſequently the Points in the Object anſwerable to *f*, *d*, in the Diſtinct Baſe, ſhall not be viſible. The ſame may be conceived if the Eye recede farther from the Eye-Glaſs than *o*; becauſe all the Rays from the ſeveral Points in the Object are mixt together, and interſect at *o*, in the Focus of the Eye-Glaſs, and thence flowing forward they ſeparate and Diverge. But then the Eye at *m* receives the Rays, that do enter it, *Parallel* or at leaſt a very little *Diverging*; and conſequently the Viſion is *Diſtinct*. T35. F.1.

Scholium 2.

From hence alſo 'tis manifeſt, that the Angle received, or Viſible *Area* of the Object, is not increaſed or diminiſhed, by the greater or leſſer Aperture of the Object-Glaſs. For the Angle *g y l* continues the ſame, though the Object-Glaſs were all covered to the very middle Point *y*. All that is effected by this *greater* or *leſſer Aperture* is the more *Bright* or *Obſcure* Appearance of the Object. But of this more fully in the next Propoſition.

Prop. LV.

Concerning the Apertures of Object Glaſſes.

By the *Aperture* of a Glaſs I mean, that part of the Glaſs which is left open and uncovered. And this ought to be various

rious according as we would have more or leſs Light admitted. It alſo varies according to the various Focal lengths of the Object-Glaſſes. For a ten Foot Object-Glaſs ſhall bear a greater Aperture than an Object-Glaſs of one Foot; and a twenty Foot Glaſs yet greater than a ten Foot Glaſs.

But at what Rate or Proportion the Apertures of Glaſſes alter in reſpect of their lengths, is not yet well ſetled.

Monſieur Auzout, (*Phil. Tranſact. N.* 4. *P.* 55.) Tells us, that he finds, *That the Apertures, which Glaſſes can bear with Diſtinctneſs, are in (about) a Subduplicate Ratio to their lengths:* Or as the Square Roots of their lengths. Whereof he intends to give the Reaſon and Demonſtration in his *Dioptrica* (which we yet want.) But this Ingenious Perſon ſhould have told us, when he ſpeaks of the Apertures of Glaſſes, whether he deſigns them for Objects on the Earth or in the Heavens. And if in this latter, whether for the *Moon*, *Mars*, *Jupiter*, or *Venus*. For each of theſe Objects will require a different Aperture of the ſame Glaſs. Becauſe the Strength of their Light is different. For to view *Venus* there is requiſite a much ſmaller Aperture, than to view the *Moon*, *Saturn* or *Jupiter*.

However till ſome better Rule can be found for ſettling the Apertures of Object-Glaſſes (which at preſent I ſhall not pretend to) I ſhall here Preſent you with Mr. *Auzout*'s Table, as 'tis to be found in the fore-cited *Philoſophical Tranſaction*, *Numb*. 4. Noting only, that his Feet are *Pariſian Feet* (which is to the *London* Foot as 1068: to 1000) and each Inch (which is the $\frac{1}{12}$ part of his Foot) is ſubdivided into twelve Lines. For it had not been worth our Pains to have reduced the whole Table to our *Engliſh* Meaſure. *Vid. Tab.* 36.

I have ſaid before (*Schol.* 2. *Prop.* LIV.) That the Angle received, or Viſible *Area* of an Object, is not Increaſed or Diminiſhed by the greater or leſſer Aperture of the Object Glaſs; all that is effected thereby is the Admittance of more or leſs

Rays;

A TABLE of the Apertures of Obiect-Glaſses.

The Points put to ſome of theſe Numbers denote Fractions.

Length of Glaſſes. Feet	Inches	For Excellent ones. Inch	Lines	For good ones. Inch	Lines	For ordinary ones. Inch	Lines
	4		4		4		3
	6		5		5		4
	9		7		6		5
	0		8		7		6
1	6		9		8		7
2	0		11		10		8
2	6	1	0		11		9
3	0	1	1	1	0		10
3	6	1	2	1	1		11
4	0	1	4	1	2	1	0
4	6	1	5	1	3	1	.
5	0	1	6	1	4	1	1.
6		1	7	1	5	1	2
7		1	9	1	6	1	3
8		1	10	1	8	1	4
9		1	11.	1	9	1	5
10		2	1	1	10	1	6
12		2	4	2	0	1	8
14		2	6	2	2	1	9.
16		2	8	2	4	1	11.
18		2	10	2	6	2	1
20		3	0	2	7	2	2.

Lengths of Glaſſes. Feet	Inches	For excellent ones. Inch	Lines	For Good ones. Inch	Lines	For ordinary ones. Inch	Lines
25		3	4	2	10	2	4.
30		3	8	3	2	2	7
35		4	0	3	4.	2	10
40		4	3	3	7	3	.
45		4	6	3	10	3	2.
50		4	9	4	0	3	4.
55		5	0	4	3	3	6.
60		5	2	4	6	3	8.
65		5	4	4	8	3	10
70		5	7	4	10	3	.
75		5	9	5	0	4	2.
80		5	11	5	2	4	5
90		6	4	5	6	4	7.
100		6	8	5	9	4	10
120		7	5	6	5	5	3
150		8	0	7	0	5	11
200		9	6	8	0	6	9
250		10	6	9	2	7	8.
300		11	6	10	0	8	5
350		12	61	10	9	0	0
400		13	4	11	6	9	8

The feet here expreſs'd are Paris-feet, and a Line is the $\frac{1}{12}$ part thereof. The Paris-Foot is to the London-foot as 1068 to 1000

Rays; and conſequently the more Bright or Obſcure Appearance of the Object. *Tab.* 35. *f.* 5. Let the greater Aperture of the Object-Glaſs *x y z* be *x z*; And the leſſer Aperture *m n*. T.35. F.5.
a b is a Remote Object Projected in the Diſtinct Baſe *d e f*. The Cone of Rays *x a z* is Projected in the Cone of Rays *x d z*; And conſequently the Cone of Rays *m a n* (as being a part of the former *x a z*) ſhall be Collected at *d* in the Cone *m d n*. But then by this latter Aperture *m n*, all the Rays that fall on the outward Ring of the Glaſs, here expreſſed by *x a m*, *z a n*, are excluded, and conſequently the Point *d* ſhall not be illuſtrated with ſo much light as were the Aperture as wide as *z x*. And therefore (ſuppoſing an Eye-Glaſs behind this Object-Glaſs, ſo as to conſtitute a Teleſcope) ſuch a vigorous Light from each Radiating Point in the Object will not be brought into the Eye.

We have the exact Natural Reſemblance hereof in the Eye it ſelf: whoſe Pupil is contracted and dilated, according as the Light of an Object is more or leſs Intenſe.

Another Particular, wherein this Contraction or Dilatation of a Glaſſes Aperture is requiſite, is this: An Object may be ſo nigh a Glaſs that the Rays from each ſingle Point, falling upon the whole Breadth of the Glaſs, may Diverge ſo much that the Glaſs is not able to Correct the Divergence of thoſe Rays that fall towards its outward Borders, ſo as to reduce them to Determine or Unite in the Diſtinct Baſe with thoſe Rays, that fall nigher the middle of the Glaſs (as before is noted after *Prop.* III.) And then 'tis requiſite to contract the Aperture of the Glaſs, ſo as to exclude theſe Exorbitant Rays. A notable Experiment of this we may make by holding a Minute Object very nigh the Pupil of the Eye, the Object ſhall appear very *Confuſed.* But by applying a Paper with a ſmall Pin-hole before the Pupil, it ſhall reduce the Appearance to much more Diſtinctneſs than before.

Prop. LVI.

The Telescope Consisting of a Convex Object-Glass, and Three Convex Eye-Glasses is Explained.

I have shewn in *Prop.* L. *&c.* the Nature and Properties of the Telescope consisting of a Convex Object-Glass, and Convex Eye-Glass. I have shewn how the Image of the Object being formed in the Distinct Base of the Object-Glass *x y z*
T.35 F.1. (*Tab.* 35. *f.* 1.) by the Rays from each single Point of the Object there uniting, and flowing forward on the Eye-Glass, are thereby all collected together and confounded in its outward Focus at *o*.

T.37. F.1. Now (in *Tab.* 37. *Fig.* 1.) Let us Combine two other Eye-Glasses *k*, *l*, with the said Telescope of *Prop.* L. And place them so, that the Distance between the first Eye-Glass *h*, and the second Eye-Glass *k*, may be the sum of their *Foci.* Also that the Distance between the second Eye-Glass *k*, and the third Eye-Glass *l*, may be likewise the sum of their *Foci.* So that all the Glasses are Distant from the next adjacent Glasses, the sum of their *Foci.* Only here it may be noted, that to cause Distinct Vision through this Telescope, 'tis not absolutely necessary that the second Eye-Glass *k* be exactly Distant from the Focus *o* of the first Eye-Glass *h*, the just length of its own Focus; For it may be more or less; but then the Visible *Area*, and Magnified Appearance of the Object shall be altered. As will be manifest after we have explained this Glass to those that consider it.

It is then evident that the first Eye-Glass *h* mixes all the Rays from different Points in the Focus at *o*; from whence they flow forward, and fall upon the second Eye-Glass *k*, each parcel of Rays parallel amongst themselves: And by the Glass *k* are formed

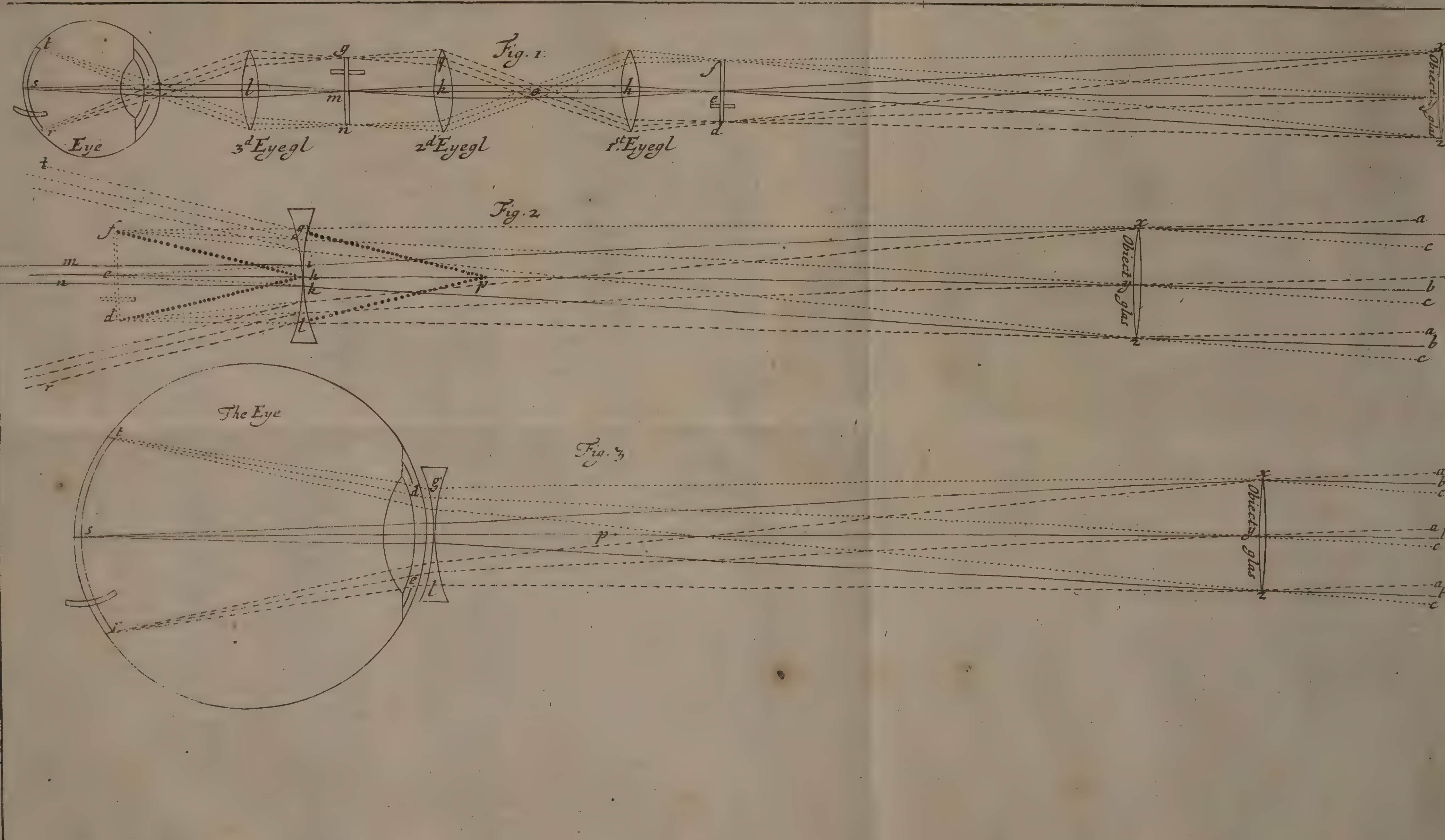


Fig. 1.
Eye
3d Eyegl
2d Eyegl
1st Eyegl
Object glas
Fig. 2
Object glas
Fig. 3
The Eye
Object glas

formed into the ſecond Diſtinct Baſe *g m n*. For we may imagine the middle Ray *o q* to proceed directly from *o*, the Focus of the Glaſs *k*; wherefore *o q* ſhall be refracted by the Glaſs *k*, and be made to run in *q g* parallel to its Axis *k m*. And then all the other Rays that are parallel to *o q* before they enter the Glaſs *k*, after they have paſſed the Glaſs *k*, do unite with *q g* in the Focus of the Glaſs *k*, and ſo the ſecond Diſtinct Baſe *g m n* is formed.

Or otherwiſe. We may conceive the Glaſs *k* to be the Cryſtalline of the Eye, looking through the Teleſcope of *Prop.* L. *h y*. And as the Cryſtalline in that Caſe does by means of the Glaſs *h* form in its Focus on the *Retina* the Image of the Diſtinct Baſe *f e d*; So may we imagine the Glaſs *k* to form in its Focus *g m n*, by means of the Glaſs *h*, the Image of the Diſtinct Baſe *f e d*.

Then from the ſecond Diſtinct Baſe *g m n* the Rays proceed as is expreſſed in the Scheme, and fall on the third Eye Glaſs *l*; By whoſe means we may imagine the Diſtinct Baſe *g m n* projected diſtinctly on the *Retina* of the Eye *r ſ t*, in the ſame manner as is ſhewn before in the Teleſcope of *Prop.* L.

And here we may obſerve that the Image on the *Retina r s t* is *Inverted*, therefore the Object ſhall appear *Erect*. *Prop.* XXVIII. *Sec.* 4, 5.

And we may conceive this ſort of Teleſcope as a double Teleſcope of *Prop.* L. For the Glaſſes *h*, *y*, make one Teleſcope; And the Glaſſes *k*, *l*, an other. And as the former by it ſelf *Inverts* the Object; ſo the latter with the former *Reverts* the *Inverted* Image; and conſequently makes the Object appear *Erect*. Yet it has been lately Publiſh'd in the *Journal des Scavans* 17. *Sept.* 1685. as a very difficult Problem in Dioptricks, why four Glaſſes in this kind of Teleſcope repreſent Objects *Erect*? I think I have ſolved this Problem to ſatisfaction, and my Anſwer is Publiſh'd *Num.* 187. of the *London Philoſophical*

phical Transactions. As also in the *Bibliotheque Universelle & Historique de l'Annee* 1688. *Tome* 3. *pag.* 329. But the Learned Author of this latter in his Translation has mistaken my Sense in one Particular. I shall therefore give it here again in the Second Part of this Work *Chap.* 2.

Dioptrick Problem solved in the Second Part of this Treatise *Chap.* 2.

Concerning the Magnifying Power of this Telescope; our *Prop.* L. will direct us how to Calculate it. For by that Proposition 'tis manifest, that if the several Eye-Glasses *h*, *k*, *l*, be of equal *Foci*, and the Distance between *h* and *k* be the sum of both their *Foci*, that then the apparent Diametral Magnitude of an Object through the Glass, is to the Diametral Magnitude viewed by the naked Eye, as the Focus of the Object-Glass, to the Focus of any one of the Eye-Glasses. But if the *Foci* of the Eye-Glasses *h*, *k*, *l*, be different, or the Distance between *h* and *k* different from the sum of their *Foci*, then to obtain the magnified Appearance, we must have Recourse to Calculation. Wherein the Cases are so very various, that to insist on them all would be very tedious, and infinitely laborious. For the Focus of *h* may be greater, equal, or less than of *k* or of *l*; and so of *k* than of *l* or *h*; and so of *l* than of *k* or *h*; As likewise the Distance between *k* and *h* may be infinitely varied. I shall therefore pass this over; A little Consideration of the several Varieties will make any of them plain, and shew how they may be easily Calculated by those versed in the foregoing Doctrine. For 'tis but considering, how the Distinct Base *g m n* is Projected, whether equal to, greater or less than the Distinct Base *f e d*; And how the Eye-Glass *l* conveys this Distinct Base *g m n* to the Eye.

In like manner, by *Prop.* LIV. may the Angle received, or Visible *Area* of an Object through this kind of double Telescope, be Determined, as in the single Telescope; Respect being had to the several Apertures, *Foci*, and Distances of the several Eye-Glasses *h*, *k*, *l*.

Scholium.

Scholium.

From the Explication of this kind of Teleſcope, and of that in *Prop.* L. may we eaſily apprehend the Theories of the various Combinations of Convex-Glaſſes in the Compoſitions of divers Teleſcopes of 3, 4, 5, 6, 7, 8, *&c.* Glaſſes.

Wherefore in explaining any kind of Teleſcope, we are firſt to obtain (by ſome Practical Rules to be deliver'd hereafter *Part* II. *Chap.* 4. *Sec.* 3.) the Focal length of each particular Glaſs by it ſelf. Then we are to conſider the Diſtances of each of theſe Glaſſes (as they lye in the Tube) from the Glaſſes before and behind it. Afterwards we are to conſider, where the Diſtinct Baſe, or Diſtinct Baſes are formed by theſe ſeveral Glaſſes, and how they are Projected as to Amplification or Diminution, which is eaſily found by the Doctrine before delivered. And then how the Eye-Glaſſes affect theſe Diſtinct Baſes; As how they Confound them, Rectifie them, Invert or Magnifie them.

For the Reſult of all is this, that the Rays from the ſeveral Images in the ſeveral Diſtinct Baſes, ſhall be Confounded on the Pupil of the Eye, in order to be rectified by the Cryſtalline, which (as has often been intimated) we may conſider as a Convex-Glaſs, whoſe Focus is on the *Retina.* And from a due Conſideration of the Premiſſes, it will appear why ſome Teleſcopes conſiſting of Convex-Glaſſes repreſent the Object *Erect*, others Inverted; in ſome *one* Glaſs is to be taken as *one*, in others two Glaſſes perform the Effect but of *one*.

And thus all the Combinations of Glaſſes expreſſed in the 8, 9, 10, 11, 12. *Iconiſmes* of *Zahn Teleſcop. Fund.* 2. *Syntag.* 3. *Cap.* 6, 7, 8, 9. are eaſily explained, with a thouſand other Varieties.

Prop. LVII.

The Telescope Composed of a Concave Eye-Glass, and Convex Object-Glass of a larger Sphere is Explain'd.

The Posture of the Glasses in this Telescope is this; The Distance of the Glasses is to be the *Difference* of their *Foci*, that is, the Concave Eye-Glass is to be placed so much nigher the Object-Glass than the Focal length of this Object-Glass, as is the virtual Focus of this Eye-Glass. And the Eye is to be placed as nigh the Eye-Glass as possible.

Note. I shall call the *virtual Focus* of the Concave, simply its *Focus*, it being well known that a Concave has no *other Focus*, but a *virtual Focus.*

I shall now shew, that through this Glass the Object appears *Distinct*, *Erect*, and *Magnify'd.* And shall shew the *Angle received* or *visible Area*, according to the several Postures of the Eye.

Only premising in this Proposition (as I have done in several former) that we suppose the Glasses of the least thickness imaginable; and especially the *Concave*-Glass in its middle Point is supposed of no thickness at all, but the two Surfaces to touch.

To explain this Glass the better, I shall express the Figure
T.37. F.2 very large, *Tab.* 37. *f.* 2. Wherein we shall first consider the Telescope it self separate from the Eye. Wherefore, let some Distant Object (as suppose a *Cross*) send the Rays *a a a* from its upper Point, *b b b* from its middle Point, and *c c c* from its lower Point. These falling on the Object-Glass *x y z* are formed thereby into the Distinct Base *f e d*. Let now the Concave-Glass *g h l*, be placed between the Distinct Base *f e d*, and the Object-Glass *x y z*, so far distant from the Distinct Base, as is the Focus of this Concave; that is, let *e h* be the virtual Focus

cus of the Concave Eye-Glaſs. Then the Rays (for Inſtance) from the middle Point, *x i*, *y h*, *z k*, falling on the Concave and Converging towards its Focus *e*, after paſſing the Glaſs become Parallel (*Corol. Prop.* XIII.) and run onwards in *i m*, *h e*, *k n*. The ſame may be conceived of the Rays from the Collateral Points; which Converge towards the Focus in *f* and *d*; *viz.* that theſe alſo after paſſing the Concave Eye-Glaſs, do proceed onwards Parallel amongſt themſelves.

But concerning theſe Rays from the Collateral Points, we muſt note alſo; That, as it is ſhewn before in *Prop.* L. *Sec.* 3, 4. Concerning the Convex Eye-Glaſs *g h l* (*Tab.* 35. *f.* 1.) that it brings the Rays of the Collateral Points *f g*, *d l*, Parallel amongſt themſelves into its Focus at *o*. So the Concave Eye-Glaſs *g h l* (*Tab.* 37. *f.* 2.) for the ſame Reaſons expreſſed before in *Prop.* L. *Sec.* 3, 4. *Mutatis Mutandis*, makes the ſeveral parcels of Rays from the Collateral Points, after paſſing it, to *Diverge*, as if they proceeded directly, from the Point *p*; *p h* and *h e* being equal, and each equal to the Focal length of the Concave Eye-Glaſs. Theſe things being fully conſidered; we proceed in the Explication of this Teleſcope. And therefore now let us apply the Eye thereto; And this alſo in a large Figure, *Tab.* 37. *f.* 3.

T.35. F.1. T.37. F.2. T.37. F.3.

I ſay firſt, the Object appears through this Glaſs *Diſtinct*; For by what foregoes the Rays from each ſingle Point, do fall on the Eye *Parallel* amongſt themſelves; And therefore each Point (by *Pr.* XXVIII.) is *diſtinctly* repreſented on the Fund of the Eye.

I ſay ſecondly, the Object appears *Erect*. For 'tis manifeſt by the Inſpection only of the Scheme, that the Rays flowing from the *lower* Point of the Object, are Terminated at *t* the *upper* Part of the *Retina*. And the Rays from the *upper* Point of the Object are Terminated at *r* the *lower* Part of the *Retina*. So that the Image is painted *inverted* on the *Retina*. And therefore by *Prop.* XXVIII. the Object appears *Erect*.

I ſay laſtly, the Object is *Magnified* by this Glaſs. For the Proof of this we are to remember, what foregoes in *Prop.* XXXVII. 'Tis there declared, that if an Object be Projected
T.37. F.2. by a Convex-Glaſs *x y z* (*Tab.* 37. *f.* 2.) in the Diſtinct Baſe *f e d*; And the Eye be placed any where between the Glaſs and the Diſtinct Baſe, as ſuppoſe at *h*, draw *f h*, *d h*, and the Object appears under the Angle *f h d*, which is much greater than *f y d*, the natural Optick-Angle. The ſame will hold, though we interpoſe the Concave-Glaſs *g l*; for the Ray *x g* from the Objects lower Point, that runs Parallel to the Axis *b y h e*, is Refracted into *g t*, as if it came directly from the Point *p*. And ſo the Ray *z l* from the Objects upper Point, that runs Parallel to the Axis *b y h e* is Refracted into *l r*, as if it came directly from the ſame Point *p*. If therefore we ſuppoſe the Rays *x g f*, *z l d*, Parallel to the Axis; then *g l* ſhall be equal to *f d*; And *p h* being by Suppoſition equal to *h e*, *p g t* ſhall be Parallel to *h f*; And *p l r* ſhall be Parallel to *h d*. And conſequently the Angle *g p l* ſhall be equal to the Angle *f h d*. Wherefore the Object through this Glaſs appears under the ſame Angle, as we may imagine the *Apices* of the Pencils *f*, *d*, would appear to the naked Eye at *h* (the Concave Eye-Glaſs being removed) And conſequently the Object appears *Magnified*.

The *Viſible Area*, or *Angle Received* by this Glaſs, is Determined by the *Aperture* or *Breadth* of the Eyes Pupil. For 'tis
T.37. F.3 manifeſt, from *Tab.* 37. *f.* 3. That if the Pupil *d e* of the Eye were not large enough to receive the Rays from the extreme Points of the Object, it would not perceive the *whole* Object through this Glaſs. Wherefore by the *Breadth* of the Pupil given, as alſo by the Focal Diſtances of the Object-Glaſs and Eye-Glaſs being given, we may eaſily obtain the *Viſible Area*, or *Angle Received*. For let us ſuppoſe that we find from theſe *Data*, that the whole Object Projecting the Diſtinct Baſe *f e d*
T.37. F.2 (*Tab.* 37. *f.* 2.) would be Projected in the breadth of the

Pupil,

Pupil, at the Diſtance *hy* from the Object-Glaſs. Then find the Angle *f y d* (as is eaſie from theſe *Data*, and the preceding Doctrine) and we have the *Angle Received.*

Corollary 1.

In the XXXI. *Prop. Sec.* 9. We have conſidered the *Locus Apparens* of an Object, Projected in the Diſtinct Baſe by a Convex-Glaſs, to the Eye placed between the Glaſs and Diſtinct Baſe. And if the *Affirmative* of the Quere, which I there propoſe, hold true; The *Locus Apparens* of the middle Point of the Object ſeen through the Glaſs of *Tab.* 37. *f.* 2. is at *p*. T. 37. F. 2.

Corollary 2.

If we ſuppoſe a Convex Eye-Glaſs, whoſe Focal length is equal to the Focal length *p h* or *e h* of this Concave Eye-Glaſs, apply'd (as directed in *Prop.* L.) to this ſame Object-Glaſs *x y z* in *Tab.* 37. *f.* 2. It would magnifie the Object equally with this Concave: And from hence it follows, that the preceding Propoſition LIII. concerning the Magnifying of a Teleſcope, may be apply'd to this ſort of Teleſcope furniſhed with a Concave Eye-Glaſs. But then the Advantage of the Teleſcope in *Prop.* L. beyond that of this Propoſition, is moſt ſignal in this particular, That it receives a very much greater Angle, or ſhews to the Eye a much greater *Area* of the Object. The *Area* in this being Determined by the breadth of the Convex Eye-Glaſs; But in that of a Concave Eye-Glaſs, the *Area* is proportioned to the breadth of the Pupil.

Corollary 3.

If the Eye recede from the Concave Eye-Glaſs of this Propoſition, it perceives not ſo great a ſpace of the Object. This

is manifest only by Inspection of the two Schemes (*Fig.* 2, T. 37. F. 2, 3. and 3. *Tab.* 37.) For if the Breadth of the Pupil *d e*, be but just sufficient to receive the Rays from the extreme Points of the Object, after they have passed the Eye-Glass, and are thereby so much Divaricated; if the Eye recede, the Pupil shall not be broad enough; and consequently shall see less of the Object. Just as in the Telescope of *Prop.* L.

Corollary 4.

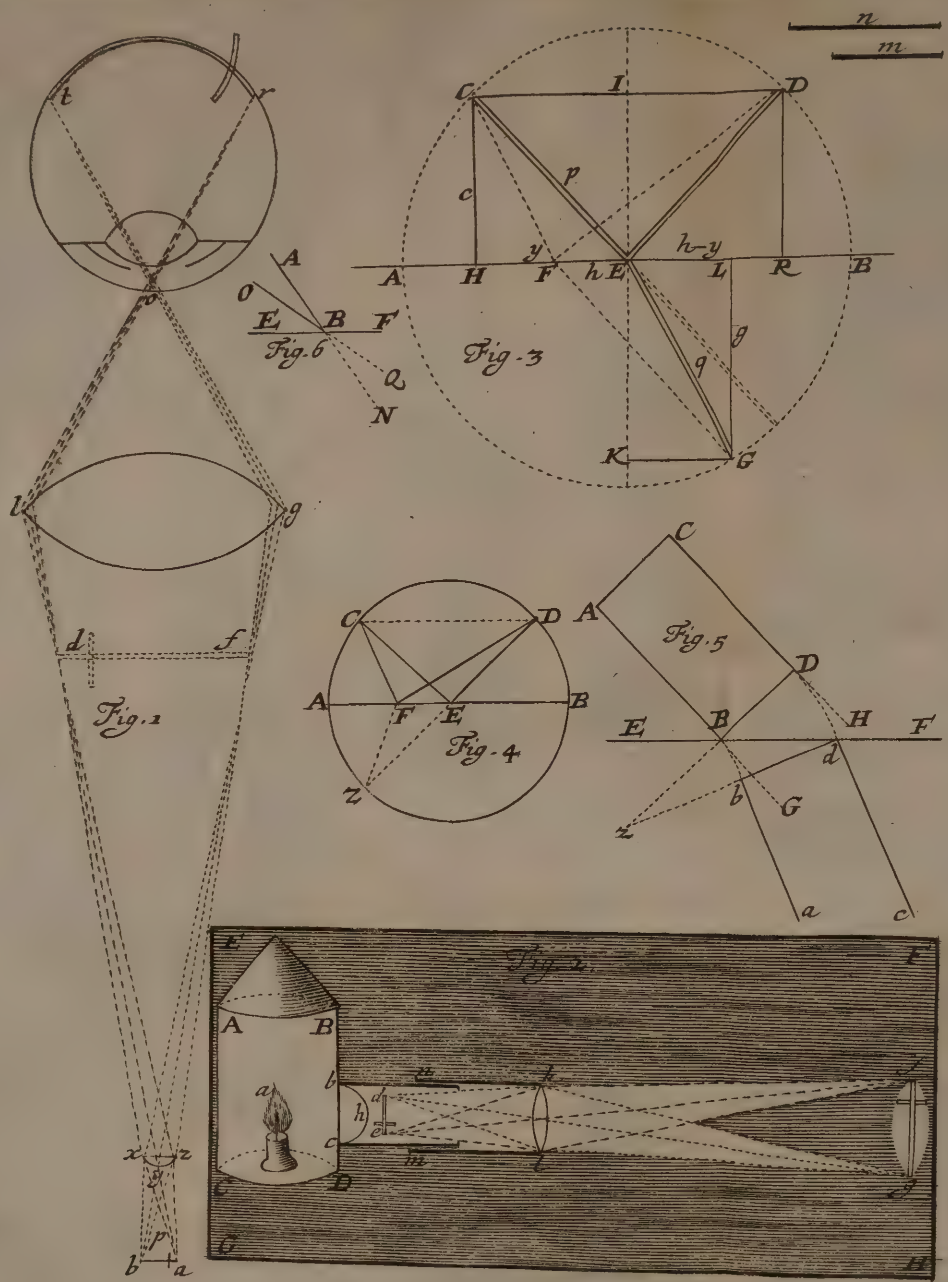
If the Eye move upwards or downwards, or to one side, or t'other of the Eye-Glass (supposing the Eye-Glass much broader than the Pupil) it perceives *consecutively* different parts T.37. F.2. of the Object. Thus suppose in *Fig.* 2. *Tab.* 37. The Pupil placed before the middle of the Eye-Glass at *h*, and not broad enough to receive the Rays from the upper and lower parts of the Object: If the Eye move upwards, it will meet with the Rays *g t* from the lower Point of the Object, which before escaped it; and so moving downwards it meets the Rays *l r* from the upper parts of the Object.

Of MICROSCOPES.

Hitherto of *single* Glasses, and of Glasses *combined* for viewing Distant Objects. We come now to treat of Glasses for viewing *minute* and *nigh* Objects, commonly called *Microscopes*. The Theory of these does so depend on what foregoes, that we shall have no occasion of insisting long upon them.

And first for *Microscopes* consisting of a *single* Convex-Glass. In these the Object is usually placed either in the Focus of the Glass, or a little nigher the Glass than the Focus; And the Eye is placed in or about the Focus on t'other side the Glass.

In



Tab. 38. pag. 181

In which Cases the Appearances were already solved. *Prop.* XXXI, XXXII, XXXIII, XXXIV, XXXV.

As to *double* Microscopes, or Microscopes consisting of more than one Convex-Glass, wherein the Object is Projected in a Distinct Base, before it be conveyed to the Eye; I explain them as follows, observing the Series of the Propositions.

Prop. LVIII.

The double Microscope composed of a Convex Object Glass, and Convex Eye-Glass is Explained.

Tab. 38. *Fig.* 1. Let *ab* be a minute nigh Object exposed before the Object-Glass *x y z*, the Segment of a very small Sphere. Let the Focus of this Object-Glass be at *p*. Then the Object being something more distant from the Glass, than its Focal length *y p*, shall be Projected in the Distinct Base *f d*, somewhere on t'other side the Glass, according to the Doctrine before delivered *Prop.* V. And of what bigness the Image shall be Projected in the Distinct Base is determined by *Prop.* XXVI. Let the Eye-Glass *l g* be placed so far distant from the Distinct Base *f d*, as is the Focal length of this Eye-Glass; And the Eye *o r t* placed where this Eye-Glass confounds all the Rays *g o*, *l o*, which shall be about the Focus of the Eye-Glass. T.38.F.1.

All things being thus combined, The Effects of this Microscope are explained in all things, as in the Telescope of *Prop.*L. As to the Magnified, Inverted, and Distinct Appearance of the Object. And therefore 'tis needless to inlarge farther thereon: To those versed in what is already delivered, the very Inspection of the Scheme is sufficient.

Only as to the extraordinary Magnifying of these Microscopes, we may farther Remark; that whereas for viewing a Minute Object, a well-constituted Eye does usually Approach thereto

thereto about the diſtance of eight Inches: could we approach the Eye thereto, and view it diſtinctly at the diſtance of half an Inch, the Optick-Angle would be wonderfully magnify'd by *Prop.* LI, LII. that is to ſay, the apparent Magnitude of the Object would be increaſed at the rate of ſixteen to one. Let us then ſuppoſe the Eye at *y*, viewing the Object *a b*, which would then appear under the Angle *a y b* equal to *d y f*; which is the ſame as were the Object increaſed to the bigneſs *d f*, and viewed by the Eye at the ſame diſtance from it as *y* in the figure is now removed from *d f*. Wherefore if by help of the Eye-Glaſs *g l*, the Eye can yet approach to the Diſtinct Baſe *d f* (which we may now repute the real Object) ſuppoſe ten times nigher than *y* is to *d f*; the apparent Magnitude of *d f* ſhall again be increaſed ten times more than before, by the nigh approach of the Eye (ſuppoſe at *y*): that is, the Object by all theſe helps ſhall be magnified 160 times in length, or Diametral Magnitude.

As to the Calculation of all theſe Angles, and apparent Magnitudes, it cannot be difficult to thoſe vers'd in what is before deliver'd.

There are various Combinations of Glaſſes in this kind of double Microſcopes; for in ſome there are two Object-Glaſſes, that is to ſay, one Object-Glaſs of a very deep Convexity, and an other of a leſſer Convexity, placed nigher the former than the Projection of the Diſtinct Baſe (according to *Schol.* 1. *Prop.* XVI.) which ſometimes is called a Middle-Glaſs. In others there are two Eye-Glaſſes, *&c.* But the Theory of all theſe depends on, and is ſo manifeſt from what has been delivered, that 'tis needleſs to enlarge.

I ſhall conclude the Firſt Part of this Treatiſe with a Piece of *Dioptricks*, which though *Ludicrous* affords an Appearance ſurpriſing and pleaſant enough. The Explication thereof is much labour'd at by ſeveral, though it be very Obvious from what foregoes. 'Tis this,

PROP.

Prop. LIX.

The Explication of the Magick Lantern, *sometimes called* Lanterna Megalographica.

The Contrivance is briefly this, *Tab.* 38. *f.* 2. *ABCD* is a Tin Lantern, from whose side there proceeds a square or round Arm or Tube *b n k c m l*, consisting of two Parts, the outermost whereof *n k m l* slides over the other, so as that the whole Tube may be lengthened or shortened thereby. In the end of the Arm *n k m l* is fixt a Convex-Glass *k l*: about *d e* there is a Contrivance for admitting and placing an Object *d e* painted in dilute and transparent Colours on a plain thin Glass; which Object is there to be placed *Inverted.* This is usually some Ludicrous or frightful Representation, the more to divert the Spectators: *b h c* is a deep Convex-Glass, so placed in the other end of the Prominent Tube, that it may strongly cast the light of the Flame *a* on the Picture *d e* painted on the plain thin Glass. And here 'tis to be noted, that the Glass *b h c* is only designed for the strong Illumination of the Picture *d e*, and has nothing to do in the Representation, and therefore in some of these Lanterns, instead of the Glass *b h c*, we shall find a *Concave-Speculum* so placed, that it may strongly cast the light of the Flame *a* on the Picture at *d e*. T. 38. F. 2.

Wherefore, Let us now consider the Picture *d e* as a very lightsome Object of distinct Colours and Parts. And let us conceive *d e* more remote from the Glass *k l* than its Focus. 'Tis then manifest, that the Distinct Image of the Object *d e* shall be projected by the Glass *k l* on the opposite white Wall *F H* at *f g*; And here it shall be represented *Erect.* For now the whole Chamber *EFGH* is dark, the Lantern *ABCD* inclosing all the Light; So that in Effect this Appearance of the *Magick Lan-*

tern

tern is no more than what is already declared concerning the Repreſentation of outward Objects in a dark Room by a Convex Glaſs, after *Prop.* IV. (*vid. Tab.* 14. *f.* 1.) And here we may obſerve, that if the Tube be *Contracted*, and thereby the Glaſs *k l* brought *nigher* the Object *d e*; the Repreſentation *f g* ſhall be Projected ſo much the *larger*; and ſo much the *more Diſtant* from the Glaſs *k l*; according to the Rules before laid down. So that the ſmalleſt Picture at *d e* may be Projected at *f g* in any greater Proportion required, within due limits. From whence the Name of *Lanterna Megalographica*. And conſequently, protracting the Tube and drawing the Glaſs *k l* more diſtant from the Object *d e*, will diminiſh the Repreſentation *f g*, and Project it *nigher* the Glaſs *k l*.

As to the Mechanick Contrivance of this Lantern, the moſt convenient Proportion of the Glaſſes, *&c.* This is ſo ordinary amongſt the common Glaſs-Grinders, that 'tis needleſs to inſiſt farther thereon in this place. 'Tis ſufficient to me that I have explained the Theory thereof.

Scholium.

On this depends the Theory of the *Optick* Experiment propos'd by Mr. *Hook* Num. 38. Pag. 741. *Philoſoph. Tranſact.* For repreſenting ſtrange Viſions and Appearances.

The End of the Firſt Part.

DIOPTRICKS.

THE
SECOND PART

Containing

Various Dioptrick Miſcellanies.

THE

CONTENTS.

To My esteemed Friend

Henry Osborn

OF

Dardys-Town in the County of *Meath*

ESQUIRE.

THE Respect which I have ever had for you since our first Acquaintance, and which on all Occasions I have expressed in private, I have now an Opportunity of declaring to the Publick: And that too so very apposite, that it would be unpardonable in me to omit it at this time; by presenting the following Sheets to you, Dedicating them to your Name, and Devoting them and their Authour to your Use and Service.

You may well remember the frequent Discourses we have had on several Subjects treated of in the following Chapters, and on account whereof I first set on this Dioptrical *Work: And particularly, I think, our Disquisitions*

quisitions concerning the Justness of Telescopick Sights adapted to Astronomical Instruments, and our Considerations of the Micrometer, were the first Occasion of my Thoughts turning this Way; And therefore the ensuing Discourses belong to you of Right. But if to this I add, the Advantage I have received by your Acquaintance, and the repeated Satisfaction I have had in your agreable Conversation; I am bound by indispensible Tyes to make this Acknowledgment.

I cannot but admire your prudent Choice of a private, retired Life; notwithstanding your great Advantages both of Nature and Fortune, that render you capable of the most publick and weighty Imploy. By this Course, you have an Opportunity of enjoying your self, and improving your Philosophical Thoughts beyond the common pitch: You can look on unconcern'd, and securely observe the froathy Sea of Business, wherein Men fluctuate; and some are shipwreckt, sink, and perish.

And because you seem careless of propagating your Name the common ways; suffer me to erect this slight Monument to it: Though I am certain at the same time, that, if you pleased, you may raise a lasting Mausolæum *to your Memory; but you seem above these Desires; yet you'll permit*

mit your Friend to Honour it as far as he can; and if the Materials or Workmanſhip do not ſeem to promiſe a long Duration to Poſterity, this only reflects on my Abilities, (which I ſhall never vindicate) but cannot leſſen the ſincere Intention of

Your moſt affectionate

April 17. 1690. Humble Servant,

WILL. MOLYNEUX.

DIOPTRICKS.

DIOPTRICKS,

PART II.

CHAP. I.

Of Refraction and Light.

(1.) *This Discourse promised.* (2.) Leibnutz's *Universal Principle in Opticks*, &c. (3.) *On Occasion whereof; Of Final Causes.* (4.) *Farther Explication of Refraction.* (5.) *Light a Body, from several Arguments.* (6.) *From its being resisted in its passage through Diaphanous Mediums.* (7.) *Its Requiring time to move from place to place.* (8.) *Impossible to be increased, but by robbing some other Place of its Light.* (9.) *Therefore impossible to be augmented uniformly.*

(1.) IN the second Experiment of *Part* I. I had occasion to mention the *Natural Cause* of *Refraction*; or why the Rays of Light passing through different Mediums are refracted at their Immersion or Emersion. I then avoided any farther enquiry into the reason thereof, as being more of a *Physical* than *Mathematical* Consideration. But at the same time, I promised a farther Disquisition thereon, to be borrowed from a most Learned Author; whose reason we shall find briefly comprehended in this, That the Different Resistance that a Ray of Light finds in passing (for instance) (1.) *This Discourse promised.*

ſtance) through Air and Glaſs, is the cauſe, why 'tis bent from its direct Courſe. But how this Refraction comes to be *from* the Perpendicular, in proceeding *from a Denſe to a more Rare Medium*; or *towards* the Perpendicular, when *from a Rare to a more Denſe Medium*; we ſhall more fully apprehend by the Diſcourſe it ſelf, which I here ſubjoyn from the *Acta Erud. Lipſiæ*, *Ann.* 1682. *pag.* 185.

(2.) Leibnutz *univerſal Principle in Opticks.*

(2.) *One Univerſal Principle of Opticks, Catoptricks, and Dioptricks. By the Learned and Ingenious* G. G. Leibnutzius.

The chief Hypotheſis common to all theſe Sciences, and by which the Progreſs of all Rays of Light is geometrically determined, may be thus laid down. *Light proceeds from the Radiating Point, to the Point to be enlightned, that way, which is of all the moſt eaſie; and this is firſt to be Determined in reſpect to plain Surfaces, and then is accommodated to Concave or Convex Surfaces, by conſidering the Planes, that are Tangents to theſe Surfaces.* But here I take no notice of ſome *Irregularities*, which perhaps may conduce to the Generation of Colours, and to ſome other extraordinary *Phœnomena*, which in practical Opticks are not at all conſidered.

Hence, *In plain or ſimple Opticks*, Tab. 38. Fig. 3. *The direct Ray proceeds from the Radiating Point* C, *to the Point to be illuſtrated* E, *by the ſhorteſt direct way*, the ſame Medium continuing all along, *that is, in the Right Line* C E.

In Catoptricks the Angle of Incidence C E A, *and of Reflection* D E B *are equal.* Let C be a Radiating Point; D the Point to be illuſtrated, and A B a *Plain Speculum*: 'Tis required to find in the *Speculum* the Point E, that reflects the Ray to D. I ſay, that it ſhall be ſuch a Point, that the whole Progreſs, Way or Journey of the Ray C E + E D, may be the *leaſt* or *ſhorteſt*, that is poſſible; or leſs than C F + F D, ſuppoſing that we take any other Point F in the *Speculum*. And this ſhall be obtained, if E be taken ſuch, that the Angles CEA,

DEB,

DEB, may be equal; as is manifeſt from Geometry. *Tab.* 38. *Fig.* 4. Produce DE to Z, and joyn FZ. T.38.F.4.

Then AZ= (DB=) AC. And FZ=FC.

Therefore CE+ED (=DZ) is leſs than CF or FZ+FD.

Ptolomy and other Antients inſiſt on this Demonſtration; and 'tis extant both elſewhere, and alſo in *Heliodorus Lariſſæus.* T.38.F.3.

In *Dioptricks,* * *The Sines* EH, EL, *of the Angles of Incidence* CEI, *and Refracted Angle* GEK *are to each other reciprocally as the Reſiſtances of the Mediums.* Let IE be Air, and EK Water, Glaſs, or any other Diaphanous Medium more Denſe than Air, C a Radiating Point in the Air, G the Point to be illuſtrated under the Glaſs: 'Tis inquired, by what Way or Path ſhall C radiate to G; or 'tis required to determine, in the Surface of the Glaſs AB, the Point E, which refracting the Ray that comes from C, ſends it to G. Here this Point E muſt be taken ſuch, that the Way, which the Ray takes, may be of all ways the *eaſieſt.* But now in different Mediums, the Difficulties of the Way or Progreſs are in a Ratioo compounded of the *Length* of the Way, and of the *Reſiſtance* of the Mediums. Let the Right Line *m* repreſent the Reſiſtance that Light finds in its paſſage through Air, and *n* the Reſiſtance of its Paſſage through Glaſs: The *Difficulty* of the Way from C to E ſhall be as the Rectangle under CE and *m*; and from E to C as the Rectangle under EG and *n*. Therefore that the *Difficulty* of the *whole* Way CEG may be the *leaſt* poſſible, the Sum of the Rectangles CE * *m*+EG * *n* ought to be the leaſt, of all poſſible; or leſs than CF * *m*+FG * *n*, ſuppoſing any other Point F taken beſides E. The Point E is now required. Wherefore, ſeeing the Points C and G, and alſo the Right Line AB are given by Poſition, therefore the Lines CH, GL, Perpendiculars to the Plain AB, and the Line HL, are given alſo. Let us call CH, *c*, and GL, *g*, and HL, *b*; but the ſought

* *Here I tranſlate according to the* 2,4. Definit. *of* Part I. *and not according to the Author.*

Line E H, let us call y; then E L ſhall be $b-y$, and C E ſhall be $\sqrt{:cc+yy}$, which we ſhall call p; and E G ſhall be $\sqrt{:gg+yy-2by+bb}$, which we ſhall call q.

Wherefore $m*\overline{\sqrt{cc+yy}}:+n*\sqrt{:gg+yy-2by+bb}$ (or $mp+$ $+nq$) ought to be the leaſt of all thoſe Quantities that can poſſibly be ſo expreſſed; and 'tis required to determine y, that ſo it may be. By my Method *De Maximis & Minimis* (*Vid. Act. Lips. Ann.* 1684. *pag.* 467. 472.) which, beyond all that are hitherto known, does wonderfully ſhorten the Calculation, it is manifeſt at the firſt ſight, almoſt without any Calculation, That mqy ſhall be equal to $np*\overline{b-y}$, or that $np.\ mq::y.\ b-y$, that is, the Rectangle of C E $*$ n ſhall be the to Rectangle of E G $*$ m:: As E H to E L. Therefore C E and E G being put equal, n the Reſiſtance of Glaſs to Light ſhall be to m the Reſiſtance of Air to Light :: As E H the Sine of the Angle of Incidence in Air C E I: To E L the Sine of the Refracted Angle in Glaſs G E K. Or theſe Sines ſhall be to each other reciprocally as the Reſiſtances of the Mediums. Which was the Aſſertion to be proved.

Wherefore, if in one Example or Experiment E L be found $\frac{2}{3}$ of E H, the ſame Proportion ſhall hold in all other Experiments, wherever C and G be taken, that in Air, this in Glaſs. If C be in Air, and G in Water, Experiment ſhews, that E L ſhall be about $\frac{3}{4}$ of E H.

Des-Cart. Notion of Refraction rectified.

Thus far this ingenious Author. The reſt of his Diſcourſe is chiefly employed in rectifying *Des-Cartes* Notion of Refraction, which tho founded on the ſame Principle here expreſſed, yet has this peculiar, that he makes Water, or Glaſs, or any other more Denſe Medium, reſiſt the Progreſs of Light leſs than Air. But in this particular he is abundantly rectified by *Leibnutzius*, who ſhews the Incongruity of that Suppoſition.

(3.) One

(3.) One thing more there is remarkable in the Learned *Leibnutzius* Discourse, which I cannot here pass over, and that is, A pious Reflection which he makes on this occasion, concerning *Final Causes.* For 'tis manifest, that the Ray proceeding from C, does not consult with it self, how it may with the greatest ease arrive at the Point E, or D, or G; neither is it carried by it self to those Points. But the *Great Creator* of all things, has so made *Light*, that this most beautiful, orderly, and admirable Event should result from its very Nature. Wherefore they are in a great Error, who reject *Final Causes* in Natural Philosophy, which, besides affording us occasion of admiring and adoring the *Divine Wisdom*, do often discover to us a curious Principle of finding out the Properties of those things whose inward Nature is not *so clearly* known by us, as that we can explain the immediate *efficient Causes* and *Instruments*, which the *Almighty Mover* imploys in producing those *Effects*, and obtaining those *Ends*. Remar on Final Causes.

Indeed I should think it an Attempt worth the Thought of some profound Philosopher, to give an Account of those admirable, orderly, and beautiful Appearances in Nature, whereof we can most plainly apprehend the *Designs* and *Final Causes*, but can hardly proceed to any farther Knowledg of them. (Thus for instance, suppose it were asked, *What is the cause of Refraction?* Were it not much satisfactory to answer, *That thereby the Ray may proceed the easiest way possible*) This surely might be able to convince the most obstinate Opposers of *Divinity*: For certainly, if we can rely upon *any Deduction* or *Consequence* drawn out by the *Mind* of *Man*, we may assuredly rest satisfied in this; that so many *Phænomena*, stupendous and surprising for their *designed Contrivance*, could not proceed but from an *Omnipotent* and *Designing Being*. But if after all, they will arrive to such an height of Extravagance, as to say, We cannot rely on these Conclusions, as being *all* in the *dark*, and Final Causes *highly deserve our notice.*

 knowing

knowing nothing; let them look to the hazard of their own Principles, who endanger their *eternal Happineſs* on confidence of their own Arguments. But to reſume our Subject.

Fermat *demonſtrates the ſame.*

The Famous Monſ. *Fermat* has written a long Demonſtration of this ſame Principle in *Dioptricks*. 'Tis publiſh'd amongſt the French Letters, at the end of his *Opera Mathematica*. *Toloſæ* 1679. Pag. 158. To which I refer the Reader.

Farther Explication of Refraction from D.Barrow. T.38. F.5.

(4.) But after all, perhaps it may not be amiſs, to illuſtrate this Buſineſs of Refraction, by ſome more familiar and ſenſible Inſtance. Wherefore *Tab.* 38. *Fig.* 5. Let the Parallelogram A B C D repreſent a Ray of Light of this breadth, and let it fall on the plain Surface of the Glaſs E F; this in ſome meaſure does ſtop its Courſe; and the Point B entering the Glaſs, ſhall endeavour (but ſlower) to proceed onwards directly to G in the Line A B produced. But all this while the Point D, continuing yet in the Air, ſhall continue its former motion in the Right Line C D H; but now 'tis impoſſible for *both* the Points to obtain what both endeavour; for each cannot perform his *direct* Motion, one *ſlower*, and t'other *quicker*. And therefore that they may *both* come higheſt to what *each* endeavours, they ſhall *both* be turned about ſome certain Point Z in the Right Line D B produced. So that whilſt the Point D in the *thinner* Medium proceeding *quicker* deſcribes the greater Arch D *d*; the Point B, proceeding more ſlowly in the more *Denſe* Medium, deſcribes the leſſer Arch B *b*; and when they have thus run through both theſe Arches, the Right Line B D ſhall obtain the Poſture *b d*. And now that the Point D alſo is emerged into the more Denſe Medium at *d*; and it alſo is now as much retarded as *b*; theſe circular Motions ſhall now ceaſe; for D is not now carried quicker than B, and therefore deſcribes not, as before, a greater Arch. Wherefore forſaking, as ſoon as they can, their former Progreſs, they ſhall both proceed in *d c*, *b a*, the Tangents to their

their Arches. And the whole Ray ABCD thus bent, and brought into the Poſture *a b c d* proceeds onwards directly in that Courſe. And here 'tis to be noted, That whatever Inclination A B has on E F, the Arches D *d*, B *b*, or their Semidiameters Z D, Z B, have always the ſame Proportion; to wit, ſuch a Proportion, as the *peculiar* Difference in the Reſiſtance or Denſity of one and t'other Medium does require (which is to be determined by Experiment): For *Tab.* 38. T38. F.6. *Fig.* 6. Let us ſuppoſe the Point B thruſt forwards towards Q or N, and that the Medium below E F is perfectly *Homogeneous*, that is, equally reſiſting in all Parts thereof; there is then no reaſon, why this Point ſhould not be carried with an equal Celerity towards whatever Part, that is, it ſhall tend equally quick towards Q in the Right Line O B Q (ſuppoſing its direction lye that way) as towards N in the Right Line A B N. And therefore the Rays of Light A B, O B, however differently inclined, ſhall find an *equal* Reſiſtance; and the Point B, whether it tend towards Q or N, ſhall be equally retarded. And alſo ſeeing the Point D (*Tab.* 38. *Fig.* 5.) continues in the T.38. F.5. firſt Medium, it ſhall be thruſt forwards with the ſame Celerity, whatever is its Inclination. Whence 'tis manifeſt, that theſe Motions, or Paths paſſed over in the ſame time, to wit, the circular Arches D *d*, B *b*, ſhall always obſerve the ſame Proportion, that is, the Proportion of their Semidiameters Z D, Z B, or Z *d*, Z *b*; which Proportion therefore principally and chiefly meaſures and determines the Refractions of Rays in the ſame two Mediums. And this is the ſame Proportion, as is between the Sines of the Angles oppoſite to Z *d*, Z B, in the Triangle Z *d* B, that is, of the Angles Z B *d*, (or ZBE) and Z *d* B. But Z B E is the Complement of the Angle ABE, and therefore (by *Def.* 2. *Part.* I.) Z B E is the *Angle of Incidence* or *Inclination* of A B to E F; and the Angle Z *d* B is the Complement of the Angle F *d c*; and therefore Z *d* B is the

Refracted

Refracted Angle (by *Def*. 3. *Part*. I.). From hence is manifest what we assert in the 7 *Experiment*, *Part* I. And here we shall note, that according to *Exper*. 6. *Part* I. Z D shall be to Z B : : As 300 To 193 ; or as 14 to 9.

Light a Body. (5.) And thus much concerning *Refraction*. The Consideration whereof does naturally suggest unto us, That *Light is a Body*. For however the Antient *Aristotelians* defined it, *Actus perspicui quatenus perspicuum*, which is perfectly unintelligible; yet so much we may perceive hereby, that they designed to exclude it from all *Corporeal Notion*. But the various Properties of Light, that do necessarily belong to a Body, are so many and evident, that they leave no room for any farther doubt in this matter. I shall mention but a few.

From its being resisted in its passage through Diaphanous Mediums. (6.) And first, by this *Affection* of being *refracted*, 'tis manifest that Light, in its passage through this and t'other Diaphanous Body does find a *different Resistance*. Now tis unconceivable, how any thing, but Body, should suffer *Resistance* ; but we may conceive the Resistance, that Light suffers in its passage through different Diaphanous Bodies, to proceed from the Medium Hindering of the *Diffusion* or *Distribution* of Light through *more* of the Parts of this Medium, and consequently it may be said to be *less illuminable*. For the Nature of Light endeavours to *diffuse* it self. And on the contrary, by how much *Light* does more equably or uniformly affect the Parts of the Medium, which it enlightens; or by how much it communicates its *Energy* to *more* of the *Particles* of the enlightened Space; that Medium may be said to be by so much the *more illuminable*, or *less to resist* the Progress of Light. Whence it is, that by how much the affected Parts of the Medium are *more solid* and *small*, and admit between them the *less Space* for any other *Heterogeneous* Matter, that suffers not by *Light* ; by so much the Medium is said to be *more enlightened*. But leaving these *Philosophical Refinements* ; 'tis manifest

nifeſt that Reſiſtance muſt proceed from *Contact* of two *Bodies*. And *Contact*, either *Active* or *Paſſive*, belongs *only to Body*; according to that of the Philoſophick Poet, *Tangere enim & tangi niſi Corpus nulla poteſt Res.* Lucretius And our *Saviour* himſelf, the Fountain of all Wiſdom and Philoſophy, Divine and Natural, ſeems to confirm this Notion; when to prove himſelf *a True Body* after his Reſurrection, He commands his Diffident Diſciple *To Touch Him.*

(7.) The Second Property, that confirms *Light* to be a *Body*, and a *Body* moved or thruſt forward, is, That it requires *time* to paſs from one place to another, and does it not in an *inſtant*, but is only of all Motions the *quickeſt*. *Requiring time to move from place to place.* For the Experiment proving this, we are obliged to the Ingenious Monſ. *Romer*, who has demonſtrated beyond all Contradiction, from the Obſervations of the *Immerſions* and *Emerſions* of the *Satellits* of *Jupiter*; *That Light requires the Time of one Second to move the ſpace of 3000 Leagues, or 9000 Miles, which is near the Earths Diameter.* He that requires a farther Account hereof, may conſult the *Journal des Scavans* 1676. Decemb. 7. *Philoſoph. Tranſact.* Num. 136. Or Mr. *Newton*'s Incomparable Piece, *Philoſophiæ Natur. Princ. Mathem. Lib.* I. *Schol. Prop.* 96. Where 'tis aſſerted, That Light requires about ten Minutes time to come from the Sun to the Earth. And 'tis moſt evident, without this Allowance for the Time ſpent in Lights Motion, the Appearances of the *Satellits* Eclipſes and Emerſions are not to be explicated by any *Excentricity* or other Hypotheſis. But by this Allowance, they anſwer to the greateſt exactneſs. And this is a Part of Aſtronomy the moſt correct and accurately determined, as well as the moſt uſeful, of all others: For hereby Geography may be rectified, the Longitude determined, and Navigation made more eaſie and ſecure. For a Confirmation of all which, I appeal to the Labours of the Ingenious Mr. *Flamſteed* and Mr. *Halley*, to

whom the Learned World is for ever obliged by their Advancements of Aſtronomy.

Light not to be increaſed, but by robbing ſome other place of its Light.

(8.) A third proof that Light is a Body, is, That it cannot by any Art or Contrivance whatſoever be *increaſed* or *deminiſhed*; that is to ſay, we cannot magnifie (for inſtance) the Light of the Sun or a Candle, no more than we can magnifie a Cubick Inch of Gold, or make it *more* than a Cubick Inch. But in this particular I deſire to be rightly underſtood, leſt I ſeem herein to advance a *Paradox*. I ſay therefore, Whenever we ſee Light increaſed, 'tis by Robbing of ſome other part of the Medium of its Light; or, by bringing the Light, that naturally ſhould have been diffuſed through ſome other part, to the more enlightened place. Thus for Example, In a Burning-Glaſs, by which the Light of the Sun is highly encreaſed in its *Focus*, or Burning-Spot: We are firſt to conſider, that in this Focus the Image of the Sun is projected, as being the Diſtinct Baſe of the Glaſs. And ſecondly we may obſerve all round about this bright Spot of the Suns Image, there is caſt the ſtrong Shadow of the whole breadth of the Burning-Glaſs. For all the Rays from the Sun, that would have fallen on this broad ſhaded ſpace, are now brought together and crowded cloſe in this bright Spot, there raiſing a vigorous *Light* and violent *Heat*. This is abundantly confirmed by an eaſie Experiment: For cover all the Burning-Glaſs, except one ſmall round ſpace in its middle, juſt the bigneſs of the bright burning Spot in its Focus; and tho there be a ſhaded ſpace round the bright Speck, as before, yet we ſhall not be ſenſible of any *Increaſe* either of Light or Heat; which plainly ſhews, that this *Increaſe* of Light (when the Glaſs is all bare) proceeds from the crowding together of thoſe Rays that would have fallen on the reſt of the Glaſs, and which (were not the Glaſs interpoſed) would have fallen on the ſhaded ſpace round about the bright Speck.

There

There ſeems but one Objection againſt what is here laid down; and that is, that Light is *increaſed* by Reflection, without depriving any place of the Light it would otherwiſe receive; or, without bringing to the enlightned part any Light that would otherwiſe eſcape it, or never come at it. But if we conſider the matter more attentively, we ſhall find it otherwiſe. For let us ſuppoſe an *Hole* of a Foot ſquare in the ſide of a Chamber, and that a Candle were placed cloſe to, and juſt before the middle of this Hole; there is but half this Candle that now enlightens this Room, the other half of the Rays proceeding directly out at the Hole: Let now a Looking-Glaſs be placed, ſo as juſt to fill up this Hole; the Rays which before would have gone out at the Hole, are now reflected into the Room; ſo that the Hemiſphere without the Chamber, which was enlightened whilſt the Hole continu'd open, is now robb'd of its Light; and all this Light is now reflected into the Room; whereby the *averſe* ſide of the Flame is made to enlighten, as well as the ſide *directly* expoſed to the Chamber. What is ſaid of this Caſe, may be accommodated to all: For ſo a Looking-Glaſs lying Horizontal, and reflecting the Sun-Beams to the Ceiling of the Room, does plainly hinder the direct progreſs of the Rays to ſome other part, and conſequently robs that part of its Light. This is evident, by ſuppoſing an Hole behind the Glaſs; as in the former Caſe.

(9.) From all which 'tis manifeſt, how vainly they attempt, who offer at *increaſing Light Uniformly*, that is, *equally* throughout the whole Sphere of a Luminous Body, or Radiating Point. Such are the Pretences of thoſe that would perſwade the World of Contrivances for making the ſmall Flame of a Lamp enlighten *ſtrongly* a whole Chappel, Hall, or Court, by being hung up in the midſt thereof. For theſe things are *impoſſible* to be effected in Nature, and they had as well pretend to *create Light*; for there is no other way of *increaſing* it; un- *Light not to be increaſed uniformly.*

Dd leſs

leſs by robbing another part of its Light; and then 'tis not *uniformly increaſed.* We have a very ſenſible Inſtance of this in the *New-invented Lanthorns*, now much uſed in *London*; which by the Convex-Glaſſes in their ſides, do ſtrongly throw thoſe Rays along the Walks of the Paſſengers, which would otherwiſe (were the Glaſſes away, and the round Holes left open) be ſpent on parts of the Streets not frequented; whereby the untrodden parts of the Streets are robb'd of their Light, more ſtrongly to ſupply and enlighten the Paths where Light is requiſite.

I have inſiſted the longer on this particular, becauſe there is nothing more commonly pretended, than this Invention of *increaſing Light uniformly*, by thoſe that do not conſider how vain the Attempt is. I muſt confeſs, could the thing be effected, it would be a piece of *Oeconomick Philoſophy*, the moſt pleaſant and uſeful imaginable, and equivalent to the *perpetual Lamp* (if ever there was any ſuch thing, as I very much doubt). The Student in his Cloſet, the Merchant in his Shop, the Houſewife in her Offices, would find a great and pleaſant Advantage therein. But above all, the *dark Northern Climates*, who are many Days, Weeks, and Months deprived of the Sun, would be infinitely obliged to the Inventor, who could make the Flame of a Lamp, no bigger than a Barly-Corn, ſupply the Preſence of that *glorious Body*.

CHAP.

Chap. II.

Dioptrick Problem.

Why Four Convex-Glaſſes in a Teleſcope ſhew Objects Erect.

I Was unwilling to burden the Firſt Part with Digreſſions; and therefore in *Prop.* LVI. thereof, I promiſed this Diſcourſe in this place.

In the *Journal der Scavans.* For *Monday* 17. *Septemb.* 1685. Pag. 466. *Amſt. Edition* We find this Paſſage.

'As Perſpectives of *One* Convex-Glaſs make Objects appear '*upright*, which thoſe of *Two* Convex-Glaſſes *invert*; and again 'thoſe of *Three rectifie*: So it ſhould ſeem, that thoſe of *Four* 'ought to *invert.* And yet Experience ſhews us that Objects 'appear *upright* through theſe Glaſſes. The ſingularity of this '*Phænomenon* obliges all skill'd in *Dioptricks*, to enquire the 'reaſon thereof; but hitherto they have found none. Monſ. '*Regis*, who applies himſelf particularly to this Part of *Na-* '*tural Philoſophy*, believes that he has hit upon the Reaſon, and 'makes us hope that he will ſuddenly publiſh it.

Thus far the *Journal*: But does not tell us whoſe Remark this is: I am apt to believe, 'twas written by Monſ. *Regis* himſelf, to the Publiſher of the *Journal.*

To me this *Phænomenon* appears very eaſily explicable from the conſideration of placing the Glaſſes in a Teleſcope. And I wonder that any one, who pretends to Skill in *Dioptricks*, ſhould make a Difficulty of it. The poſture of the Glaſſes in the Tube is thus; After the Object-Glaſs, the firſt Eye-Glaſs is placed ſo much diſtant (towards the Eye) from the Focus of the Object-Glaſs, as is this Focus of the Eye-Glaſs;

 then

then the ſecond or middle Eye-Glaſs is placed ſo much diſtant from the Focus of the firſt Eye-Glaſs, as is the Focus of this middle Eye-Glaſs. Laſtly, the neareſt or third Eye-Glaſs is placed ſo much diſtant from the Focus of the middle Eye-Glaſs, as is the Focus of this neareſt Eye-Glaſs; and the Eye, looking through them all, is placed in the Focus of this neareſt Eye-Glaſs.

I ſay therefore firſt, that *one ſingle Convex-Glaſs*, cannot properly be ſaid *by it ſelf*, to ſhew Objects *erect* or *reverſe*; but only *in reſpect of placing the Eye* that looks through it; and *in reſpect of the Objects Diſtance from it.* For if the Eye that looks through ſuch a ſingle Convex-Glaſs, be placed nigher thereto than the Glaſſes Focus or *Diſtinct* Baſe, diſtant Objects appear *erect*: If the Eye be placed juſt in the Focus or Diſtinct Baſe, diſtant Objects are neither *erect* or *reverſed*, but all in *Confuſion between both.* And if the Eye be placed *farther* from the Glaſs than the Focus or Diſtinct Baſe, diſtant Objects are *reverſed.*

This being laid down, I aſſert, ſecondly, That the Object-Glaſs of a Teleſcope, conſiſting of a Convex Object-Glaſs and Convex Eye-Glaſs, *reverſes* the Object, both to the Eye-Glaſs, and to the Eye that looks through it. For the Eye-Glaſs is placed *farther* from the Object-Glaſs, than is the Focus of the Object-Glaſs. And the Eye-Glaſs does nothing towards the *rectifying* or *reverſing*. Thus we ſee the *reverſing* of Objects in a Teleſcope of two Convex-Glaſſes, proceeds wholly from the *Object-Glaſs* and its *Poſition*; and the Eye-Glaſs has nothing to do in the Affair: For were the *Eye* it ſelf in the place of the *Eye-Glaſs*, it would ſee the Object *inverted* through the ſingle *Object-Glaſs.*

I come now to conſider the ſecond Eye-Glaſs placed after the firſt Eye-Glaſs. And here it is manifeſt, that placing this as it ought in a Teleſcope; if we place our Eye nearer to this middle Eye-Glaſs than its Focus the Eye ſees the Object *in-*

verted

verted and confuſed : Place the Eye in the Focus, it ſees the Object all in confuſion, neither *erect* nor *reverſed.* For here again, there is a diſtinct Repreſentation of the Object to be received on a piece of Paper, as in the Focus of the Object-Glaſs ; and the Eye being placed at any time in this place, being in the *Diſtinct Baſe*, ſees all in confuſion, (all which is manifeſt from the Firſt Part, and therefore but lightly touch'd here). But then let the Eye be placed farther from this middle Eye-Glaſs than its Focus (for ſo is the third or immediate Eye-Glaſs, it being always diſtant from the middle Eye-Glaſs, the Aggregate of both their *Foci),* it perceives the Object *erect* and *confuſed.*

Laſtly, the third or immediate Eye-Glaſs does nothing towards the *erecting* or *reverſing* the *Species*, which it receives *erect* from the middle Eye-Glaſs ; no more than, in a Teleſcope of two Convex-Glaſſes, the Eye-Glaſs does to the *Species* it receives from the Object-Glaſs ; as is ſhewn before. All which will be manifeſt from inſpection only of *Tab.* 37. *Fig.* 1.

Wherefore we are to conſider the Teleſcope conſiſting of a Convex Object-Glaſs and three Convex Eye-Glaſſes, as *two Teleſcopes*, each conſiſting of two Convex-Glaſſes. The firſt conſiſts of the Object-Glaſs and firſt Eye-Glaſs, and this *inverts* the *Species* ; that is, the *Species* is *inverted* in the *Diſtinct Baſe* of the Object Glaſs, and is ſo brought to the Eye. The *ſecond Teleſcope* conſiſts of the two *immediate Eye-Glaſſes*, and this *erects* what the former *inverted* ; that is, the *Species* in the Diſtinct Baſe of the middle Eye-Glaſs is *erect*, and is ſo brought to the Eye, by the Eye-Glaſs. The Eye-Glaſſes themſelves, in *either* Caſe, having nothing to do with the *erecting* or *inverting*, but merely in *repreſenting* in the ſame poſture the Image immediately before them.

The *French* Problem ſhould not therefore have broken a Teleſcope of four Convex-Glaſſes into *four Pieces*, but *into*

two ;

two; and the Caſe would then have been plain; whereas, by breaking it into *four* Perſpective-Glaſſes, that is attributed to *two* of them, which *neither* of them does, *viz.* *Inverting* and *Erecting*.

Therefore I ſay laſtly, That *one* Convex-Glaſs (that is the Object-Glaſs) as poſited in this Teleſcope, *inverts*: The *ſecond* (that is the *firſt* Eye-Glaſs) does nothing towards *erecting* or *reverſing*; but *repreſents* the Image as it is in the *Diſtinct Baſe* of the Object-Glaſs before it, that is, *inverted*. The *third* Glaſs *erects*, or rather *reſtores* what was before *inverted*. The *fourth* repreſents the Image, as it receives it from the *Diſtinct Baſe* of the *third*, that is, *erect*.

And this I think a ſufficient and eaſie Anſwer to what the *French* Man makes a great Difficulty.

As a Corollary to what has been laid down in this Chapter, we may deduce this Practical Rule for combining or putting together a Teleſcope of *Prop.* LVI.

Take the two firſt Eye-Glaſſes, and combine them by Tryals, ſo as to make a Diſtinct Inverting Teleſcope of *Prop.* L.

Then take the Object-Glaſs and firſt Eye-Glaſs, and by Tryals combine them likewiſe.

Laſtly, take both theſe Teleſcopes, and without altering the Diſtances of their Glaſſes in either of them ſingly, by Tryals combine both theſe Teleſcopes, till the Appearance be clear and diſtinct.

What is here done by Tryals, may be effected by actual Menſuration, or deſigning out the Diſtances of the Glaſſes from each other by knowing their Focal Lengths.

CHAP.

Chap. III.

Of Glasses for defective Eyes.

(1.) Spectacles for old and pur-blind Men an Invention in Dioptricks of great use. (2.) Some Rules for choosing Spectacles both for old and pur-blind. (3.) Observations on Mr. Hook's *Invention for helping* Myopes *by Convex Glasses. (4.) Telescopes and Microscopes adapted to defective Eyes.*

(1.) Were there no farther Use of *Dioptricks* than the Invention of *Spectacles* for the Help of defective Eyes; whether they be those of *old Men*, or those of *pur-blind Men*; I should think the Advantage that Mankind receives thereby, inferiour to no other Benefit whatsoever, not absolutely requisite to the support of Life. For as the *Sight* is the most noble and extensive of all our Senses; as we make the most frequent and constant use of our Eyes in all the actions and concerns of human Life; surely that Instrument that relieves the Eyes when decay'd, and supplies their Defects, rendring them useful, when otherwise almost useless, must needs, of all others, be esteemed of the greatest Advantage. In what a miserable condition do we count those, in whom it hath pleased the *great Contriver of the Eyes and Sight*, to shut those two little Windows of the Soul? And we may imagine, that they, in whom these Lights are but *partly* obscured, do in some measure partake of the Misery of the blind. How melancholy is the condition of him, who only enjoys the Sight of what is immediately about him? With what Disadvantage is he ingaged in most of the Concerns of human Life? Reading is to him troublesome, War more than ordinary dangerous, Trade and

Spectacles for old and pur-blind Men, an Invention of great use.

Condition of old Men deplorable without Spectacles.

and Commerce toilſome and unpleaſant. And ſo likewiſe, on the other hand; How forlorn would the latter part of moſt Mens Lives prove, unleſs *Spectacles* were at hand to help our Eyes, and a little form'd piece of Glaſs ſupply'd the Decays of Nature? The curious Mechanick, engaged in any minute Works, could no longer follow his Trade than till the 50th. or 60th. Year of his Age: The Scholar no longer converſe with his Books, or with an abſent Friend in a Letter. All after would be melancholy Idleneſs, or he muſt content himſelf to uſe an other Man's Eyes for every Line. Thus forlorn was the ſtate of moſt *old Men*, and many *young*, before this admirable Invention; which, on this very account, can never be prized too highly.

Rules for Choosing Spectacles both for old and pur-blind.

(2.) And becauſe in the Firſt Part hereof, *Prop.* XXVIII. and XLV. I have but ſlightly touched on the proper Method for helping *defective Eyes*; I think it convenient to proſecute that matter more fully in this place. Always ſuppoſing what foregoes in the Firſt Part as underſtood.

Firſt therefore for helping the Eyes of *old Men*, as being more frequently and univerſally requiſite than the Relief of pur-blind Eyes.

In the Firſt Part (*Prop.*XXVIII. *Sec.*8. and *Prop.*XXXI.) we learn, that a *Convex-Glaſs* is here to be uſed: For theſe ſeeing *diſtant* Objects *diſtinctly*, and *nigh* Objects *confuſedly*, muſt uſe ſuch Glaſſes for reading, &c. as make *nigh* Objects appear *as diſtant*; or, which bring the Rays from each ſingle Point in a *nigh* Object, as if they came from a more *diſtant* Point. Or thus, ſeeing the Cryſtallines of *old Men* are *too flat* for *nigh* Objects, that is, want *Convexity*; we are to help them by adding to them an artificial or adventitious Convexity of a Glaſs. But then our enquiry muſt be, What is a *Proper* Convexity to this or that *particular* Eye? And becauſe reading, or working curious ſmall Works, as being engaged upon *nigh* Objects, are the

the chief Imployments wherein *Spectacles* are requisite. I shall suppose our Enquiry chiefly designed for this purpose. And indeed, tho there can hardly be any Rules laid down, *strictly* to determin this matter; for *distinct* Sight may consist within a *great Latitude*: Yet if we observe the following Directions, we shall be apt to err less, and to fit our Eyes better with *Spectacles*, than if we observed no Rule at all, but chose at a venture.

First, When we first find our selves begin to require *Spectacles*, let us make choice of the *flattest Convexities*, that will possibly help our Eyes. These are usually called *Young Spectacles.* There are many ways of finding out and trying *such*; but none more ready, obvious, and easie, than trying with which one can read a small Print distinctly, with the Book *farthest* from the Eyes; or try which *Spectacles* burn at the greatest distance; for these *Spectacles* are the most proper for those Eyes to use, and shall prejudice the Sight less, and preserve it longest of any.

We may note likewise, that the distance of the Print, or Object from the Eye continuing the *same*, a Convex-Glass may be said to be *older* or *younger*, according at it is removed *farther from* or *nigher to* the Eye or Object. This is manifest from the Doctrine in the First Part, concerning the *Locus Apparens* of an Object through such a Glass. To which therefore I refer.

Secondly. If your naked Eyes can read a moderate Print at the full extent of your Arms, or at the distance of about two Feet or two and an half; and you desire a Pair of *Spectacles* to read with, at the usual Distance of reading, *viz.* about a Foot or little more: Procure a Pair of Glasses of such a Convexity, that an Object being exposed before them at the Distance of about a Foot, they may have their *Imaginary Focus*, or the *apparent Place* of the Object, distant from them about two, or two Feet and an half. All which may be easily obtained and effected by the Doctrine in the First Part. Barrow, *Lect. Opt.* 14. *p.*103.

In the next place, for relieving the Eyes of the *ſhort-ſighted*, *pur-blind*, or *Myopes*. We muſt conſider, that theſe, laboring under the contrary Defect with *old Men* (for they ſee *nigh* Objects *diſtinctly*, but *diſtant* Objects *confuſedly*), muſt be relieved with a Remedy of a contrary effect; and therefore they are helped by *Concave-Glaſſes*, which bring the Rays of *diſtant* Objects into the Eye, as if they were *nigh*. And becauſe we may conceive the Cryſtallines of theſe Eyes as too *protuberant* or *convex*, therefore we are to take off from this too great *Convexity*, by adding an adventitious *Concavity*. But then, there is nothing ſo univerſally complained of by thoſe who are thus affected, as the Difficulty they find in fitting proper Glaſſes to their Eyes. For the removal whereof, the following Rules may be obſerved.

Firſt, That for viewing *diſtant* Objects, according to what is noted in the Firſt Part, *Prop.* XLV. If a *ſhort-ſighted* Perſon can read diſtinctly, or ſee Objects at the diſtance of a Foot from his naked Eye; a *Concave-Glaſs*, whoſe *Virtual* Focal-Length is a Foot, makes ſuch a Perſon ſee diſtant Objects diſtinctly. And ſo of any otherwiſe diſpoſed Eye. So that knowing the Diſtance at which a pur-blind Perſon reads diſtinctly with *unarmed* Eyes, 'tis eaſie, by the Doctrine in the Firſt Part, to aſſign him a proper Glaſs for his Eye to ſee *diſtant* Objects.

Secondly, For Glaſſes proper for *Myopes* to *read* by, or to ſee Objects at the diſtance of about a Foot and half. Let us ſuppoſe Eyes ſo affected, as not to be able to read, but at the diſtance of Four Inches; and that we deſire Glaſſes for theſe Eyes to read by, at the ordinary diſtance of about a Foot and half: Let us form ſuch *Concave*-Glaſſes, which, being expoſed to an Object at the diſtance of a Foot and half, may have their *Virtual reſpective* Focus at the diſtance of four Inches. And ſo likewiſe for any other Diſtance. All which is eaſily performed by thoſe verſed in the Firſt Part of this Work.

Barrow, Lect. Opt. 14. p. 102.

Thirdly,

Thirdly, We are to note, that *Myops* ſhall require *different* Glaſſes for viewing Objects at *different* Diſtances. But the viſive Faculty not being contained within ſuch ſtrict and determined Limitations; that Glaſs which is uſeful at an Object an hundred Foot diſtant, ſhall ſerve likewiſe at an Object diſtant fifty Foot; but then 'tis not ſo helpful at one diſtant five Foot. But another may be had proper even for this Diſtance, and not ſo uſeful at an Object diſtant an hundred or fifty Foot. All which is manifeſt from the Doctrine in the Firſt Part.

Fourthly, There are ſome Eyes ſo ill conformed, that no Glaſſes whatever will relieve them. Of this I have often heard a very ingenious Man and great Philoſopher Sir *William Petty* often complain in his own particular. But then this proceeds not from a *too little*, or *too great* a Convexity in the Cryſtalline; but from ſome other Indiſpoſition or ill Configuration, not to be relieved by Glaſſes.

Laſtly, Perſons *pur-blind* labour under this great Inconvenience, that the Glaſſes which relieve them in one particular, do hinder their ſtrong Viſion of diſtant Objects in another particular. For as Concave-Glaſſes do order the Rays from any one ſingle Point, properly to be received by the Eye of a *Myops*: So at the ſame time, they *diminiſh* the Appearance of the whole Object. And from hence it is, that tho theſe ſort of Eyes may be well enough relieved for *Reading* and *Writing* at a *convenient* diſtance, and for ſeeing pretty large Objects at the diſtance of 100, 200, or 500 Foot; yet for Objects *much farther*, unleſs they be very *large* indeed, they are not ſo eaſily ſupply'd.

M. Hook's Contrivance for helping Myopes conſidered.

(3.) And here I cannot but take notice of an ingenious intimation of Mr. *Hooks* (to whom the World is certainly much obliged for his curious Contrivances in *Mechanicks*, and his other Philoſophick Indeavours) publiſhed in *Num*. 3. of the *Philoſophical Collections. Lond*. 1681. which he calls *Myopibus Juvamen*.

'Tis briefly this, That ſome ſort of *ſhort-ſighted* Perſons, who cannot be relieved by *Concave* Glaſſes, may perhaps find ſome help in *Convex*-Glaſſes; their Eyes being removed at a convenient diſtance farther from theſe Glaſſes than their *Diſtinct Baſes*. As for reading by theſe Glaſſes; the Book muſt be *inverted*, and then the Image in the *Diſtinct Baſe* ſhall be *erect*, and the Eye ſhall perceive it *erect*. As for Writing, the Difficulty is greater than mentioned by the *Learned Author*: For the *Myops* muſt not only learn to write *inverted*, but alſo *retrograde*, *viz.* from the Right to the Left Hand; and, what is yet more inconvenient, from the bottom towards the top of the Page; which is hardly practicable on account of blotting the wet Writing. As for viewing *diſtant Objects* with theſe Glaſſes; I acknowledg with the ingenious Author, that much of the Diſagreableneſs of the *inverted Proſpect* is taken off by uſe and cuſtom, as I my ſelf have experienced by my frequent uſe of inverting Teleſcopes. But yet I cannot go ſo far with the Author, as to aſſent to his Deduction from hence; which is, that 'tis only uſe and cuſtom that makes us judge Objects *erect* that are perceived by an *inverted* Image on the Fund of the Eye. For then a Man *ſtanding on his Head*, ſhould judge the Trees and other Objects he ſees *inverted*; which no one in his Senſes will do, but rather judge what is right, *viz.* that he himſelf is *inverted*, whilſt the circumjacent Objects continue *erect*. This will be more evident to us, by conſidering the caſe of an adult Perſon, who has been blind from his Birth, and now ſuddenly reſtored to his Sight: He is not prejudiced by cuſtom, and yet (doubtleſs) would judge as is uſual. But laſtly, an other great Difficulty that will attend the uſe of theſe Spectacles for *Myopes*, is, that they muſt be carried at ſuch a *diſtance* from the Eyes, that it will be very troubleſom to manage them commodiouſly. And if to this again we add the diſtance requiſite for the Object, I queſtion whether ſome Mens Arms will be long enough

to

to manage a Pen for Writing, or turning the Leaves of a Book.

I have been the longer on this Propoſal, becauſe the *Worthy Author* does candidly invite all, to communicate to the Publick, what real Benefit by it, or Objections againſt it, they ſhall find.

(4.) I ſhall conclude this Chapter with the way of Adapting *Teleſcopes* and *Microſcopes* to *defective Eyes*. Which is briefly thus; *Adapting Teleſcopes and Microſcopes to defective Eyes.*

A Teleſcope compoſed of a *Convex* Object-Glaſs and *Concave* Eye-Glaſs, being apply'd to an *old* Eye, the Eye-Glaſs may be a little *farther* removed from the Object-Glaſs than ordinarily: On the contrary, in ſuch a *Teleſcope* for the Eye of a *Myops*, the Eye Glaſs may be removed a little *nigher* to the Object-Glaſs; the reaſon hereof is manifeſt from what has been delivered in the Firſt Part. But then, *how much farther*, or *nigher* they are to be removed, is only to be determined by Experiment, and by every ones fitting the Glaſs to his own Eye.

And ſo likewiſe in Teleſcopes compoſed of a *Convex* Object-Glaſs, and *Convex* Eye-Glaſs: For the Eye of an *old Man*, the Eye-Glaſs may be removed a little *farther* from the Object-Glaſs, or from the *Diſtinct Baſe*. And for the Eye of a *Myops* a little *nigher* to the Object Glaſs, than for Eyes *naturally* and *orderly* affected.

In like manner for *Microſcopes*; Firſt the ſimple or ſingle Convex for an *old* Eye, is to be removed a little *farther* from the Object; and for a *Myops*, a little *nigher* the Object, than the uſual poſture.

And ſo in *Double Microſcopes*. For an old Eye, the Eye-Glaſs is to be a little *farther* from the Object-Glaſs or *Diſtinct Baſe*; and for a *Myops*, a little *nigher* the Object-Glaſs or *Diſtinct Baſe*. And becauſe in theſe, the *Diſtinct Baſe* is brought *nigher to*, or *farther from* the Eye-Glaſs (without altering the

Diſtance

Diſtance of the Eye-Glaſs and Object-Glaſs), only by removing the whole Microſcope *nigher to* or *farther from* the very Object: This latter Motion effects the ſame as the former; and therefore may be uſed for *old* or *ſhort* Sights inſtead of the former.

CHAP. IV.

Of Mechanick-Dioptricks,

(1.) Chief Authors that have treated of Grinding or Forming Optick-Glaſſes. (2.) For trying whether a Glaſs be not plain. (3.) For finding the Focal Lengths of Glaſſes. (4.) Concerning the Centre of a Glaſs. (5.) For trying the Regularity and Goodneſs of an Object-Glaſs. (6.) Managing great Glaſſes, and the Author's Treating thereof. (7.) Proportioning Glaſſes in Teleſcopes (8.) Mr. Hooks's *Contrivance for making an Object-Glaſs of a ſmall Sphere ſerve a long Tube.*

Authors treating of Glaſs-grinding. (1.) I Deſign not in this Chapter to deliver at large the ſeveral ways of Grinding and Forming *Optick-Glaſſes*, the manner of making the *Forms*, *Tools*, or *Diſhes* wherein they are ſhaped; or the various *Machines* contrived by ſeveral ingenious Heads for this pnrpoſe: For this may juſtly require a particular Treatiſe by it ſelf. And becauſe nothing of this kind has ever yet appeared in Engliſh, according as I find the preſent Work accepted, I may perhaps hereafter attempt ſomething in this way, for the ſatisfaction of ſome ingenious *Engliſh* Spirits, who may be inclinable to offer at theſe Exerciſes. In the mean time, they that are Maſters of the Languages wherein they are written, may apply themſelves to *Pere Cherubin*'s *Dioptrique Oculaire*, and *Zahn*'s *Oculus Artificialis*; wherein they will find an abundance on this Subject. But after all that can be writ concerning it, Practice and Experience will find out many

Cherubin.

Zahn.

many Conveniencies and Inconveniencies, which can hardly be committed to Words, or deſcribed; and therefore this is at laſt the beſt Inſtructour. But here I ſhall mention the chief Modern Authors that have contrived *Engines* for Grinding Glaſſes, that thoſe who pleaſe may conſult them. Mr. *Hook* in his *Micrographia* deſcribes an Engine for this purpoſe. *Hevelius* in his *Selenographia*, *Chap.* I, II. deſcribes another for Modelling the Forms or Diſhes for Grinding Spherick Glaſſes. And in the Firſt Part of his *Machina Cœleſtis*, *Chap.* XXIII. deſcribes one for Grinding *Conick-Glaſſes.* *Ant. Mar. Schyrleus de Rheita* in his *Oculus Enoch & Eliæ*, *Lib.* IV. has a Machine for *Conick-Glaſſes.* *Maignan* at the end of his *Perſpectiva Horaria* deſcribes Machines both for Spherick and Conick-Glaſſes. *Des-Cartes* in his *Dioptricks* has another for *Conick-Glaſſes.* Monſ. *Borelly* has given the World the Secret of his manner of Grinding great Glaſſes in Cypher, *Journal des Scavans*, *Ann.* 1676. *July* 6. but has not yet obliged us with the Diſcovery: Tho he be a Perſon of the greateſt Candor and Freedom, and the moſt communicative; as I am obliged to expreſs with much Gratitude for his Civilities ſhew'd me in *Paris* 1685. at which time he gave me an Object-Glaſs formed by this way for a Teleſcope 24 Foot long. The celebrated Monſ. *Fatio de Duillier* (of whom Dr. *Burnet* gives deſervedly ſo excellent a Character in his Letters of Travails) in the *Journal des Scavans*, *Ann.* 1684. *Novemb.* 20. deſcribes an Invention of his own, for exactly forming the Diſhes for Grinding Spherick Glaſſes; which indeed is very ingenious, and perfectly new: But I cannot tell how eaſily it may be practiſed, without ſome farther Improvement; for his Contrivances of the Block *c*, and of the Screws *d*, *g*, (I refer to his Figure) ſeem not to have all the Motions requiſite, to keep the Point *e* exactly *true* to the Axis of the little Teleſcope *h k*.

Hook. Hevelius. De Rheita Maignan. Des-Cartes Borelli. DeDuillier

The

Wren.

The Incomparable Sir *Christopher Wren*, our *English Archimedes, Apollonius, Diophantus*, proposes his Contrivance for Forming *Hyperbolick* Glasses, *Num.* 48. and 53. of the *Philosophic. Transact.*

Conick-Glasses not better than Spherick. From Mr. Newton.

But all farther Endeavours for Forming *Conick*-Glasses, which have hitherto been wholly frustrated and unsuccessful, may now be put to a full stop; when we hear in this matter the Opinion of as great a Philosopher and Mathematician, as this or any Age could ever boast of, *The Celebrated Mr.* Newton *of* Cambridge, who in his profound Treatise, *Philosophiæ Naturalis Principia Mathematica*, has fathom'd the greatest Depths of Nature, and laid a Foundation for Posterity to raise an infinite Superstructure. Thus he, in the First Book, *Schol. ad Prop.* XCVIII. *Ad usus autem Opticos, &c.* In *English* as follows, *But for all Optick Uses, Spherick Figures are the most commodious. If the Object-Glasses of Telescopes were composed of two Spherick-Glasses containing Water between them, perhaps the Irregularity of the Refractions that are made on the Surfaces of the Glasses towards their edges, may be accurately enough corrected by the Refractions of the Water. And such Object-Glasses are preferable to Elleptick or Hyperbolick-Glasses; not only because they are easier and more accurately to be formed; but also because they refract more accurately those Pencils of Rays that are* (collateral or) *out of the Glasses Axis. But the different Refrangibility of different Rays, will for ever hinder us from perfecting Opticks by Glasses either of Spherick, or any other Figures whatsoever. And unless we can correct the Errors that arise from hence, all our Labour is lost in other Corrections.*

And indeed if we consider it right, we shall find it impossible, by whatever Figures to render the Appearance of the *Collateral* Parts of an Object so *distinct* as the *direct*; for the very natural Eye does it not; and therefore we are forced to apply it successively *directly* before the Parts of any Object we design to view: And we may well despair to perform by Art, more

more than what the Almighty Framer of the Eye has given us a Pattern for.

What this *Great Man* means by the *Different Refrangibility* of *Different Rays*, We may find, *Num.* 80. *p.*3075. *N.* 83. *p.*4059. *N.* 84. *p.* 4087. *N.* 85. *p.* 5004. *N.* 88. *p.* 5084. *N.* 110. *p.* 217. *N.* 121. *p.* 499. *N.* 123. *p.* 556. *N.* 128. *p.* 692. of the *Philosoph. Transactions*; wherein he lays down a perfectly new and most ingenious Theory of *Light*.

Different Refrangibility of Rays.

In the same Tracts, *Num.* 81, 82, 83. we may find an Account of a new *Cata-Dioptrical Telescope*, invented by this same excellent Person.

Cata-Dioptrick Telescope.

(2.) All that I shall offer at more in this Chapter, is, to lay down some practical Rules for finding the *Foci* and *Centres* of Glasses; with some other *Accidental Remarks*, that may be requisite to the clearer understanding and performance of some Precepts delivered in this Treatise.

Trying whether a Glass be not plain.

The Object-Glasses of Telescopes are generally of so little Curvity on their Superficies, that by *looking on* them or by *feeling* them, it cannot be discovered, whether they are *plain* or of a *Spherick* Figure; or which is formed on a Sphere of a *greater*, which of a *lesser* Radius. To find this (as I have said before, in *Schol. Prop.* XXXI. *Part* I.) we are to shake the Glass nimbly at our Arms length before the Eye; and if the Objects seen through it seem to *dance* or *move*, the Glass is not plain. And that Glass which makes the Objects seem the most to move is formed on the *less* Sphere, whether *Convex* or *Concave*.

(3.) When we have thus found our Glasses to be *Spherick*. Then supposing them *Convex*, there are several Methods for finding their *Foci*. I shall lay down some of the plainest and most certain.

For finding the Foci of Glasses.

First, for Glasses of pretty deep Convexities (that is, of small Spheres), apply them to the end of a Scale of Inches and Decimal Parts, and expose them before the Sun; and upon the

Scale, we ſhall find the bright Interſection of the Rays exactly meaſured out. Or expoſe them in the Hole of a dark Chamber; and where a white Paper receives the diſtinct Repreſentation of *diſtant* Objects, there is the Focus of this Glaſs. This is an univerſal and certain way for all Convexes. For a Glaſs of a pretty long *Focus*, obſerve ſome diſtant Objects through it, and recede from the Glaſs, till the Eye perceive all in *confuſion*, or till the Objects begin juſt to appear *inverted*; here the Eye is in the Focus. If it be a Plano-Convex Glaſs, make it reflect the Sun againſt a Wall; we ſhall on the Wall perceive two ſorts of Light, one more *bright* within an other more *obſcure*; withdraw the Glaſs from the Wall, till the *bright* Image is at its ſmalleſt; the Glaſs is then diſtant from the Wall about the fourth part of its *Focal* Length. If it be a double Convex, expoſe each ſide to the Sun in like manner, and obſerve *both* the Diſtances of the Glaſs from the Wall: The firſt Diſtance is about half the Radius of the Convexity turned from the Sun, and the ſecond Diſtance is about half the Radius of t'other Convexity likewiſe: Thus we have the Radii of the two Convexities; whence the Focus is determined by *Prop.* III. *Part* I. The reaſon hereof depends on the Doctrine of *Catoptricks*.

But the moſt exact way of determining the juſt Focal Length of the Object-Glaſs of a Teleſcope, is what I ſhall lay down in *Chap.*V. *Sec.* 4. of this Part, which, becauſe I muſt neceſſarily deliver in that place, I will not here anticipate.

Foci of Concaves. The Foci of Concaves are obtained by *Reflection*; for as a *Concave Miroir* or *Speculum* burns at the diſtance of about half the Radius of the Concavity; ſo a *Concave-Glaſs* being ſuppoſed a *Reflecting Speculum*, ſhall unite the Rays of the Sun, at the diſtance of about half the Radius of the Cavity.

Concerning the Centres of Glaſſes. (4.) Before we proceed to the *Centration* of Glaſſes, we are to recollect the 18th. *Definition* of the Firſt Part, wherein we define the *Axis of a Glaſs*. I ſay therefore, when the *Axis* of

a Glaſs paſſes directly through the Centre of the Glaſſes Breadth, or roundneſs of its Aperture, that Glaſs is ſaid to be *truly centred*. And this will always be ſo when the Glaſs is *equally thick* round the edges of its *Aperture*. But this will be more intelligible by a Scheme *Tab*. 40. *Fig*. 1. Let *a g d e a* be a Plano-Convex Glaſs, much thicker towards the Edge *d e*, than towards *a*; let *a g* be equal to *g d*, *g* is the *Centre* of this Glaſs, for 'tis the middle Point of its Aperture or Breadth. But then this Glaſs is not *truly centered*: For, to *l* the Centre of the Convexity *a g d* draw *g l*, I ſay *g l* is not the Axis of this Glaſs; but if there may an other Line *l k* be drawn, whoſe Portion within the Glaſs *i k* is greater than *h g* (as in this Caſe it may eaſily be demonſtrated. For $lk = lg$ and *l h* more than *l i*. For $\angle lih =$ Rect. therefore *h g* leſs than *k i*) and which being perpendicular to the plain Surface *a e*, as well as to the Convex Surface *a k d*; *l k* muſt conſequently be the Axis of this Glaſs. Wherefore *k* is the *true Centre* of this Glaſs. And becauſe, to compleat this Glaſs, there is wanting the Portion *d f e*; therefore to make the true Centre of the Glaſs *k*, coincident with the middle Point of its Breadth or Aperture; we are to make *k b* equal to *k d*; and then we are to cut off, or to cover the Portion of the Glaſs *b a c* equal to *d f e*; and ſo we obtain the *compleat* and *truly Centered* Glaſs *c b k d e c*. The ſame may be underſtood of *double Convexes*, without farther Explication. T.40.F.1.

But becauſe in Object-Glaſſes of even moderate Lengths, 'tis impoſſible by the Eye, or any Admeaſurement of their Thickneſs, to know whether they be *truly Centered*: Therefore we muſt have recourſe to ſome other Methods, that may ſhew this. And theſe are various.

Firſt, Holding the Glaſſes at a due diſtance from the Eye, let us obſerve the two reflected Images of a Candle; and where theſe two Images *unite* or *coaleſce*, there is the *true Centre* of the Glaſs; if this be in the *middle* of the Glaſſes *Breadth*, the Glaſs

 is

is *truly Centred*; if not, we are to rectifie it, ſhall be declared hereafter.

A ſecond way is, By preſenting the Glaſs before the Sun, and making it reflect the Light on a Plain nighly parallel to its Surface, at a proper diſtance; and we ſhall perceive two ſorts of Light reflected; one *ſmaller*, but much more *ſtrong* and *vigorous*, within another more faint and large. Then by a due poſture of the Glaſs (found by Tryals) both theſe Lights are to be projected as round as poſſible; and at a proper diſtance from the Wall on which they are reflected; the *round brighteſt* Spot is to be brought into the ſmalleſt compaſs that it can (Tryal will make all this plain). When the Glaſs is in this poſture, if the *bright* Spot be projected juſt in the middle of the fainter Light, the Glaſs is *well centred*. If it be projected to the Left Hand of this middle, the Glaſs is thickeſt towards the Left Hand Edge. And ſo to whatever ſide of the faint Light, this bright Spot is projected, on *that ſide* is the Glaſs *thickeſt*; and on *that ſide* lies the *true Centre*. The reaſon hereof I ſhall explain in a Plano-Convex-Glaſs, and is the ſame in a double Convex; *mutatis mutandis*. For the bright Spot is the Image of the Sun projected by the curve Surface of the Glaſs conſider'd as a *reflecting Speculum* (whereof more hereafter), and the faint Light is the Reflection of the Sun from the other plain Surface. T. 40. F. 2. *Tab.* 40. *Fig.* 2. *a d e* is a Plano-Convex-Glaſs, thicker towards the ſide *d e* than towards *a*. Let the Curvity *a d* (whoſe Centre is *c*) be expoſed directly to the Sun: By the known Laws of *Catoptricks*, the Parallel Rays (from the middle Point of the Sun, for inſtance) falling on the Curvity (which parallel Rays in the Figure are expreſſed by continued Lines), are united in the Focus at *f*, diſtant from the Curvity about half its Radius, there cauſing a brisk Light and Heat. In the mean time theſe parallel Rays do fall on the plain Surface *a e obliquely*, and

and are reflected thereby alongst the prickt Lines; and these are they which cause the *faint* Light. Now 'tis manifest, that if a Plain at *f*, parallel to the plain Surface *a e*, received these two Reflections, the *bright* Light at *f* would not be found in the middle of the *faint* Light: For here we see the *faint* Light is thrown much to one side. And the *bright* Light *f* is projected upwards from the middle of the *faint* Light; for the Glass is thickest upwards towards its Edge *de*.

The third Way of Examining the *Centres* of Glasses is yet more *compleat* than the former; for it does not only discover the Fault (if there be any, as in long Object-Glasses 'tis very rare but there is; especially if they be wrought in the Form by the unguided Hand, and not by Engine), but withal, it rectifies the Fault. 'Tis thus; Cover the Surface of the Glass with a thin piece of Paper, in which there is cut a round Hole of about an Inch diameter, and round about this Hole there are to be struck two or three Concentrick Circles; move this Paper upon the Glass, till you see, on the plain that receives the reflected Light, that the *bright* Spot is exactly in the middle of the other fainter Light round it. This also one may measure by a Pair of Compasses, having, to that end, slightly fixed the Paper to the Glass, that we may more nicely determine, whether this *bright* Spot be exactly in the middle. This therefore being carefully adjusted by gently sliding the Paper on the Glass (if it be requisite), we are, without the least altering this *true* Position of the Paper, to fix it more firmly *to* the Glass. And laying it thus on a Table, let us mark on the Glass (by the Point of a Diamond) three Points in one of the Circumferences Concentrick to the round Hole in the Paper. And sticking a small piece of Cement on the Glass about the middle of the round Hole; by means of the three marked Points, let us find the exact Centre of this round Hole. Then uncovering the whole Glass (except only the Ce-

ment in which the Centre is marked), with a Diamond-pointed Compaſs, let us ſtrike as large a Circle on the Glaſs, as its Breadth will bear. Then *round* the Glaſs according to this Circle, and 'tis as exactly *centred* as the Senſe can judge.

But note, That whereas I have ſaid that the *brighter* Spot is ſmaller than the *fainter*; and that the *brighter* Light is to be reflected *into* the *middle* of the *fainter*. This is to be underſtood, ſuppoſing the Breadth of the Glaſs will *allow* it. For the Glaſs may be ſo narrow, that the Projection of the bright Image of the Sun may be *broader* than the Breadth of the Glaſs. But this happens ſo ſeldom in Practice, that I paſs it over.

Pere Cherubin, who is often very nice in matters of little moment, and looſe enough in thoſe of greater weight and abſolute neceſſity, deſcribes an implicated Contrivance for true Centring of Glaſſes. *Viſion Parfait. Tom.* II. *p.* 109.

For trying the Regularity and Goodneſs of an Object-Glaſs.

(5.) The ſame Frier lays down a Way of Examining the Regularity and Goodneſs of an Object-Glaſs, *pag.* 25. which it may not be amiſs here to inſert. After we have *centred* the Object-Glaſs as well as we can, by the foregoing Method: To try the Regularity of its Form to the greateſt exactneſs, *ſays he*, We muſt do thus. On a Paper ſtrike two Concentrick Circles; one whoſe Diameter is the ſame with the Breadth of the Object-Glaſs; t'other of half that Diameter. This inward Circumference divide into ſix equal Parts, by the known way of applying the Radius ſix times in the Circumference, and making ſix fine ſmall Holes therein with a Needle. Let us cover one ſide of the Glaſs with this Paper; and then expoſing it to the Sun, we are to receive the Rays that paſs through theſe ſix Holes on a Plain at a juſt diſtance from the Glaſs. And by withdrawing or approaching this Plain from or to the Glaſs, we ſhall find, whether the Rays that paſs through theſe ſix Holes unite exactly together at any diſtance from

from the Glaſs; if they do, we may be aſſured of the Regularity of this Glaſs, that is, of its *juſt Form.* And at the ſame time we obtain exactly the Glaſſes *Focal Length.*

But after all, there is no better way for trying the Excellency of an Object-Glaſs, than by placing it in a Tube; and trying it with ſmall Eye-Glaſſes at ſeveral diſtant Objects. For that Object-Glaſs that repreſents the Objects the *brighteſt* and moſt *diſtinct*, and bears the *greateſt Aperture*, and *moſt* Convex or Concave Eye-Glaſs, without colouring or Hazineſs, is ſurely the beſt. The moſt convenient Object to try them at, is the *Title-Page* of a large Book; wherein there are generally Letters printed of divers Magnitudes, and therefore affords variety of ſmall Objects; whereby the comparative Excellency of Object-Glaſſes may be nicely eſtimated. This the Celebrated Monſ. *Caſſini*, the *French* King's Aſtronomer, ſhew'd me when I viſited him at the *Obſervatory* in *Paris*, *Ann.* 1685. who tryed all his Glaſſes by the large Title-Page of a Book, fixt inverted on the Jaume of a Steeple Window more than ½ of a Mile diſtant from the *Obſervatoire.*

(6) The next Piece of *Mechanick-Dioptricks*, which I ſhall mention, is, The *Managing Great Glaſſes.* And herein I ſhall not ſwell this Volume with deſcribing thoſe ſumptuous Contrivances and coſtly Machines invented for this purpoſe: It ſhall ſuffice me to refer the Reader to the Original Authors, where he may find them deſcribed at large. *Manageing great Glaſſes.*

Hevelius in his *Machina Cœleſtis*, *Part.* I. *Cap.* 19, 20, 21, 22. deſcribes the Engines he uſed for his Teleſcopes. And amongſt others, a Contrivance for managing his Tube of 60 Foot, and another of 150 Foot long. Hevelius.

The deſervedly Celebrated Monſ. *Hugens*, one of the chief Mathematick Luminaries of the preſent Age, has publiſh'd a ſmall Tract, *Aſtroſcopia Compendiaria*, deſigned only for Deſcribing his way of Managing great Glaſſes with very little trouble Hugens.

ble, and without a Tube. This I am ſure is no barren Speculation of the Ingenious Author's, but ſucceſsfully practiſed by him; as I can gratefully teſtifie, having had the favour of being ſhewn the whole Contrivance by the Excellent Author himſelf in his Garden at the *Hague*, *Ann.* 1685. at which time I had the happineſs alſo of ſeeing his *Planetary Clock*, or *Moving Ephemeris*, a Machine that cannot be ſufficiently admired.

Planetary Clock.

Cuſſet.

Monſ. *Cuſſet*, an ingenious *French Man* of *Lions*, has publiſh'd his Contrivance for managing great Glaſſes, in the *Journal des Scavans*, *Ann.* 1685. *May* 18.

Monſ. *Caſſini*, when I was with him at the *Obſervatoir* in *Paris* (amongſt other Curioſities, which according to his uſual Candour and Civility he communicated to me) ſhew'd me two very pretty Contrivances for managing great Glaſſes; which, becauſe not yet publick, I ſhall deſcribe as well as I can by Memory at this diſtance of time. The firſt was a plain Piece of Clock-work, moved by a Spring and regulated by a Pendulum Vibrating half Seconds. This carried an Arm that ſtood ſomething prominent from the Body of the Work; which Arm at its extremity carried the Object-Glaſs fixt in a Ring. This Arm and the Object-Glaſs, by means of graduated Arches in the Clock-work could be turned *ad Libitum* directly to any Star. Thus ſuppoſe *Saturn* were to be obſerved; by a common *Ephemeris*, knowing his Longitude and Latitude, together with the Time of Day or Night, the Arches of the Clock being put to ſuch and ſuch Diviſions, and the Machine it ſelf placed with ſuch or ſuch a part horizontal or perpendicular; the Object-Glaſs was of courſe directly expoſed to the Star. Then the Pendulum being put in motiou, the Machine kept the Glaſs conſtantly expoſed directly to the Star in its diurnal Motion. This whole Machine then, being placed upon an Height, carried the Object-Glaſs in its due poſition; and the *Obſerver below* managed the

the Eye-Glaſs by his free Hand, aſſiſted with a *Reſt*. Tho I cannot retain ſo exact a Remembrance of this Engine, as to venture at a Scheme of the particular Parts; yet this Deſcription perhaps will be ſufficient to give the ingenious Aſtronomer an Idea thereof, ſo as to apprehend the Contrivance in general. Something much of the ſame kind may we find deſcribed in Mr. *Hooks Animadverſions on Hevelius Mach. Cœleſtis*. p. 66, 67, 68, *&c.*

The other Contrivance he mentioned to me, if I forget not, he or Monſ. *Borelly* told me, was due to Monſ. *Azout*. It is thus; From the Tube of a ſmall Teleſcope there ſtands out an Arm perpendicular to the ſide of this ſmaller Tube: This Arm carries the great Object-Glaſs; then an Obſerver upon an Height manages this ſmall Teleſcope, following therewith the Motion of a Star; by which means the great Object-Glaſs (being parallel to the Object-Glaſs of this leſſer Teleſcope) is kept conſtantly in proſecution of this ſame Star: Then the other Obſerver *below* manages the Eye-Glaſs as before.

Theſe two Contrivances are indeed pretty Thoughts; but I cannot promiſe that they can be ſo eaſily practiſed; unleſs the Machines that carry the great Object-Glaſs be made to *riſe* and *fall* at pleaſure, as the Star riſes or ſets. For otherwiſe, the Obſerver that manages the Eye-Glaſs, ſhall ſoon loſe the ſight of his Object; unleſs he have the opportunity of *riſing* and *falling*, by ſome ſuch Contrivance as the forementioned Monſ. *Cuſſet* propoſes in the fore-cited place. But this is vaſtly chargeable.

'Tis now above ſeven Years ſince Monſ. *Boſſat* of *Thorouſe* has promiſed the World his Contrivance for managing great Glaſſes; of which he has given a ſmall Specimen in the *Journal des Scavans* 1682. *Dec.* 28. but we hear nothing farther of it; perhaps becauſe the Contrivance requires *reflecting Speculums*, which much weaken the light of the Object, and are therefore found uſeleſs. *Boſſat.*

Propor-tioning of Glasses in Telescopes and Mi-croscopes.

(7.) I may reckon another Piece of *Mechanick Dioptricks*, The *proportioning* of the Glasses in Telescopes and Microscopes. We have said before, that, of two or more Object-Glasses of the same Focal Lengths; that is the best, which will bear an Eye-Glass of the greatest Convexity (this is usually called the Deepest Charge): But yet there are some Proportions to be observed, which Experience has found out, as the most convenient and best adapted for most Mens Eyes, that are well disposed. For so a good Object-Glass of 12 or 13 Feet, will bear a *Charge* of 3 Inches, better than a *Charge* much *deeper* or *shallower*:

But moreover, In adapting an Eye-Glass to an Object-Glass, respect is likewise to be had to the *Object* we contemplate; for Objects of a sedate Light, as *Saturn*, *Jupiter*, &c. will allow deeper *Charges*, than those of a more brisk and strong Light, as *Venus*, &c.

Wherefore this whole Affair being only the Subject of Experiment, to that I shall refer, and only hint by the bye; That for Telescopes of three Convex Eye-Glasses, *Cherubin* advises (*Dioptrique Oculaire*, III. *Par. Sec.* 2. *Cap.* VII. *pag.* 188.) that, of the three Eye-Glasses, that next the Object-Glass should be of the *deepest* Charge; the middle Eye-Glass ought to be something *shallower*; and the immediate Eye-Glass the *shallowest* of all.

For Proportioning Glasses in *double* Microscopes, we may consult the same Author. But 'tis tedious to transcribe.

The LV. *Prop.* of the First Part, relates to the *Apertures* of Object-Glasses. After which, we have nothing to add in this place; which otherwise might have been challenged by a Discourse thereon, as being of a *Mechanick* Consideration.

M. Hooks *Contri-vance to make a Glass of a small Sphere serve a long Tube.*

(8.) The last Piece of *Mechanick Dioptricks* I shall mention, is an ingenious Thought of Mr. *Hooks*, for making a short Object-Glass perform the part of one formed on a much larger Sphere.

'Prepare

"Prepare (says he) two Glasses, the one exactly flat on "both sides, the other flat on the one side, and Convex on "the other, of what Sphere you please. Let the flat Glass be "a little broader than t'other. Then let there be made a "Cell or Ring of Brass very exactly turned, into which these "two Glasses may be so fastened with Cement, that the plain "Surfaces of them may lye exactly parallel, and that the Con- "vex-side of the Plano Convex-Glass may lye inward, but "so as not to touch the flat of the other Glass. These being "cemented into the Ring very closely about the edges: By a "small Hole in the side of the Brass Ring or Cell, fill the "interposed space between these two with *Water*, *Oyl of Tur- "pentine*, *Spirit of Wine*, *Saline Liquors*, *&c.* then stop the Hole "with a Screw: And according to the differing Refraction of "the interposed Liquors, so shall the Focus of this compoud "Glass be longer or shorter. *Vid. Philosoph. Trans. Num.* 12. *pag.* 202.

This, I must confess is an ingenious Hint: But I doubt the desired Effect will not be so successfully attained thereby, so as to constitute an Object-Glass for a Telescope. For certainly, were it effectual; 'tis so easie and withal so useful, that before this time it would have obtained, and been practised *universally*. And this makes me question, whether it would be of any better effect, than a *Meniscus-Glass*, or a Combined Glass of *Prop.* XVII. *Part.* I.

CHAP. V.

Of Telescopick-Instruments.

(1) *Controversie between* Hevelius *and* Hook *concerning Telescopick Sights.* (2.) Hevelius's *Objections against them. His Mistake concerning them.* (3.) *Their Fabrick or Contrivance.* (4.) *Adjusting them to plain Rulers or Tubes.* (5.) *To Quadrants, Sextants, &c.* (6.) *Dioptrick Reason of their Performance.* (7.) *Adapting the* Micrometer *to a Telescope.*

Controversie between Hevelius *and* Hook *concerning Telescopick-Sights.*

(1.) THE Fame of *Johannes Hevelius, Consul of Dantzick,* is deservedly celebrated by all that delight in Astronomy. His Performances herein are highly extoll'd by all; and the sumptuous Volumes of his Labours and Studies, which he has published, have procured him an immortal Name of Honour amongst the Literate. But notwithstanding all his commendable Endeavours, he has not yet arrived at the height of perfection; but something is yet wanting and deficient even in his most costly Machines. Whoever peruses the large and elegant Volume of his First *Machina Cœlestis,* will admire at the vast Treasure he has expended on Astronomical Instruments of all sorts; when he sees even the very *Description* of them so very sumptuous. And yet, at the same time, whoever peruses a small Book of *Animadversions on this Machina Cœlestis,* by the ingenious Mr. *Hook,* will find one grand *Defect* does attend the noble *Hevelius* Instruments, which renders them (I will not say, useless, faulty, or no better than *Ticho's,* yet) not so compleat and perfect, as otherwise they had been by the Addition of *Telescopick Sights.* For *Hevelius* wholly used *Plain Sights,* which certainly are not so accurate as Telescopick.

copick. And tho I muſt confeſs ingeniouſly, that this renouned Aſtronomer, by his extraordinary Diligence, great Care, and perpetual long-continued Practice, but chiefly by his peculiar ſharpneſs of Sight, had arrived to a great exactneſs of Obſervation by plain Sights (as I find by comparing the Obſervations made by the moſt curious Aſtronomers of our Age, *Flamſteed*, *Halley*, *Caſſini*, *&c.* by Teleſcopick Sights, with thoſe Obſervations made by *Hevelius*;) yet this we are to attribute more to the peculiar acuteneſs of his Eye, and to his extraordinary Diligence and Care in Obſervation, than to the exactneſs of *plain Sights*. For to me it ſeems manifeſt, from what the Learned Mr. *Hook* lays down in the forementioned Book, that the naked Eye cannot ordinarily perceive an Angle (or Object that ſubtends an Angle) leſs than a Minute, or half a Minute at the ſmalleſt. For tho we perceive Stars of that magnitude that their Diameters are not half a Minute; yet this is by a ſort of *adventitious* or *glaring* Light, that is cauſed by the Refraction of their Rays in the Air; which makes them appear to us much bigger than really they are; as is manifeſt, when we come to look at them with a Teleſcope that takes off this *glaring* Light.

(2.) The want therefore of Teleſcopick Sights is what Mr. *Hook* chiefly inſiſts upon, as defective in *Hevelius* coſtly Aſtronomical *Apparatus*: But yet, in his whole Book of *Animadverſions*, he takes no notice of the chief Objections, which *Hevelius* uſes againſt them. And I am perſwaded the Candour of that Noble Aſtronomer (whoſe Memory muſt now be ſacred) was ſo great, that upon the removal of theſe Difficulties, he would have given up the Cauſe; for it ſeems the Controverſie was long agitated between them. Hevelius *Objections againſt them.*

Theſe Objections we ſhall find in the Firſt Part of the *Machina Cœleſtis, Cap.* XIV. *pag.* 296. *Accedit, ſi quando Obſervator non æquè directè & precisè ſemper, ut ſæpius, crede, contingeret,*

tingeret, per Centra lentium collineat, facile diverſitas aliqua aſpe-ctûs obſervationibus poſſit induci, quæ ſuo tempore Cœli ſcrutatores jugiter ſeduceret. Cæterum, cum Acus vel Fila adeo prope lentem Ocularem ad obſervatoris oculum vix in remotione aliquot digitorum ſubſiſtunt; dubito an Dioptra hæc oculo tam propinqua, multo accuratius Stellas quaſvis minimas, quam Pinnacidia noſtra, ad ſex novemve pedes ab invicem remota, poſſit detegere. Nam etiamſi objectum diſtinctius videas; in eo tamen, quod Dioptra tua oculo propius adheret, plus à verò deflectere poteris, quam nos circa noſtra Pinnacidia, quæ tanto ſpatio ab invicem removentur. Ut taceam, quod interſectio filorum minimas Stellas tibi tegat, &c.

For Engliſh Readers thus,

"Add to this, That if at any time the Obſervator chances "not to look directly and preciſely through the midſt of the "Glaſſes (as believe me it may often happen) ſome Varieties "may eaſily intermingle with the Obſervations, which in time "may egregiouſly deceive the Aſtronomer. Moreover, ſeeing "the Needle or croſs Threads, do ſtand ſo cloſe to the Eye-"Glaſs, and near the Eye of the Obſervator; I queſtion whe-"ther theſe Sights, ſo near the Eye, can diſcover the ſmalleſt "Stars much more accurately, than our plain Sights, which "are diſtant from each other Six or Nine Feet. For, tho by "theſe Teleſcopick Sights, one may ſee the Object more di-"ſtinctly; yet becauſe they are ſo nigh to the Eye, one may "err, more than 'tis poſſible by our plain Sights, that are ſo "far aſunder; ſo that I ſhall take no farther notice of another "Inconvenience, which is, that the Interſection of the Threads "ſhall cover the ſmalleſt Stars from your Sight.

His Miſtake concerning them.

Thus far the Learned *Hevelius*. Which ſhews plainly, that he had no right apprehenſion of the Nature of theſe Sights. And therefore the beſt way of reconciling him to them, had been, fairly to have laid down the *Dioptrical Reaſons* of their Performance and Exactneſs. Upon a right underſtanding whereof,

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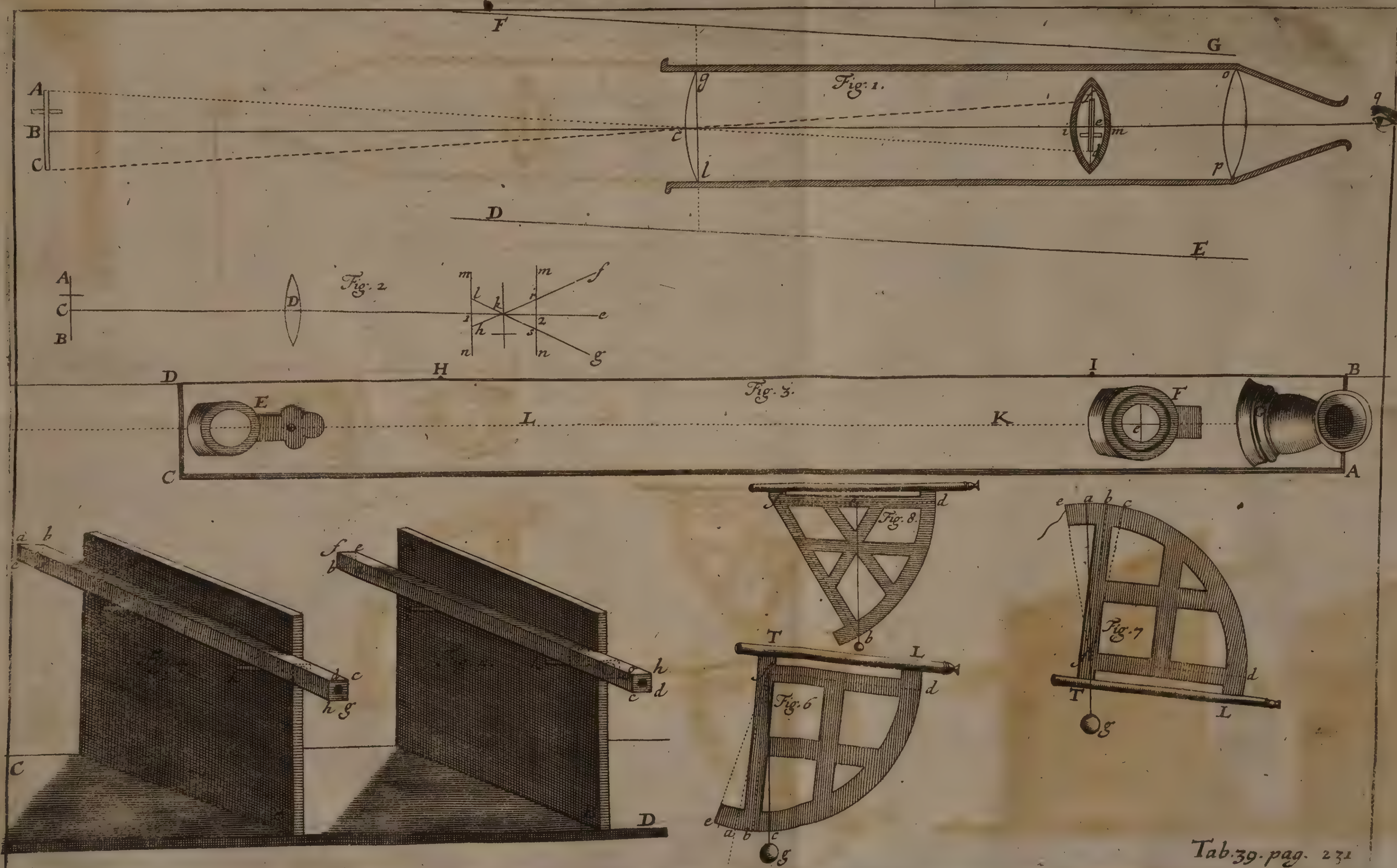


Tab. 39. pag. 231

whereof, all thoſe Objections would be anſwered, and would naturally vaniſh. This had been the right Method of proceeding amongſt *Candid Philoſophers*: Whilſt vilifying his Inſtruments, and ſlighting his Performances with them as no better than thoſe in the Age before him, did but exaſperate the Noble old Man, and made him adhere more obſtinately to his former Practice.

That *Hevelius* did not rightly apprehend the Nature of *Teleſcopick Sights*, is manifeſt by this Objection which he makes againſt them, from the *ſhortneſs* of the *Line* of *Collimation*, which he imagins no *longer*, than between the Eye or Eye-Glaſs, and croſs Hairs; but is really as *long* as between the Object-Glaſs and croſs Hairs: As ſhall be evident from what I ſhall now lay down.

Wherein I ſhall briefly explain their uſual Fabrick or Contrivance; their adjuſting to a plain Ruler, Cylindrick, or ſquare Tube; their adjuſting to Quadrants, Sextants, and other Inſtruments; and the Dioptrick Reaſon of their Performance and Exactneſs.

(3.) And firſt, the Fabrick or contrivance of theſe Teleſcopick Sights is briefly thus, *Tab.* 39. *Fig.* 1. Chooſing an Object-glaſs *g c l* and convex-Eye-glaſs *o p* proper for the length of the Ruler or Tube *g o p l* which we are to uſe. Let us take care that the Object-glaſs be pretty well *centred* (by Chap. 4. Sect. 4. of this part) but in this particular the greateſt exactneſs is not requiſite (whatever *Pere Cherubin* d'Orleans may ſay to the contrary, in the ſecond Tome of his *Viſion Parfait*. Paris 1681. *fol.* in his deſcription of Levels. *pag.* 23, 81, 106, 108, 109. &c. but chiefly *pag.* 107. wherein the Friar is moſt groſly miſtaken) 'tis ſufficient, if the Glaſs be pretty nigh the matter, as uſually moſt Glaſſes are, immediately out of the Workmans hands. This Object-glaſs and Eye-glaſs are each to be fixed ſtrongly on a braſs Ring in the Ruler at their proper diſtance.

Their Fabrick or Contrivance. T.39. F.1.

True Centration of Glaſſes not abſolutely requiſite.

And

And exactly in the Focus of the Object-glass *f m d i* in an other brass Ring are strain'd the finest cross-Hairs *f d*, *i m*; This Ring is contrived to be moveable to the Right and Left hand, towards *f*, or towards *d*, and also (if the nature of the Instrument, to which we affix these Sights, require it) *upwards* and *downwards*; and to be steadily fixed in any posture by screws or otherwise. The Mechanick contrivance where of is obvious enough, and needs not here be described, every one pleasing himself in his own way.

By the Doctrine in the first part 'tis manifest, that this Tube being thus disposed and presented before a Distant Object *ABC*, the Picture or Image of the Object is projected in the Focus of the Object-glass *f e d*; and this Image in the distinct Base is at the plain of the Cross-hairs. Wherefore all the Rays that compose this Image, which escape, or do not fall on, the Cross-hairs, shall arrive at the Eye freely, and distinctly. But the Points in the Image, which are projected, and fall just on the Cross-hairs, are hid by the Cross-hairs from the Eye: and the Hairs themselves appear as if they were *really* stretch'd upon the *very Object*. For they are extended in the distinct Base, which, in this kind of Telescope, is the *Locus apparens* of the Object, by *prop.* 50. *schol.* At the same time the Hairs themselves appear very distinctly to the Eye *q* in the outward Focus of the Eye.glass by *prop.* 32, 33. *schol.*

We may then conceive, that there is *some one* Point in the Object, as suppose B, which sending a Cone of Rays on the Object-glass, the Principal Ray of this Cone, or the Axis thereof, after passing the Object-glass, runs parallel in *c e* to the side of the Ruler or Tube *l p*. Wherefore, if the intersection of the Cross-hairs, *e*, without removing or stirring the Object-glass in the Tube, be brought to meet with this Line, or to cover this Point B in the Object, and be there strongly fixed: Whatever Point, in any Object, shall hereafter be found covered by this crossing of the

Hairs;

Hairs; we may be aſſured that this line of Collimation, *viz.* the line from the Eye to this point in the Object, or the line from this Interſection *e* to the point in the Object, runs parallel to the ſide of the Ruler or Tube, and therefore the ſide of the Ruler is directly pointed towards that mark in the Object. Reſpect being had to the Breadth of the Ruler.

In like manner, ſuppoſing the ſides of the Tube were in the lines D E, F G; and the Object-glaſs fixed upon it in the poſture expreſſed in the Figure. We may conceive ſome point A in the Object, the Axis of whoſe Cone of Rays *A c d* after paſſing the Object-glaſs, runs parallel to the ſides D E, F G. If then the Interſection of the Croſs-hairs *e*, without ſtirring the Object-glaſs in the Tube, be brought downwards to *d*, ſo that it may meet with the line *c d*, and cover the point A in the Object, and the Ring of the Croſs-hairs be there fixed. Whatever point in an Object ſhall hereafter be found covered by this croſſing of the Hairs, the line from the Interſection of the Hairs to this point in the Object runs parallel to the ſides of the Tube or Ruler D E, F G. And therefore we may be ſure, the ſides of the Ruler are directly pointed towards the mark A in the diſtant Object. Reſpect being had to the Rulers Breadth.

For from whatever point on the inner ſurface of the Object-glaſs (how thick ſoever it be, or how *ill* ſoever *centred*) the Ray *c d* emerges; no other Ray can emerge from that ſame point, and fall on the point *d*, but it muſt neceſſarily be a principal Ray or Axis of ſome Cone, and muſt run parallel to the ſide of the Ruler D E. For whatever Optick Angle the length A B in any Object ſubtends; that length ſhall be projected by this Object-glaſs in *d e* ſubtending the ſame Angle *d c e*.

(4) Wherefore we now come to ſhew how to find out the Ray *c d*, that runs parallel to the ſide of the Ruler D E; or how to rectifie the Croſs-hairs on the Ruler. *Adjuſting them to plain Rulers or Tubes.*

 And

Exactly determining the Focus of the Object-Glass.

And first for, adjusting the Cross-hairs at their *exact distance* from the Object-glass, that is, in its *exact Focus*. This is easily performed thus, Let us look at some Object distant 3 or 4 miles; and moving or shaking our Eye before the Eye-glass upwards and downwards, or to one and t'other hand, let us observe whether the Cross-hairs seem to *move*, or *dance*, upon the said Object: for if it do, then the Cross-hairs are *not* at their *exact distance* from the Object-glass; but they must be moved farther from or nigher to the Object-glass; till the Eye, looking at such a distant Object, and moving before the Eye-glass, perceives the Cross-hairs, as it were, *fixed* and *immoveable* on the Object.

(Note. *This is the way for exactly determining the Focal length of an Object-glass, to which I have referred in* Chap. 4. Sec. 3.)

If in *raising* the Eye, the Object seems to *fall down* on the Cross-hairs, or if in *depressing* the Eye, the Object seems to *rise* on the Cross-hairs, then are the Cross-hairs *too nigh* the Object-glass: but if in *raising* the Eye, the Object seems to *rise* on the Cross-hairs; or in *depressing* the Eye, the Object seems to *sink* or *fall* on the Cross-hairs; then are the Cross-hairs *too far from* the Object-glass. All which will be evident from *Tab.* 39. *f.* 2. wherein let A B be a distant Object, whose middle point C is projected by the Object-glass, D at *k*. Let *m n* 1 be the Cross-hairs *too nigh* the Object-glass and *m n* 2 the same *too far* from the Object-glass *e*, *f*, *g*, the Eye placed at three different stations. In the case of the first Cross-hairs, if the Eye *rise* from *e* to *f*, it perceives the point *k depressed* from 1 to *k*, or if the Eye *fall* from *e* to *g*, it perceives the point *k* raised from 1 to *l*: and here the Cross-hairs are *too nigh* the Object-glass. But in case of the second Cross-hairs; if the Eye *rise* from *e* to *f*, the point *k* seems to *rise* on the Cross-hairs from 2 to *r*; or if the Eye *fall* from *e* to *g*, it perceives the point *k fallen* from 2 to *s*: and in this case the Cross-hairs are *too far distant* from the Object-glass. But if the Cross-hairs are exactly in the *Focus* at *k*, let the Eye *rise*

or

or *fall*, the Croſs-hairs ſeem *fixed* and *ſteddy* on the Object. And this is the firſt thing requiſite for adjuſting theſe Sights.

This I have borrowed from my own *Sciathericum Teleſcopicum*, Publiſhed at *Dublin* 1686. *quarto.* And I hope, one may be allow'd to tranſcribe from himſelf without being called a *Plagiary*.

When this Affair is well adjuſted; we may proceed to the ſecond Rectification, which conſiſts in making the *line of ſight*, *mire*, or *collimation*, exactly *parallel* to the Sides of the Tube or Ruler, to which the Teleſcopick Sights are to be adapted. And for the eaſier obtaining of this, we are firſt to be aſcertain'd, that even the *two ſides* of the Ruler or Tube are exactly parallel. And hereof we may be informed after this manner. *Tab.* 39. T.39. F.3. *Fig.* 3. On an even board draw the right line D H I B, let A B C D be a Ruler, to which the Teleſcopick Sghts are to be fitted, E the Ring carrying the Object-glaſs, F the Ring carrying the Croſs-haits, G the Snout carrying the Eye-glaſs. To the line B D apply the ſide B D of the Ruler, and looking through the Glaſſes obſerve the point in an Object diſtant a mile or two whereon the Croſs-hairs fall. Then remove the Ruler, and apply its other ſide A C to the line B D, and obſerve whether the Croſs-hairs fall on the ſame point of the Object, as before. If they do ſo, then are the ſides of the Ruler A C, B D, parallel; if no1, then the ſides are not parallel. The reaſon that ſo remote an Object muſt be choſen, is, that the Breadth of the Ruler may ſubtend an imperceptible *Angle* in a circle whoſe *Radius* is the diſtance of the Object from the Object-glaſs.

Or otherwiſe, In the plain Board ſtrike two round braſs-wire Pinns. Suppoſe H, I, which having their *roundneſs* from their *being drawn*, muſt needs have their ſides parallel. To theſe Pins apply *one* and *t'other* ſide of the Ruler, and obſerve as before.

Having thus *found*, or *made* the *two ſides* of the Ruler (or four ſides of the Tube, if need be) parallel; the next thing is to make the line of Collimation L K parallel to theſe ſides; or to

bring the Interſection of the Croſs-hairs *e* to meet with the Axis L K of ſome of the *Radious Pencils*, which Axis, after paſſing the Object-glaſs, runs parallel to theſe Sides.

Tab. 39. *Fig.* 4, 5. To effect this (*Tab.* 39. *f.* 4, 5.) we are to raiſe the plain Board A B edgwiſe on the Board of a window or table C D, ſo that we may reſt or ſuſtain the Ruler or parallelipiped Tube *a b c d e f g h*, on the round Pins *i*, *k*: then looking through the hole in the end *g h d c* deſigned for the Eye, let us obſerve the point in a far diſtant Object, which falls exactly on the Interſection of the Croſs-hairs. Afterwards, let us invert the Ruler or Tube (as we have it in *fig.* 5.) making the ſide *a b c d*, which in *fig.* 4. was *uppermoſt*, now undermoſt in this *fig.* 5. by which means, the ſide of the Tube *a d h e*, which in *fig.* 4. was *fartheſt* from the Board, in *fig.* 5. is *next* the Board. Then looking through the Tube, let us obſerve, whether the Croſs-hairs fall now on the ſame point in the Object, as in the firſt poſture: If they *doe* agree exactly, then is the *line* of *collimation* parallel to the ſides *a h*, *b g*; if they do *not*, but fall to the *right* hand apparently of the ſaid point in the Object, then are the Croſs-hairs to be removed (by whatever Contrivance they are made moveable) to the *Left* Hand (the contrary requiring the contrary), ſo that the Tube continuing in this latter poſture, the croſs-Hairs may cover a Point in the Object, middle between the Point covered in the poſture of *Fig.* 4. and in the poſture of *Fig.* 5. at the firſt ſight. And thus by frequent Repetitions and Tryals, we at laſt bring all to rights. After the ſame manner that we have rectified the Line of *Collimation* to run *parallel* to any two parallel Sides of the Tube, we may rectifie it to a *Paralleliſm* with the other two parallel Sides of the Tube (ſuppoſing the Ring that carries the croſs-Hairs to have all the Motions requiſite to ſuch Rectification). And ſo we fix all ſtrongly, chiefly the Object-Glaſs and croſs-Hairs; and the Operation is compleat.

By

By this Method the *Line of Sight* of any Cylindrick or ſquare Tube may be made to run parallel to its Sides, for many Operations and Obſervations Mathematical and Natural: Amongſt others, for finding the *Declination* of the *Magnet*, according to the Methods lately propoſed by Monſ. *Hautefeville*, and M.*Sturmius* in the *Journal des Scavans*, 23.*Aug*.1683. And in the *Acta Eruditorum*, *Lipſiæ*, *Ann*. 1684. *Decemb.* And for want of this Method; what Monſ. *Sturmius* ſays in the foreſaid *Act. Lipſ*. pag. 579. is very *defective*. For thus he, *Sola Tubi locatio, ut Axis Viſionis per medias Lentes excurrens Meridianæ Lineæ exacte reſpondeat, difficultatis quippiam habere videbatur; verum & huic infirmitati præſens, uti credo, inventum eſt Remedium, &c.* And the Remedy he tells us is, That the Tube be made a *Parallelipiped* of Wood or Braſs; for then, *ſays he*, Applying the Side of your Tube to the *Meridian* Line, the Axis of Viſion will be *parallel* to the ſaid *Meridian Line.* But with the Leave of ſo Great a Man, *I deny this*; unleſs firſt it be *rectified*; ſo that this *Axis* runs *parallel* to the Side of the Tube. And let us take what care ſoever poſſible for truly centring the Object-Glaſs, and placing it, and the croſs-Hairs exactly in the Tube; we muſt after all rectifie theſe Sights by ſome ſuch Method as I have laid down, or elſe we may be egregiouſly deceived. And on this account, all the *Levels* and *Inſtruments*, to which *Pere Cherubin D'Orleans* has adapted Teleſcopick-Sights, and which he has ſo neatly and ſumptuouſly deſcribed by curious Schemes, and a large Volume, *La Viſion Parfait. Tome* II. *A Paris* 1681. *Fol.* are *deficient* and *uſeleſs*. For he places the whole *Rectification* of this *Line of Collimation* in the *true Centration* of the Glaſſes, *pag*. 107. And rejects the Right Rectification by moving of the croſs-Hairs as erroneous, *pag*. 107. But in this the Friar betrays his Ignorance; for tho the *true Centration* of the Object-Glaſs be of good *Convenience* and *Advantage*; yet it does not *perfect* the Inſtrument

Sturmius *Miſtake.*

Cherubins *Groſs Error.*

without

without farther Rectification; as being impoſſible to be obtained to ſufficient accuracy; and therefore we muſt have recourſe to ſome ſuch Method as foregoes.

I have hitherto mentioned only two of the fineſt croſs-Hairs to be extended as a Menſurator in the Focus of the Object-Glaſs: But for ſome Uſes, perhaps the *fineſt Silver*, or *Gold Wire*, is better; as not being diſordered by *Heat* and *Cold*: Or elſe, the *Point* of the ſmalleſt and moſt curious *Needle*, on whoſe Extremity, the ſmalleſt Teleſcopick-Star may be viſible.

See the end of this Chapter.

Conſtancy and Security of theſe Telecopick-Sights.

When theſe Teleſcopick Sights are rightly adjuſted in the Tube, and ſtrongly fixt in their due Poſture by Screws, and all covered over from outward Injuries and Accidents, they are of all Sights the moſt conſtant and laſting, and the leaſt ſubject to be diſordered: So that, when one finds the *Great Hevelius* objecting againſt them, their *Aptneſs to be out of order*, one would think the moſt commodious Fabrick of them was never explained to him; tho I am ſure, his Inſtructor Mr. *Hook* was as able, as any in the World, to inform him rightly in this Matter.

To Quadrants, Sextants, &c.

(5.) I come now to the Rectification of theſe Sights on *Quadrants* and *Sextants*, for taking Angles. This is done either *before* or *after* the Diviſions into Degrees, *&c.* are made on the Limb of the Quadrant. If it be done *before*, then we ſuppoſe the Teleſcope T L (*Tab.* 39. *Fig.* 6, 7.) fixt to the Quadrant, which we ſuppoſe continued a little farther than the Fourth part of a Circle. Chooſing then an Object pretty near the Horizon; let us look through the Teleſcope, in the uſual Poſture of Obſervation, as *Fig.* 6. and obſerve the Point in the Object marked by the croſs-Hairs; and at the ſame time we are to note moſt nicely the Point *c*, which the Plumb-Line *f c g*, hung from the Centre *f* of the Quadrant, cuts on the Limb. Then we are to invert the Quadrant into the Poſture of *Fig.* 7. (which is eaſily done by the uſual Contri-

Tab. 39. Fig. 6, 7.

vances

vances for managing great Quadrants, by tooth'd Semicircles and endleſs Screws) keeping ſtill the Teleſcope T L nighly upon the ſame height from the Ground, as before) unleſs the Object we look at, be ſo far diſtant, that the Breadth of the Quadrant ſubtends but an inſenſible Angle. But yet for certainty, 'tis better to keep the Teleſcope, as 'tis ſaid, upon the ſame height from the Floor); then direct the Teleſcope T L, that the croſs Hairs may cover exactly the ſame Point in the Object, as before in the Poſture of *Fig.* 6. And hanging now the Plumb-Line *a f g* on the Limb of the Quadrant; let us remove it *to* and *fro*, till we find out the exact Point *a*, from which the Plumb-Line being hung, ſhall moſt nicely hang over the Centre of the Quadrant *f*. Then carefully marking the Point *a*, let us divide the Arch *c a* into two equal Parts in *b*; and drawing *b f*, the Point *b* is the Point from which we are to begin the Diviſions of the Quadrant: And the *Line* of *Collimation* through the Teleſcopick-Sight, ſtands exactly at Right Angles to the Line *b f*. So that the Quadrant *b f d* being compleated and divided, the ſaid Line of Sight through the Teleſcope runs exquiſitely parallel to the Line *f d*.

In the next place, ſuppoſing the Quadrant *b f d* truly compleated and divided; and that we deſigned to fix thereto the Teleſcopick-Sight T L; ſo that the *Line* of *Sight* may run exactly at Right Angles to the Line *b f*, or parallel to the Line *d f*. We are to do as in the foregoing Praxis. And if in dividing the Arch *a c*, we find its half exactly coincident with the Point *b*, we have our deſire. But if it differ from the Point *b*, and fall *between* *b* and *d*, then the *Line* of *Collimation* through the Teleſcope ſtands at an *obtuſe* Angle with the Line *b f*; and the Inſtrument errs in *exceſs*: If this half Arch fall *without* *b* and *d*, then the *Line* of *Collimation* makes an *acute* Angle with the Line *b f*; and the Inſtrument errs in *defect*.

defect. And by often Tryals, we are to remove the cross-Hairs within the Tube, so much, as is requisite to correct this Error. And when we have thus rectified them to their due place, there they are to be strongly fixt. Or else, in Observations taken by this Instrument, we are to *make allowance* for this Error; by *subtracting from* (if it be in *excess*) or by *adding to* (if it be in *defect*) each Observation so much, as we find the Error to be.

The reason of this Rectification is most plain; for 'tis manifest, that $c f d$ (*Fig.* 6.) *wants* of a full Quadrant, as much as $a f d$ (*Fig.* 7.) *exceeds* a Quadrant. So the difference of the two Arches in the two Postures being $a c$; half this difference $b c$ *added* in *Fig.* 6. or $a b$ *subtracted* in *Fig.* 7. makes $b d$ a compleat Quadrant.

If we find our Instrument *err* in taking Angles, and we desire to know the Error *more nicely*, than perhaps the Divisions of the Instrument it self will shew it: We are to do thus; Let us observe diligently the Object pointed at, in the Posture the Instrument discovers its Error; and the Object pointed at when the Instrument lies *truly*. Then, with a large Telescope and Micrometer (as is used in taking the Planets Diameters, as shall be declared hereafter, *Sec.* 7.): Let us take the Angle subtended at the Object-Glass of the Quadrants Telescope by the length between these two Objects, and we obtain the Error of our Instrument most *nicely*. Thus for Example; Supposing the Quadrant $b f d$ already accurately divided, and that the Plumb-Line, *Fig.* 6. plays over the Point c: And upon the Inversion of the Instrument, *Fig.* 7. we find that before we can get it to play exactly over the Centre f, we must hang it over the Point e; so that the Arch $e b$ exceeds $b c$ by the Arch $e a$; 'tis plain that the Angle $e f a$ is the Error of the Instrument: For had the Plumb-Line hung over a, and over the Centre f in this latter Posture, the In-

strumen

ſtrument had been *exact*; becauſe *a* is as much on one ſide *b*, as *c* is on t'other ſide *b*. Wherefore *e f a* being the Angle, by which our Inſtrument *errs* in obſervation: Let us turn the Inſtrument into the uſual Poſture of Obſervation, as in *Fig.* 6. and hanging the Plumb-Line on the Centre *f*; let us bring it to play nicely on the Point *e*, and obſerve what diſtant Object is covered by the croſs-Hairs: Then let us bring it to play exactly on the Point *a*, and obſerve likewiſe what diſtant Object is pointed at by the Teleſcope-Hairs. Laſtly, by a large Teleſcope and Micrometer, let us meaſure the Angle between theſe *two Objects*, and we ſhall have the Angle of Error much more nicely, than 'tis poſſible the Angle *e f a* ſhould be given by the Diviſions on the Limb of the Quadrant *e a*. And thus much for adjuſting a Quadrant.

A *Sextant* is rectifi'd in like manner; If we conſider (*Tab.* 39. *Fig.* 8.) that if from the Centre *f* to the beginning of the Diviſions *d* there be drawn the Radius *f d*; and it be divided equally in *c*; and from *c* there be ſupended the Plumb-Line *c b*: When the Plumb-Line hangs over the 60th Degree at *b*; then the Line *f d* lies horizontal: And conſequently, if the Line of Collimation through the Tube be parallel to *f d*, this Line alſo lies horizontal. To try which, Whilſt the Sextant ſtands in this Poſture, obſerve the Object marked by the croſs-Hairs; then invert the Sextant; and over the Point *b* hang the Plumb-Line; and when from the Point *b* the Plumb-Line hangs over the middle Point *c*, then again is the Line *f d* horizontal in this Poſture. Mark then, whether the croſs-Hairs cover the ſame Object as before: If they do, then the *Line* of *Collimation* is parallel to *f d*: If they do not; but the Point in the Object marked in this latter Poſture be *higher* than the Point marked in the firſt Poſture, the Inſtrument errs in *exceſs*; if it be *lower*, the Inſtrument errs in *defect*. And either we are to remove the croſs-Hairs, till we

T 39 F 8. *Rectification of a Sextant.*

 bring

bring all to rights, and there fix them: Or by the Methods before laid down in the *Rectification* of the *Quadrant*, we are to find the Quantity of this erroneous Angle, and to allow for it in Obſervation.

Rectification of a movable Sight.

In Inſtruments furniſhed with two pair of Teleſcopick-Sights, one on a *fixt* Arm, and t'other on a *moveable* Arm (by the Ancients termed an *Alidade*); 'tis eaſie rectifying the Sights on the *moveable* Arm thus: After the Sights on the *fixt* Arm are rectifi'd by what foregoes; bring the Index of the moveable Arm to the beginning of the Diviſions on the Limb of the Inſtrument, be it Quadrant or Sextant, *&c.* 'tis then manifeſt, that the *Line of Collimation* through the *movable* Teleſcope (if it be right) ſhould lye *parallel* to the Line of *Collimation* through the *fixt* Teleſcope. Obſerve therefore, whether the croſs-Hairs in *both* Teleſcopes do at the *ſame time* cut the *ſame* Star, or fall on the *ſame* Point in an Object diſtant three or four Miles. If they do, then the *movable* Teleſcope agreeing with the *fixt*, and the *fixt* being *ſuppoſed rectifi'd* to the Diviſions on the Inſtrument, the *movable* is *right* likewiſe. But if the Hairs in the *movable* Teleſcope do *not* agree in marking the *ſame* Point with the croſs-Hairs in the *fixt* Teleſcope; then the Hairs in this *movable* Teleſcope are to be *removed* (by whatever Contrivance there is for that purpoſe) and brought to *rights*, and there *fixt*.

There are other Methods propounded for rectifying Teleſcopick-Sights on other ſorts of Inſtruments, by means of Obſervations towards the *Zenith*, as our former Methods have been imployed towards the *Horizon*. But 'tis ſufficient here to lay down only what foregoes, as being of the greateſt and moſt frequent uſe: Referring for the others to M. *Picard's* Treatiſe of the *Meaſure of a Degree of a great Circle of the Earth*; publiſh'd at the end of *Memoirs for a Natural Hiſtory of Animals*, &c. By the *Academy Royal at* Paris; lately tranſlated into Engliſh, and printed at *London*, 1688. *Fol.*

Before I quit this Point, it may not be amiſs to intimate one *Uſe*, to which a plain Ruler furniſhed with Teleſcopick-Sights (ſuch as is expreſſed *Tab.* 39. *Fig.* 3.) may be apply'd; and that is, not only for trying the exquiſite *ſtraitneſs* of *either* of its *own* Edges, and *paralleliſm* of its *own* two Sides; but alſo, for the ready Tryal of the ſame in any *other* Ruler: For 'tis but affixing (by a little Cement, or otherwiſe) this *Teleſcopick-Ruler* over the Ruler to be try'd, and reſting the Edge of this latter againſt the Pins H, I, and gently ſliding the Edge alongſt theſe Pins, and always touching them; looking all the while through the Teleſcope, obſerve whether the croſs-Hairs do ſteadily adhere to the ſame Point in an Object: For if the Edge of the Ruler have the leaſt irregular Crookedneſs, tht croſs-Hairs will move from the Point firſt obſerved. And this ſhall detect the leaſt Curvity in the Edge of a Ruler (eſpecially if the Ruler be long, and the Diſtance of the Pins be conſiderable) that ſhall eſcape the moſt exquiſite Eye of a Workman. The way of trying the Paralleliſm of the two Sides of this latter Ruler, is the ſame with what foregoes for the Teleſcopick-Ruler it ſelf: For when the Teleſcopick-Ruler is adjoyned over the other, they may both be taken but as one Ruler with Teleſcope-Sights affix'd.

Further Uſe of a Teleſcopick-Ruler.

(6.) I come now to the laſt thing propoſed concerning Teleſcopick-Sights; and that is, To ſhew the *Dioptrick-Reaſon* of their Performance and Exactneſs. But herein there will be little requiſite to be added to what foregoes, both in the Firſt Part concerning Teleſcopes in general, and to what is laid down in this Chapter concerning Teleſcopick-Sights. 'Tis manifeſt by Experiments, that the *ordinary Power* of Man's Eye extends no farther than perceiving what ſubtends an Angle of about a Minute, or ſomething leſs. But when an Eye is armed with a Teleſcope, it may diſcern an Angle

Dioptrick-Reaſon of their Performance:

 leſs

leſs than a Second. The Teleſcope that magnifies diſtinctly the Appearance of *Body*, magnifies alſo diſtinctly the Apppearance of *Extenſion*, *Space*, and *Motion* through this Space; ſo if the Minute-Hand of a Watch, which can but juſt be percieved to move, be looked upon with a Magnifying-Glaſs, we ſhall ſee it give a conſiderable Leap at every Stroak of the Balance. And thus likewiſe the ſlow diurnal Motion of the Sun or Stars, which is hardly perceivable by the bare Eye, unleſs aſſiſted by an Inſtrument of a vaſt Radius, is moſt eaſily perceived through an ordinary Teleſcope of 18 Inches long: Inſomuch that we may determine to the greateſt Niceity and Exactneſs, when a Star paſſes juſt over the croſs-Hairs, even to the ſingle Beat of a Second-Pendulum. And let an Object in the Heavens riſe never ſo little, the Image in the Diſtinct-Baſe falls correſpondently at the croſs-Hairs; and the Eye, by means of the Eye-Glaſs, perceives this Motion, be it never ſo ſmall. Thus ſuppoſe (*Tab.* 39. *Fig.* 1.) that a Star *riſe* from B to A, the Image *falls* at the croſs-Hairs in the Diſtinct Baſe from *e* to *d*; then by means of the Eye-Glaſs *o p*, the Space *e d* is mightily magnified, and conſequently the Angle B *c* A, equal to *e c d* by which the Star is riſen, is made moſt ſenſible to the Eye *q*. By whatſoregoes in the Firſt Part concerning the magnifying of this ſort of Teleſcope.

Hevelius Miſtake farther manifeſt.

By this we may perceive, how the Noble *Hevelius* was miſtaken in his Eſtimate of theſe Sights; when he imagin'd the *Line of Collimation* therein was no *longer* than between the croſs-Hairs and Eye-Glaſs: Whereas this Diſtance is not at all to be conſider'd in their Performance; the *Line of Collimation* being full as *long* as the Diſtance between the Object-Glaſs and croſs-Hairs. I am perſwaded, had he been rectifi'd in this particular, he would never have adhered ſo obſtinately to the Uſe of *plain Sights* upon his moſt coſtly Inſtruments. Tho I muſt confeſs, 'tis difficult to wean a Man from the

Uſe

Uſe of what he has been accuſtomed to for ſo many Years; and upon the Exactneſs of which, the Accuracy of all his former Labors did depend.

As to all other Objections which he makes againſt them, as that they are eaſily diſordered, that the Glaſſes are eaſily vitiated by the Breath of the Obſerver, &c. They are not of any the leaſt moment. For 'tis manifeſt, they may be contrived ſo, as to be *more ſecure*, and *leſs* ſubject to Injuries, than any other plain Sights whatſoever: And in this particular, Teleſcope-Sights are ſo far from being obnoxious, that certainly they are preferable to the beſt contrived plain Sights; for what can be more ſimple and eaſier preſerved, than the forementioned ſmall (but ſtrong) Braſs Rings defended by a Tin or Braſs Tube covering all? When once theſe are adjuſted and fixt, nothing can poſſibly injure them. 'Tis true, the Breath of the Obſerver, if puft into the Teleſcope, will ſully the Eye-Glaſs; but how eaſily is this avoided? Who is it goes purpoſely to make a *ſpeaking-Trumpet* of a *Teleſcope*? The other moſt conſiderable Objection againſt their Uſe is, That in dark Nights, at the ſmaller Stars, the croſs-Hairs in the Teleſcope require a little *enlightening*, or elſe they are *inviſible*, and cannot be ſeen when the Star juſt applies to them. This is ſo eaſily remedi'd, by admitting to them, through a ſmall opening purpoſely left in the ſide of the Tube, the leaſt glimmering Light of a Lanthorn; or by placing a Lanthorn a little aſide before the Object-Glaſs; that 'tis not worth mentioning as a Difficulty, much leſs is it to be made an Argument for their utter rejection. As to what he ſays of the Hairs being ſo groſs as to cover the ſmaller Stars, this only relates to the Material we employ; and the fineſt Silk-Worms Clue will be found ſmall enough almoſt to biſect the ſmalleſt Stars: If not, let us uſe the *fineſt Needle*, on whoſe ſlender Point we may diſtinctly receive the moſt minute Star

His other Objections anſwered.

Ar

Farther Uses of Telescopick-Sights.

And thus much concerning *Telescopick-Sights*; from whose application to Mathematick-Instruments, Astronomy, and Geography may expect their utmost Advancements. And even Natural Philosophy it self may hereby receive the greatest Help, when we consider how Telescopes may be apply'd to many Experiments therein; amongst others, to make the most nice *Hygroscope*; and has already been used for accurately determining the capricious *Variations* of the *Magnet*. *Telescopick-Sights* have been already successfully apply'd to most exquisite Levels; wherein Monf. *Picard* in his Curious Treatise *Du Nivellement* has prevented any farther Explication: And I doubt not, but every Day will find new Uses for these Sights. Amongst others, I'll presume to mention my own *Telescopick-Dial* already publish'd, *Anno* 1686. A Contrivance, which, without Vanity I may say, has not displeased *at Home*, and has been well received *Abroad*.

Adapting the Micrometer to a Telescope.

(7.) The next Telescopick Instrument which I shall explain, is the *Micrometer*. Concerning the *Invention* of this Ingenious Instrument, I have only this to say, That for the Honour thereof, there are several Competitors: Monf. *Petit*, Surveyor of the Fortifications in *France*, was the first that publish'd to the World the rough Draught hereof, 12. *Mar.* 1667. *Vid. Journal des Scavans*, 16. *May* 1667. After him Monf. *Azout*, another Ingenious *Frenchman*, publish'd a Tract concerning the exact Mensuration of the Planets Diameters, wherein he seems to challenge the Invention of this Instrument to himself and Monf. *Picard*, *Journ. des Scavans*, 28. *Juin.* 1667. and *Philosoph. Transact. Num.*21. *pag.*373. But last of all a Candid *Englishman* of our own, Mr. *Rich. Townley*, does vindicate the first Contrivance hereof to its *true* and *original Author*, Mr. *Gascoigne* an *English* Gentleman, who was kill'd in King *Charles* I. Service, *Vid. Philosoph. Transact. Num.* 25. *pag.* 457. Wherein Mr. *Townley* (who is of undoubted Credit) asserts,

Inventor of the Micrometer.

that

that Mr. *Gaſcoigne* made and uſed this Inſtrument before the Civil Wars in *England*: And that Mr. *Townley* had then in his Cuſtody two or three of theſe Inſtruments firſt deviſed by Mr. *Gaſcoigne*; to which Mr. *Townley* himſelf had added ſome conſiderable Improvments. All which, with the exact Fabrick, and fitting of the Body of the Inſtrument to a Teleſcope, we ſhall find accurately deſcribed in *Num.* 29. *p.*541. *of Philoſoph. Tranſact.* to which I ſhall therefore refer the Reader; and ſhall hint only ſuch things concerning it in this place; as may be *there wanting* for the clearer Inſtruction of the unexcerciſed Beginner.

Firſt therefore for a brief Deſcription thereof (as much as is requiſite to maintain the order of our Diſcourſe); 'tis in ſhort this. In the Focus of the Object-Glaſs of a Teleſcope, there are placed two fine parallel Hairs, or ſmooth Edges of Braſs Plates; theſe are made by Screws to open or cloſe at pleaſure, as wide as the Teleſcope admits. The Turns of theſe Screws are reckon'd out by proper *Indices*; ſo that in opening the Edges of the Micrometer, the Indices do ſhew, how many *Revolutions* of the Screws, and Parts of a *Revolution* are compleated in that Opening. Suppoſe therefore the Screws to be of ſo fine a Thread, as to contain 30 Threads in an Inch length; then every Revolution of the Screw opens or cloſes the Edges of the Micrometer a thirtieth part of an Inch. By one Revolution of the Screw, the *Index* receives one Revolution: Then, the Circumference of the Plate, over which the *Index* moves (as the Hand of a Watch over the Hour-Plate) being divided into 100 Parts; when the *Index* moves one of theſe Parts, the Screw moves the Edges a three thouſandth part of an Inch; (or the one thirty ſix thouſandth part of a Foot; by which we find how eaſie 'tis to divide a Foot into thirty ſix or forty thouſand Parts) And this Motion, tho every Minute, is made, by the Eye-Glaſs of the Teleſcope, perceivable.

The

The way of taking ſmall Angles by this Inſtrument is thus; Suppoſe it were the Diameter of the Moon; Open the Micrometer till the two Edges do juſt claſp or touch the Moons Edges; then obſerve by the Indices how many Revolutions and parts of a Revolution were compleated to this opening; and by a proper Table (the way of compoſing which I ſhall ſhew preſently), convert theſe Revolutions and Parts into Minutes and Seconds. In like manner, for obſerving ſmall Angles on the Earth, the Diameters of the other Planets, the Diſtances of *Jupiter*'s *Satellits* from his Body, or the Moons Spots, &c.

But now for making the Table, Firſt we are to fix the Micrometer exactly in the *Focus* of the Object-Glaſs (by the Rules before given, *Chap.* 5. *Sect.* 4.) if it be at very diſtant Objects we deſign to uſe it: Or otherwiſe in the *reſpective Focus*, if it be deſigned for nigh Objects. We may then compoſe the Table two manner of ways: The firſt is more eaſie, tho not ſo very certain and accurate, yet exact enough for moſt Uſes. Meaſure by Inches and Decimal Parts the Diſtance between the Object-Glaſs and Micrometer, taking into the Account two Thirds of the Object-Glaſs's Thickneſs: Let us ſuppoſe the Diſtance 10 Foot, or 120 Inches, or 120000 Parts; and we deſire to know what Angle is ſhewn by the Micrometer, being open 2 Inches, or 2000 Parts.
T. 25. F. 4. The Computation is plain (*Tab.* 35. *Fig.* 4.) $ey=120000$, $fd=2000$, then $ed=ef=1000$. And As ey : To Rad. :: So ed : To Tang. $\angle eyd=\angle eyf=0^\circ\ 28'\ 38''$, and therefore $\angle fyd$ is equal to $0^\circ\ 57'\ 16''$. Then finding by accurate Admeaſurement, how many Revolutions of the Screws or *Index*, are requiſite to open the Edges 2 Inches; the ſame compleats the Angle $0^\circ\ 57'\ 16''$. Suppoſe therefore 60 Revolutions open the Micrometer 2 Inches; then 60 Revolutions ſhew, that the Object, that juſt appears through the

Edges

Edges at that opening, ſubtends an Angle of 0° 57′ 16″. Then 30 (*viz.* half 60) Revolutions give 28′ 38″, *viz.* half 57′ 16″. And one Revolution gives an Angle of 57″ 16‴; and the hundredth part of a Revolution gives 34‴+. And thus the Table is compoſed to any Number of Parts and Revolutions requiſite. But this Way, depending on the exact Admeaſurement of the Diſtance of the Micrometer's Edges (which can hardly be obtained to ſufficient Accuracy, unleſs we know moſt nicely what Number of Threads in the Screw there were in an Inch length; for then we know what Number of Revolutions compleat an Inch), 'tis not ſo accurate as what follows, which is,

The ſecond way for compoſing the Table, is this: Having fixt the Micrometer at its due Diſtance from the Object-Glaſs; on the ſide of a Wall or Houſe far diſtant mark out two conſpicuous Objects, that may both at a time be received into the Teleſcope: Meaſure nicely the diſtance of theſe Objects from each other; and alſo the diſtance of either of them from the Object-Glaſs (which we ſuppoſe directly before the Point in the Wall middle between the two Objects). And by Trigonometry calculate the Angle, which the diſtance between theſe two Objects ſubtends before the Object-Glaſs. Then looking through the Teleſcope, open the Micrometer, till the two Edges thereof exactly meet with or embrace theſe two Objects; and obſerve, how many Revolutions and parts of a Revolution are performed in this opening; for ſo many compleat the Angle before calculated. And having the Revolutions and Parts that compleat any one Angle, we may eaſily find all the reſt, as aforeſaid. For in theſe ſmall Angles, the Angles and Revolutions are proportional; that is, if a *certain* Number of *Revolutions* give a *certain Angle*; *half* this *Number* gives *half* this *Angle*; and the hundredth part of this Number gives the hundredth part of the Angle, *&c.*

In the first Method that I proposed for adapting the Micrometer, and composing the Table, I have allowed for the Object-Glasses Thickness in measuring its distance from the Micrometer. But this Nicety is hardly requisite; unless it be in short Tubes. For at the Radius of 10 Foot, 1 Inch is the Tangent of 28′ 38″; and at the Radius of 10 foot + one tenth of an Inch, 1 Inch is the Tangent of 28′ 37″; so there is but *one Second* difference; tho we should err one tenth of an Inch in admeasuring the distance between the Object-Glass and Micrometer.

Other Telescopick-Instruments. I might now mention the Application of a *Lattice of fine Hairs* in the Focus of the Object-Glass of a Telescope, as an help to draw distant Objects in *Perspective*: And of applying there a pretty contrived Parallelogram for the same purpose. But the first is obvious enough by the least intimation thereof; and the latter is so amply described by *Pere Cherubin d'Orleans* in his *Dioptrique Oculaire*; that 'tis needless to add any thing farther in this place.

Instead of cross-Hairs. I conclude this Chapter with a brief hint of what I have found very commodious for many purposes; that is; instead of the forementioned cross-Hairs, I have often used a curious piece of clear, thin, flat Glass, whereon there are drawn two very fine cross-Lines by the curious Point of a Diamond, smaller than the most fine Wyre or Hair; not easily disturbed by a sleight Touch (unless we break the Glass), nor alterable by Heat and Cold. Thus also may we make a Lattice.

CHAP.

Chap. VI.

Of the Invention, Discoveries made by, and other Applications, of Optick-Glasses.

(1.) *Optick-Glasses unknown to the Ancients.* (2.) *Pretended Passage in* Plautus. (3.) *An other Passage in* Pliny. (4.) *Probably invented about* 1300. (5.) *Friar* Bacon's *Pretence.* (6.) *Inventers of the Telescope.* (7.) *Optick-Glasses long known before the Telescope. Remark thereon.* (8.) *Celestial Discoveries by the Telescope.* (9.) *In the fixt Stars.* (10.) *In* Saturn. *Examination of* Gallets *Hypothesis.* (11.) *In* Jupiter. *Motion of his Satellits diligently prosecuted by* Cassini *and* Flamsteed. *Satellits all disappearing.* (12.) *Reflection on the Motions of* Saturn's *and* Jupiter's *Satellits.* (13.) *In* Mars. (14.) *In the* Sun. (15.) *In* Venus *and* Mercury. *Hence the Falsity of the* Ptolemaick *Hypothesis.* (16.) *In the* Moon. (17.) *Planets whether inhabited.* (18.) *Telescopes Use on Earth.* (19.) *Uses of the Celestial Discoveries of the Telescope.* (20.) *Microscopick Discoveries and Writers.* (21.) *Viewing nigh Objects with a Telescope. Use thereof in Miniature-Painting.* (22.) *Measuring Distances at* one Station *by the Telescope.*

(1.) THat the *Ancients* had no knowledg of *Optick-Glasses*, is most evident from their universal silence in this Matter: Their most learned and inquisitive Philosophers makeing no mention, or the least hint thereof, in their Writings. And doubtless a Contrivance of that universal Use, beneficial to all old Men, both in Reading and Writing, could never have been so concealed, as that not the least Footsteps *Optick-Glasses unknown to the Ancients.*

thereof ſhould remain to Poſterity. The only Reliefs they had for their decayed Sights were certain *Collyria* or *Eye-Salves*; and when theſe fail'd them, they were left almoſt in the *dark* for *minute* and *cloſe Objects*.

We hear indeed mighty Stories of *Archimedes* burning the Ships of *Marcellus*, at a great diſtance from the Walls of *Syracuſe*. But whether the Matter of Fact be *true* or *falſe* (as I am very inclinable to believe it *falſe*), yet there is no mention of his performing this admirable Effect by *Optick-Glaſſes*. Perhaps, if there were any ſuch thing done at all; it was performed by *Concave Speculums*: And no one denies the Ancients the knowledg of *Catoptricks*. For *Archimedes* himſelf writ a Book (as 'tis ſaid) *De Speculis Uſtoriis Parabolicis*; but it has never yet ſeen the Light.

And yet there are in the World a ſort of Men, ſo devoted to the paſt Ages, that they will not allow any Improvements of Arts in the modern Generation, unknown to the Ages ſome Centuries before us. Of this Claſs was he, (whoever he was) that, rather than the Ancients ſhould be ignorant of *Optick-Glaſſes*, would forge a Paſſage in *Plautus* (which really is not at all to be found in him), for Confirmation of his Opinion.

Pretended Paſſage in Plautus.

(2.) *Pancirollus* (who ſurely was too candid a Perſon to be the firſt Author of this Fiction) in the Second Book *De Rebus Inventis*, *Tit.* 15. quotes this Paſſage from *Plautus*, *Cedò Vitrum, neceſſe eſt Conſpicilio uti*: Which, ſays he, cannot poſſibly be meant of any other thing but of the Glaſſes which we call *Spectacles*. And his Commentator *Salmuth* takes ſome pains to cite *Chriſtianus Becmannus* (I ſuppoſe in his *Oratio de Barbarie & Superſtitione ſuperiorum Temporum*) for clearing this Paſſage of *Plautus*: But yet he is ſo hard preſſed with it, that by no Art, but by main ſtrength he breaks through it, and ſays, That notwithſtanding that Paſſage, yet certainly *Optick-Glaſſes* are a modern Invention.

Whereas

Whereas, had he been aware, that that Quotation from *Plautus* is a mere *Fiction*; and that no ſuch Paſſage can be found in all his Writings; he might eaſily have avoided its Force, without all that ſtir. For ſo we ſhall find it anſwered in the *Lettere Memorabili del Abbate Michele Giuſtiani Parte Terza, Let.* 16. *Paſſage out of Plautus forged.*

(3.) Another place cited for the Antiquity of *Optick-Glaſſes*, is that of *Pliny*, *Lib.* 7. *Cap.* 53. *Hiſt. Nat.* wherein we find the word *Specillum.* To this Paſſage we have this Anſwer in the forementioned Letters of *Giuſtiani*; that *Specillum* cannot poſſibly be here meant of a *Spectacle*-Glaſs, ſeeing we find the Expreſſion, *Inungit Specillum*; which, ſays he, cannot be underſtood of *Spectacles*, which we rather *wipe* and *cleanſe*, than *anoint* and *greaſe*. But this Conſtruction of the Learned Authors is much *forced* and *unnatural*: For the plain ſenſe of that Paſſage in *Pliny* is this. *Pliny* in that Chapter is giving Inſtances of the *ſudden Deaths* of many Men; and telling how they were ſeized, whilſt they were doing *ſo* or *ſo*, and wholly thoughtleſs of that fatal moment. Amongſt many other Examples, he has this; *Super omnes C. Julius Medicus dum inungit, Specillum per Oculum trahens.* The meaning whereof is no more, than that the *Phyſician* C. Julius *was on a ſudden ſeized by Death, whilſt he was applying an Unguent to his Patients Eye, and drawing his Probe* (called *Specillum*) *through it.* Whereas, to joyn *inungit* and *Specillum*, ſpoils the Grammatical Senſe of the whole, and renders it unintelligible. *Paſſage in* Pliny. Specillum *a Chyrurgeon's Probe.*

'Tis evident therefore, that from neither of theſe Paſſages can we draw any Argument for the *Antiquity* of *Optick-Glaſſes*. *Optick-Glaſſes probably invented about* 1300.

(4.) Wherefore ſeeing we muſt neceſſarily allow this Invention due to the Modern Age of the World; our next Enquiry ſhall be, Where firſt to fix it. But herein we ſhall find but faint Traces to direct us.

Monſ: *Menage*;

Menage's Opinion.

Monsieur Menage a learned and ingenious French-man, in his *Origini della Lingua Italiana, Genevæ*, 1685, commenting on the Word *Occhiali del Galilæi*, discourses there of the Time of the invention of *Spectacles*: And after relating the known Story of *Frier Jordan*, (of which more anon) he has this notable Passage; That *Monsieur du Cange* had told the Authot (*Monsieur Menage*) of a Greek Poem, the Manuscript whereof is now in the *French* King's Library, wherein the Poet, who lived *An.* 1150, Jesting on the Physicians of those Times, says of them to this Purport in *French*; *Qu'ils tatent le Poux, & qu'ils Regardent les Excremens du Malade aver une Verre. That they observe the Excrements of their Patients with a Glass.* But Mons. *Menage* is of Opinion, that this was a Transparent Glass, whelm'd over the Vessel, more for the Relief of their Nose against the Stench, than of their Eyes.

But however we may doubt of *Spectacles* being so ancient as 1150. We may be certain that about the Thirteenth Century, they were commonly known and used. For (beside what we shall say hereafter of our Country-man *Frier Bacon*) the most learned Mons. *Spon* in his *Recherches Curieuses D' Antiquitè, Dissert.* 16. inserts a Letter of Signior *Redi* to *Paulus Falconerius*, concerning the Time when *Spectacles* were invented; and this he fixes between 1280 and 1311. from the Testimony of a Manuscript Chronicle in Latin, in the Library of the *Friers Preachers* of St. *Katherine* at *Pisa*, Fol. 16.

Spina's Pretense.

Wherein 'tis said, that *Frater Alexander de Spina, Vir modestus & bonus, quæcunque vidit aut audivit facta, scivit & facere. Ocularia ab aliquo* primo *facta, & communicare nolente, ipse fecit & communicavit corde hilari & volente.* And this *Alexander de Spina* was a Native of *Pisa*, and dyed there, *An.* 1313.

Another Authority

Signior *Redi* has in his Library a Manuscript written *An.* 1299. *Di Governo della Famiglia de Scandro di Pipozzo.* In which there is this Passage; *Mi truovo cosi Gravoso di Anni che non arei Valenza*

lenza Di Leggere e Scrivere ſenza Vetri appellati Okiali, Truovati novellamente per Commodità delli Pouveri Veki, quando affiebolano del Vedere. Thus in Engliſh, *I find my ſelf ſo preſſed by Age, that I can neither read or write without thoſe Glaſſes they call* Spectacles, *lately invented, to the great Advantage of poor Old Men, when their Sight grows weak.*

The *Italian* Dictionary, *de la Cruſca*, on the Word *Occhiale*, makes this remark, That *Frier Jordan de Rivalto*, who dyed at *Piſa*, *An.* 1311. in a Book of Sermons which he writ *An.* 1305. tells his Auditory in one of them, that it is not Twenty Years ſince the Art of making *Spectacles* was found out, and is indeed one of the beſt and moſt neceſſary Inventions in the World. Frier Jordan's *Authority.*

About the ſame time *viz.* 1305. *Bernard Gordon* a famous Phyſician of *Montpelier*, in his *Lilium Medicinæ*, thus commends a certain *Eye-Salve*: *Et eſt tantæ Virtutis, quod decrepitum faceret legere Literas minutas abſque Ocularibus.* And *An.* 1363. *Guido de Chauliac*, in his Book entituled *Grand Chirurgery*, after propoſing ſeveral *Collyria*, ſaith; If theſe or the like will not do, you muſt make uſe of *Spectacles*. *Gordon.* *Chaueiac.*

From all which we may be pretty certain, That *Spectacles* were well known in the 13th. Century, and not much before. But who the Happy Man was, that firſt hitt upon this lucky Thought, may yet be queſtioned. 'Tis true indeed, if we credit the forementioned Chronicle of the Convent at *Piſa*, Frier *Spina* makes as fair a Challenge to the Invention, as the firſt Author, who refuſed to communicate it. But I am apt to believe, That, whoever this cloſe Man was that would not impart to *Spina*, He was a Frier; and that theſe Monkiſh Men, and *Jordan* amongſt the reſt, had this Invention whiſpered amongſt themſelves, before it was publick; and that they all had the *Firſt Hint* thereof from our Country-Man *Frier Roger Bacon.*

Bacon's Pretence.

(5) That this learned *Frier Bacon* who dyed *An.* 1292. and lyes buryed at *Oxford*) did perfectly well underſtand all ſorts of *Optick-Glaſſes*, ſhall be plainly made out, from the natural and eaſie ſenſe of his own Words, in his Book of *Perſpective*: Whereby we ſhall find, that he not only underſtood the Effects of ſingle *Convex* and *Concave-Glaſſes*; but knew likewiſe the way of *combining* them, ſo as to compoſe ſome ſuch Inſtrument as our *Teleſcope*. This perhaps will be looked upon as a *Great Paradox*, and as great Partiality in an *Engliſh* Author to his Country-Man; eſpecially conſidering, how univerſally the contrary has prevail'd; the Votes of moſt learned Men having conferr'd the Honor of this Invention on other Pretenders. But if, from the unconſtrain'd Words of his Books, we plainly make out this Aſſertion, I hope the Attempt may not be counted unreaſonable or partial.

And Firſt in his Book of *Perſpective* Part III. Diſ. 2. C. 3. he has theſe Words; *Si vero Corpora non ſunt plana* (having treated of them before) *per quæ Viſus videt, ſed ſphærica; tunc eſt magna Diverſitas, nam vel Concavitas Corporis eſt verſus oculum, vel Convexitas, &c.* By which 'tis manifeſt, he knew what a *Concave* and *Convex-Glaſs* was. Moreover, in the ſame Place *Diſ. ult.* he proceeds thus; *De Viſione fractâ majora ſunt, nam de facili patet, maxima poſſe apparere minima, & è contra; & longè diſtantia videbuntur propinquiſſimè, & è converſò: Sic etiam faceremus Solem & Lunam & Stellas deſcendere ſecundum Apparentiam hic inferius, &c.* Thus in Engliſh, *Greater Wonders than all theſe are performed by refracted Viſion; For thereby, 'tis eaſily made appear, that the Greateſt Object may be repreſented as very little, and contrarily; And ſo likewiſe, the moſt diſtant Objects as juſt at hand, and contrarily. Hereby alſo may we bring the Sun and Moon and Stars down here below in Appearance*, &c. This, I think, is ſo expreſs in the Point, that it leaves no room to doubt, but that he had ſome admirable Secret in Optick Glaſſes. Add

to

to this what he has in his Epiſtle *ad Pariſienſem*, of the Secrets of *Art* and *Nature*, Cap. 5. *Poſſunt etiam ſic figurari Perſpicua, ut longiſſimè poſita appareant propinquiſſima, & è contrario; Ita quod ex incredibili Diſtantia legeremus literas minutiſſimas, & numeraremus Res quantumcunque parvas, & ſtellas faceremus apparere quò vellemus. Glaſſes* or Diaphanous Bodies, ſays he, *may be ſo formed, that the moſt remote Objects may appear as juſt at hand, and contrarily; So that we may read the ſmalleſt Letters at an incredible Diſtance, and may number things though never ſo ſmall, and may make the Stars appear as near as we pleaſe.*

And that theſe Things may not ſeem *incredible* of this *Great Man*; who, in that dark, ignorant Age could be maſter of theſe admirable Inventions; I ſhall refer the Reader, for a more compleat Account of him, to *Ant. a Wood Hiſt. & Antiquit. Univerſit. Oxonienſis*, Lib. 1. Pag. 136. and to Dr. *Plott*'s *Nat. Hiſt. of Oxfordſhire*, Cap. 9. Sect. 2, 3, *&c.* and Sect. 39, 40, 41. Where we may find, how he was perſecuted by the ignorant malicious *Friers* of his Order, as practiſing *Magick* and *Necromancy*: for which they caſt him into Priſon, and there detain'd him for a long time, ſome ſay to his Death, in the 78th. Year of his Age. There we ſhall find, how he was the firſt Promoter of the *Emendation of the Calendar*. compleated afterwards in the Time of Pope *Gregory* II, But above all, his Pretenſe to the firſt *Invention* of *Gunpowder* ſeems as well founded, as poſſible, on this Paſſage in his *Epiſtola ad Pariſienſem*, Cap. 6. (a Hundred years before *Barthold. Swartz.* lived) *In omnem Diſtantiam quam volumus, poſſumus artificialiter componere Ignem comburentem, ex ſale Petræ & Aliis*; (Theſe *Alia*, in another Manuſcript Copy, are, *Sulfur & Carbonum Pulvis*) And ſoon after he adds, *Præter hæc* (*i. e. Combuſtionem*) *ſunt alia ſtupenda Naturæ, nam ſoni velut Tonitrus & Coruſcationes poſſunt fieri in Aere, imò majore Horrore*

Bacon *invented Gunpowder.*

 quam

quam illa quæ fiunt per Naturam: Nam modica materia adapta, ſc. ad Quantitatem unius Pollicis, ſonum facit horribilem, & Coruſcationem oſtendit violentem, & hoc fit multis modis, quibus Civitas aut Exercitus deſtruatur.: Igne exſiliente cum Fragore ineſtimabili. Mira hæc ſunt, ſi quis ſciret uti, in debita Quantitate & Materia. By which laſt Paſſage we may gueſs, he had not the way of applying it to a Gun; though 'tis manifeſt, he was ſenſible that ſome ſuch Uſe might be made of it. But the particular Manner did not offer it ſelf to him at firſt.

Note. I Confeſs, I have not by me at this time the Originals, from whence theſe Paſſages are quoted; the preſent Diſtractions of our miſerable Country having ſeparated me and my Books; and the Place, where I am, affords not the Copies: Therefore, if in theſe Quotations I am any wiſe miſtaken, I muſt not be blamed, acknowledging that I have them *at ſecond hand* from the forenamed Authors.

But to return to our *Optick-Glaſſes.* 'Tis evident that *Bacon* was acquainted with them; and probably knew how to adapt them in a *Teleſcope.* But the long and cloſe Impriſonment he ſuffer'd before his Death (for 'tis ſaid no one was permitted to ſpeak to him; and that all his Writings, Books and Inſtruments were ſeized and burnt; except only thoſe few Fragments of his which we have ſaved accidentally) was the Reaſon, that we have no farther Advancements of his in this kind transmitted to Poſterity. But 'tis very probable that the uſe of ſingle Glaſſes in *Spectacles,* as being an Invention of immediate Advantage to Human Life, and in it ſelf very eaſie and ſimple, might therefore be preſently catched at by the World, and put into Practice: Whilſt his other more curious Combinations of Glaſſes might be loſt and forgot. And this I am the more inclinable to believe; Firſt, becauſe *Frier Bacon*'s Time agrees ſo well with *Frier Jordan*'s forecited Teſtimony *An.* 1305. That it was not then twenty years

years ſince the Invention of *Spectacles*: And ſecondly, becauſe we find this ſort of *Monkiſh Men* firſt take notice of the Invention, before all other Men; which ſhews, they had it delivered amongſt themſelves only, for a while before others.

(6.) And thus much concerning *Frier Bacon*'s Pretenſe. But that I may not ſeem altogether partial, I ſhall here add the Opinions of others, concerning *Other Inventors* of the *Teleſcope*. For I find no other Pretenders to the Invention of ſingle *Convex* and *Concave* Glaſſes, but the forenamed. *Borellus* has written a ſmall Tract purpoſely on this Subject, *De vero Teleſcopii Inventore*: Wherein, Cap. 12. he ſeems to give the Invention to *Zacharias Joannides* of *Middleburg* in *Zeland*, *An.* 1590. Another Candidate for this Diſcovery, he names *Johannes Lipperhoy*, or *La Prey*, *An.* 1609. A *Dutchman* alſo, whom *Surturus* calls *Lipperſein*. *Adrianus Metius* Mathematick Profeſſor at *Franequer* ſays, his Brother *Jacobus Metius* of *Alkmaer* was certainly the firſt Inventor of the *Teleſcope*. And if we believe the *Italians*, we ſhall have the Honour of inventing this Inſtrument conferr'd on the incomparable *Galileo*. But he himſelf in his *Nuncius ſidereus* confeſſes, that the firſt Intimation he received of this Inſtrument, was, that a *Dutchman* had then lately made one; which ſet him (*Galileo*) upon the thought how to effect it; which, he ſays, he ſucceſsfully diſcover'd by the conſideration of *Refraction*, and found that a *Concave* and a *Convex* Glaſs rightly adapted would perform what he only heard in general of the *Dutch* Invention.

(6.) *More Modern Inventors of the* Teleſcope.

Zacharias Johnſon.

Jo. Lipperhoy.

Ja. Metius.

But certainly the firſt Publick notice of this Contrivance came from ſome of the forementioned *Dutchmen* (for *Frier Bacon*'s Hint mentions not the particular Combination of the Glaſſes) and therefore the Inſtrument is deſervedly called *Tubus Batavus*. Though we muſt confeſs at the ſame time, that *Galileo*, *An.* 1610. (ſee his *Nuncius ſidereus*) did firſt apply this curious Inſtrument to Celeſtial Obſervations; and had then

made ſuch wonderful Diſcoveries in the Heavens thereby, that all his Philoſophick Succeſſors have ever ſince attempted to climb higher, by lengthening their *Ladders*, and advancing this Inſtrument by many Degrees. However I muſt not here conceal the Pretenſe of *Baptiſta Porta*, who, in his *Magia Naturalis*, Lib. 17. Cap. 10. Printed *An.* 1589. has theſe Words, *Si utramque (Lentem ſc. Concavam & Convexam) rectè componere noveris, & longinqua & proxima, majora & clara videbis.* But *Porta*'s Character is ſo well known, that we may eaſily imagine, he had got this Hint from *Holland*.

Baptiſta Porta.

Fontana.

Franciſcus Fontana a *Neapolitan*, in his *Obſervationes cœleſtium terreſtriumque rerum*, contends that he himſelf *An.* 1608. firſt invented the *Teleſcope*, compoſed of a *Convex Object-Glaſs* and *Convex Eye-Glaſs*: For the *Tubus Batavus*, and *Galileo*'s Tube was furniſh'd with a *Concave Eye-Glaſs*; and *Fontana* confeſſes, it was before his; and that *An.* 1618. he firſt invented the double *Microſcope*. *Rheita* in his *Oculus Enoch & Eliæ*, Lib. 4. towards the end, pretends to be the firſt Diſcoverer of the *erecting Teleſcope* of three Convex Eye-Glaſſes, as alſo of the *Teleſcope* for looking with both Eyes, called *Teleſcopium binoculum*: Of which latter, *Cherubin* has writ his whole Volume, *La Viſion parfait*, &c.

Rheita.

(7.) Optick Glaſſes long known before the Teleſcope.

(7.) Thus we ſee, how long the Uſe of ſingle Optick Glaſſes was common in the World (even about 300 years) before Men rightly underſtood their due Application, in the Compoſition of this admirable Inſtrument. They had them in their hands, they look'd through them, *now* a Convex, *then* a Concave, and admired their Effects, and the Help they gave to diſorder'd Eyes: but ſtill were ignorant of the vaſt Advantage the moſt *acute* Eye might receive by them, even to the Increaſe of its Power, ſome Thouſands of Degrees beyond its natural Abilities. This was reſerved for ſome lucky Chance in a future Age, to be diſcovered by him that ſhould

firſt

firſt be ſo fortunate, as to adapt theſe Glaſſes at their *due Diſtance*: for to ſome ſuch happy Hitt, I imagine the Invention is due; and not to any profound Thought on the nature and properties of Glaſſes, that firſt ſuggeſted the Contrivance to the *Dutch Mechanick*, that was its Author.

Remark thereon. Uſes of Diſcoveries not immediately known.

And this does naturally ſuggeſt a Thought to us, of ſome incouragements in natural Enquiries, by the method of *experimental Philoſophy*; that perhaps we are every day ingaged amongſt ſome *particular Things*, which we commonly ſee, handle, uſe, and are converſant with; and which have in them ſome latent, hidden Properties, which, upon a right Application, (to be diſcovered perhaps by ſome lucky Hitt) may be of the moſt uſeful and ſurpriſing Effects. And that therefore, we ſhould not deſpair of making the greateſt Diſcoveries about even the *meaneſt* Things. Who could expect to ſee ſuch Wonders from an eaſie Compoſition of three ſuch plain, ſimple Bodies, *Niter*, *Sulfur*, and *Charcole*, as we daily ſee from *Gunpowder*? And the Property of the *Magnet's* drawing Iron was commonly known many generations, before it was ſo happily applyed to guiding a Ship: Who could have thought, by looking upon that dark unpromiſing ſtone, that future Ages ſhould uſe it to ſuch a ſtupendous and advantageous a Purpoſe, far exceeding the Virtues of the moſt illuſtrious Gemms? Hence may we learn, not to deſpiſe the Products of Nature, even of the meaneſt Appearance. And let us not ſay, that any Diſcovery is *uſeleſs*, ſince we know not what Time and Poſterity may produce from the ſimpleſt Truth. And this naturally leads me to the diſcoveries made by *Optick* Glaſſes.

(8.) Celeſtial Diſcoveries of the Teleſcope.

(8.) *Galileo* (as is noted before) is deſervedly reputed the firſt that raiſed up this *Gigantick* Inſtrument, that ventures to climb Heaven and from thence brings down the Stars. He firſt, was ſurprized and ſtruck with wonder, to ſee *four little Moons* dancing round *Jupiter*, that from their firſt Creation

to

to the lucky Moment when he firſt diſcovered them, had never ſtruck the eye of any mortal Inhabitant of this Globe. Were theſe then made for the *Uſe* of *poor Man*, from whoſe Knowledg they were concealed for 5000 years together ? Vain Man! that thus preſumes to confine the Deſigns of the *Almighty Creator* to miſerable Duſt and Aſhes; when his *infinite Power* can make Millions of *intelligent Beings*, and all intelligent after different ways, to ſerve and praiſe him: And theſe perhaps are the Inhabitants of theſe *diſtant Worlds*, and of thoſe again infinitely extended beyond theſe. 'Tis true indeed, now theſe little Planets are diſcovered, we have happily applyed them to an advantageous purpoſe (as ſhall be ſhew'd hereafter) But this we are to eſteem as a particular Benefit of Providence to theſe latter Generations; and reſpects not all the general Race of Mankind, that lived and were buſie for 5000 years together; and knew nothing of them. But in this ſtupendous Enquiry I ſtop, as not being able to reach it with the *longeſt Teleſcope*.

Whether all for the uſe of Man.

To keep therefore to our Subject: I ſhall take the Heavens in order, as they lie; conſidering firſt the *uppermoſt*, and ſo deſcend *down* to our Earth, and ſhall briefly declare the *Diſcoveries* made in each, and (as far as I can attain it) by *whom* and *when*; with farther References to thoſe Authors, where each particular may be found more fully treated of.

(9.) In the fixt Stars.

(9.) And Firſt, for the *Fixt Stars*: That whitiſh Band or Zone, the *Galaxia* or milky Way, that ſo irregularly incompaſſes a great ſcope in the Heavens, and of which the Ancients could give no tolerable Account, is found by the Teleſcope to be no other, than an heap of very minute Stars thickly ſet together; which, by their great Diſtance, Smalneſs and Cloſeneſs, appear to the naked Eye, as one united whitiſh Cloud. In like manner, the *Nebuloſa Orionis*, *Præſepe Cancri*, &c. are found to be a Congeries of ſmall Stars cloſely ſet together, but eaſily diſtinguiſhable by the *Teleſcope*. The *Pleiades* or

ſeven

seven Stars (tho scarce more than six appear) are found by an ordinary Glass to be nigh forty. And in the single Constellation of *Orion*, the Telescope discovers more Stars than the naked Eye can number in all the Heavens. On this Account, the Seed of *Abraham*, that was to be made *numerous* as the Stars in the Firmament, may yet (for ought we know) admit of Propagations through many future Generations, before it comes up to its Limits. And the number, which *Archimedes* demonstrated greater than that of the Grains of Sand composing this Globe of Earth, may perhaps fall short of the Stars in the Heavens: For hardly any Corner of the Firmament so dark; But the Telescope, turn'd towards it, descries Multitudes of *glittering Spangles* therein.

(10.) From the *fixt Stars* let us contract our Prospect, and in a vast, long, and almost immense Course homewards, we first meet with *Saturn*. By his slow Motion he takes State upon him, as carrying about him something more weighty than ordinary. But the short sight percieves nothing thereof, and sees only a plain round Globe, as the rest of the *Chorus* dancing round the Sun. All his Equipage and Attendants are hid from our View, 'till surveyed more closely by the Telescope: And then behold a mighty Ring parallel to the Equator, bright as the Planets own Face, encompassing round his Body; very thin, and separated in all Appearance on all sides from his Globe: sometimes appearing broader, sometimes narrower, and sometimes almost vanishing; then again returning by a regular Period, and resuming by Degrees its former Shape; which again by degrees it looses according to his own Periodical Motion. But this is not all his Equipage, for besides this Throne of Light, this Majestick Planet is constantly attended by a Guard of *five Satellits*, that follow his Motion and dance round him continually in a Circle. (10) *In Saturn.*

Galileo

Galileo.

Galileo was the firſt that obſerved any thing extraordinary in *Saturns* Appearance *An.* 1610. *Octob.* as he tells us in ſome of his *Italian* Letters: But his Glaſſes were too ſhort to give the true Shape of this Planet. All that he deſcry'd was ſomething appendent on each ſide of him, which he took to be two Globes much leſs than *Saturns* own Body; and therefore he firſt publiſh'd (and at the ſame time conceal'd), this Diſcovery, by tranſpoſing the Letters of this Sentence, *altiſſimum Planetam tergeminum obſervavi.* But when the Teleſcope was better advanced, (as what Invention is it, that receives not Advancements in Time?) the true and genuine Appearance of *Saturn* began to ſhew it ſelf, and its regular Changes were taken notice of. But though ſeveral Authors writ Treatiſes of this ſurpriſing Appearance, and particularly the celebrated *Hevelius* (*de nativa Saturni facie*) *Hodierna*, &c. yet all their Obſervations were imperfect and deficient; and chiefly for want of excellent Glaſſes: Till the incomparable *Chriſt. Hugenius* has put the laſt hand to this Affair; and in his ingenious Treatiſe, *Syſtema Saturnium, Hag. Comit.* 1659. 4°. has publiſh'd to the World a compleat Hiſtory of all Obſervations of this Planets Appearances with a moſt ingenious Theory for their Explication. In the beginning of the year 1655. his excellent Perſon firſt diſcovered the biggeſt of *Saturns Satellits* with a Teleſcope of 12 feet, charged with an Eye-Glaſs of 3 Inches; afterwards, *An.* 1656. he doubled that Length, retaining the ſame Eye-Glaſs. The *Satellit* he diſcover'd, is the *Fourth* from *Saturn*; and in the forenamed Treatiſe, he gives us the *Epochæ* and *Tables* of its Motion; But our moſt ingenious Countryman, Mr. *Halley*, deſervedly celebrated for his Aſtronomical Labours, diſcovered in the year 1682. that *Hugenius*'s Numbers were conſiderably run out; and therefore he ſet himſelf to correct the Period of this *Satellit*, which he has done accordingly, Num. 145. Pag. 82. *Philoſoph. Tranſact.* And in Num. 187. Pag. 299. we

Hevelius.

Hugenii *Syſtema Saturnium.*

we ſhall find Monſ. *Caſſini*'s Tables of the Motions of all *Saturns Satellites*, together with their diſtances from *Saturn* correſpondent to their Periodical Times: Of which more hereafter.

The other four *Satellites* were all diſcovered by Monſ. *Caſſini* in the Order following. The third and fifth were firſt ſeen by him, *An.* 1671, 72 and 73. by a 17 Foot Glaſs of *Campani*, and 36 Foot Glaſs of *Divini*, and by ſuch another of *Borelli*. An Account whereof may be ſeen at large in Num. 92. of the *Philoſoph. Tranſact.* But the innermoſt or firſt, and the ſecond were not ſeen by him till the year 1684. at which time, having procured Glaſſes of an extraordinary length, as 80, 100, 150 and 200 Feet; the vaſt diſtance and ſmalneſs of theſe Planets could no longer conceal them from his ſight. *Vid. Philoſoph. Tranſact.* Num. 181.

Caſſini's *Diſcoveries about* Saturn.

The laſt thing I ſhall take notice of,relating to this Planet, is, That Monſ. *Gallet*, *Provoſt* of S. *Symphorian* at *Avignon*,in the year 1684. has advanced an Hypotheſis for ſolving its Appearances; which, as it relates to the Teleſcope, may ptoperly be here conſidered. I ſhall therefore briefly propoſe ſome of the chief difficulties, that ſeem to attend this Theory: And that I may not be prolix, I ſhall ſuppoſe the Reader acquainted with what Monſ. *Gallet* lays down in the *Journal des Scavans*, *An.* 1684. *May* 15. & *June* 12. and in Latin in the *Acta Lipſiæ*, *An.* 1684. *Septemb.* Pag. 421.

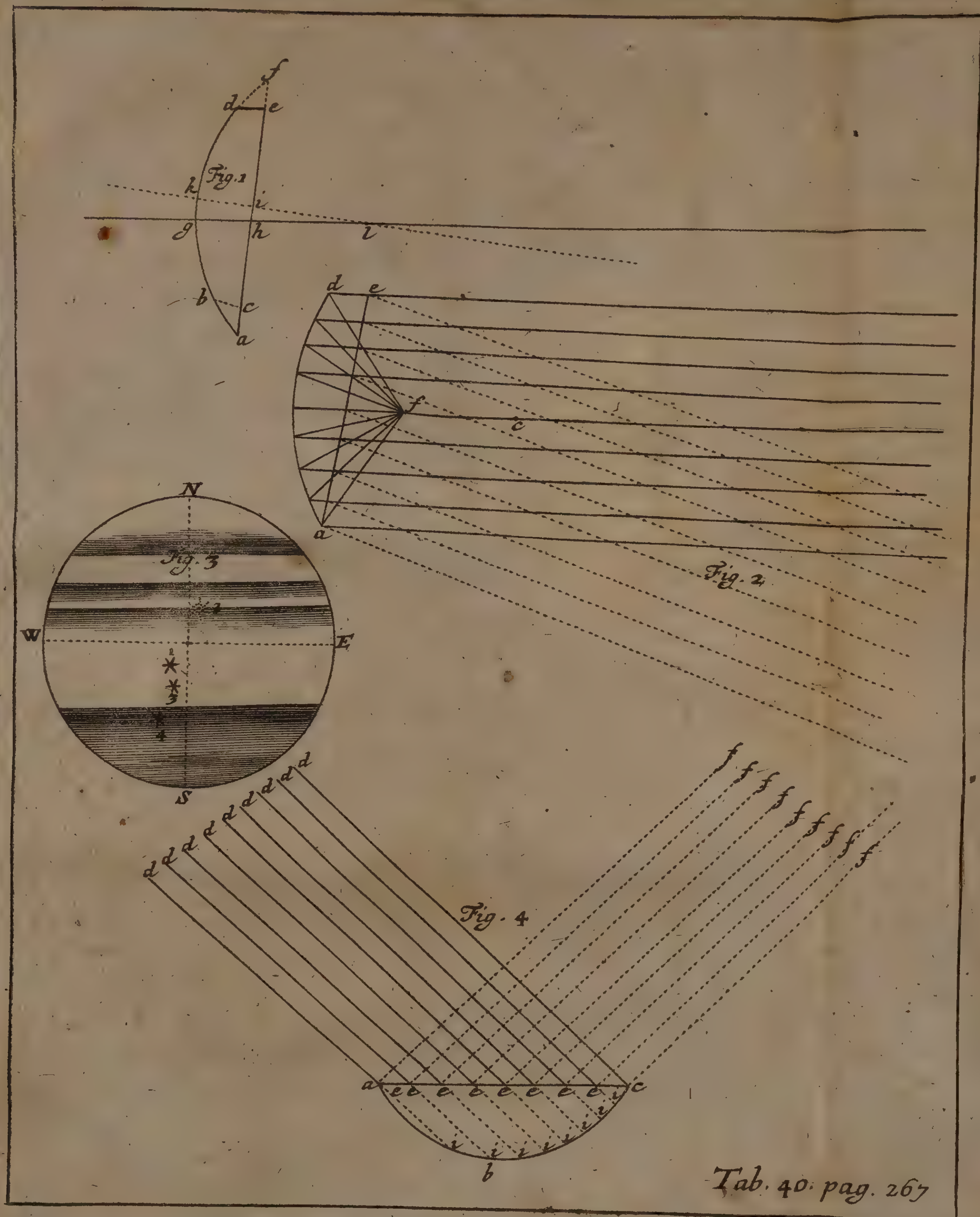
Examination of Gallet's *Hypotheſis.*

Firſt therefore, he ſuppoſes *Saturn* and the other Planets, except the Moon, *polite Globes*, reflecting the Image of the Sun as a *Convex Speculum*. Which ſeems not at all to be founded on more than mere *Conjecture*: For we have no Reaſon to think them different in this particular from the Moon; which is found of a rugged uneven Surface.

But ſecondly, granting that they (and eſpecially *Saturn*) may be *polite Spheres*, (for we will not confine the infinite Variety of the Creation;) and granting that *Saturn* reflects two

 ſorts

ſorts of Light; one whereby his whole Body becomes viſible; and the other the bright Image of the Sun from his Convex Surface; as we ſee a Convex Speculum is it ſelf viſible, by the Rays it reflects diſorderly from its whole Surface; (which ſo far partakes of a little Roughneſs) and at the ſame time reflects a bright and orderly Image of the Sun, from one certain part of this Surface to the Eye rightly poſited: Yet this bright Image of the Sun, which is reflected from *Saturn*, (how far ſoever *Saturn* be removed) can never be projected by an Object-Glaſs, in its diſtinct Baſe, *greater* than the Projection of the whole Body of *Saturn* in the ſame diſtinct Baſe. And yet (if I miſtake not) this is the Foundation of Monſ. *Gallet*'s Theory. This is ſo evident to any one the leaſt verſed in *Dioptricks* and *Catoptricks*, that 'tis needleſs to inſiſt upon it any longer. We may make a convincing Experiment hereof: Expoſe a reflecting Convex-Speculum before the Sun, and by a Convex-Glaſs project the Image of this Speculum on a Paper in a dark Room: we ſhall there ſee the Repreſentation of the Speculum it ſelf, and of the bright Image of the Sun on the Speculum. And indeed by the leaſt Conſideration of the matter, it will be evident to us, that 'tis impoſſible it ſhould be otherwiſe: For the little Image of the Sun, reflected from the Convex Speculum, poſſeſſes but a very ſmall part of the Speculum's Surface; and therefore cannot poſſibly be projected, by the Object-Glaſs in its diſtinct Baſe, *greater* than the Projection of the Convex Speculum itſelf. The ſame may we conceive of *Saturn*, by ſuppoſing him a Convex, polite, reflecting Speculum; for though he ſhould then, beſides the Figure of his own Body, reflect the bright Image of the Sun, from a ſmall part of his Spherick Surface on the Object-Glaſs; yet the Object-Glaſs could never project, in its diſtinct Baſe, the Repreſentation of this Image of the Sun *greater* than the Repreſentation of the whole Body of



Tab. 40. pag. 267

Saturn. And, notwithſtanding the Evidence hereof, Monſ. *Gallet* affirms, the Appearance of *Saturns Ring* or *Anſæ* proceed to from hence; that the Object-Glaſs projects, in its diſtinct Baſe, the repreſentation of the bright Image of the Sun, reflected from *Saturn*'s *Convex polite Surface*, *greater* than (or clearly *without*) the Repreſentation of his *Body* itſelf.

Thirdly, Monſ. *Gallet* affirms, that the Reaſon why *Jupiter*, *Mars*, *&c.* are not projected by an Object-Glaſs, in its diſtinct Baſe, with a *Ring* or *Anſæ*, is becauſe theſe Planets are *nearer* to us than *Saturn*: And therefore in the Projection of *Jupiter*, the bright Image of the Sun reflected from its polite Surface, is repreſented by the Object-Glaſs in its diſtinct Baſe, *equal* to the Repreſentation of *Jupiters* whole Body; in the Projection of *Mars* it is *leſs*, *&c.* But the Diſtance of theſe Planets (even of the nigheſt) is ſo very great, (and as it were infinite) in reſpect of the ſmall Breadth of an Object-Glaſs; that in compariſon to this ſmall breadth of an Object-Glaſs, we can make no difference between the Diſtance of *Saturn* and even *Mercury*.

Laſtly, the Experiments, on which Monſ. *Gallet* founds his whole *Hypotheſis*, ſeems not at all to confirm, or in the leaſt wiſe to reſpect what he builds thereon. The Phænomenon ariſing from reflecting the Sun-Beams by an Object-Glaſs on a Wall poſited *obliquely*, proceeds from the Object-Glaſs being conſider'd as a *Concave Reflecting Speculum*, having alſo another Surface either Plain or Convex beſides the Concave, (as I have noted before, Chap. 4. Sec. 4.) and yet he ſuperſtructs hereon a Theory, for explaining the Appearances of *Convex Polite Surfaces*, ſuch as he makes the Planets. This will be manifeſt by *Tab.* 40. *Fig.* 4. wherein, for eaſe ſake, we take a T.40. F.4.
Plano-Convex Glaſs *a b c*, and expoſing its plain ſide *a e c* obliquely to the Suns Rays *d e*, *d e*, *d e*, ſome of them ſhall be reflected by the plain Surface *a e c* into *e f*, *e f*, *e f*; but others of them

 entring

entring the Glaſs run on in *e i*, *e i*, *e i*, (we do not here conſider the Refraction they ſuffer) and ſo falling on the Concave Surface *a i b i c* (for ſo I'll call it) are *reflected* by it, according to the Laws of *Spherick Catoptricks*: But the Reflections of theſe immerged Rays I have not expreſſed, for avoiding Confuſion in the Scheme. I acknowledge, the *Phyſical* Cauſe of this latter Reflection, from the Surface of the Glaſs *a i b i c*, is perhaps not to be accounted for by Human Underſtanding; but the matter of Fact is certain. One ſhould think, when the Rays are arrived at the extreme Points *i*, *i*, *i*, of the hindmoſt Surface of the Glaſs, they ſhould, without any of them being reflected at all, emerge from the Glaſs: But 'tis manifeſt *ſome* of them are *reflected*, and that too, juſt in the ſame manner, as if the Surface *a i b i c* were a Polite, Opake, Concave Surface; and not covered by the Surface *a e c*. They that deſire to enquire farther into the natural Cauſe hereof, may conſult *Grimaldi Phyſico-Matheſis de Lumine & Coloribus*, *Bonon.* 1665. 4°.

'Tis then by the Reflection from this Concave Surface *a i b i c*, (give me leave ſo to call it) that the ſimilitude of *Saturns Anſæ* are repreſented in *Gallets* Experiment; and by the Reflection from the *Plain* Surface *a e e c*, that the Similitude of his Body is repreſented in the ſaid Experiment. And how this can be accommodated to the Reflection, which *Saturn* himſelf makes from his own Body, and to his Appearance through a Teleſcope, I confeſs I cannot apprehend. Moreover *Gallet*'s Hypotheſis gives no Account of the two dark Spaces on each ſide the Globe of *Saturn*, between his Body and the *Anſæ*. For the Experiment, on which he founds his Fancy, ſhews no ſuch Diſtinction; all being inlightened therein: As will be viſible to thoſe ſhall try it.

'Twere too tedious in this Place to conſider particularly Monſ. *Gallet*'s Scheme, and his *Particular Syſtem* of *Saturn*; which

which might eaſily be ſhewn defective. But thus much I thought requiſite to ſay in this Place; becauſe the Theory he propoſes is of a *Dioptrical* Conſideration: Becauſe it has never yet been taken notice of by any other: And becauſe he advances it in Oppoſition to Monſ. *Hugens Syſtem*, which carries with it ſo much Probability.

(11.) *Jupiter* next preſents himſelf leſs incumbred than *Saturn*, yet not wanting a Courtly Train: For tho his Guards are but *four* in number, yet their ſize and brightneſs ſhew their *Strength*, and their quick Motion round him ſhews their Diligence. *(11.) In Jupiter.*

Galileo was certainly the firſt Inhabitant of this Globe, that ever ſaw theſe *Satellites*, *Jan.* 7, 1610. And from that Moment to this, no more could ever be diſcovered about him: Though Vanity and deſire of being the Author of ſome Novelty, made Frier *Ant. de Rheita* proceed ſo far, as to write a Tract of 5 more *Satellits* (9 in all) about *Jupiter*. But *Hevelius*, in the fourth Chap. of his *Selenography*, has demonſtrated the *Frier* to be miſtaken, and has ſhewn the ſmall fixt Stars (only diſcoverable by the Teleſcope) that deceived him. Galileo. Rheita.

Theſe *Satellites* are eaſily ſeen by a 3 Foot Glaſs, and I have juſt perceived them with one of 15 Inches. But to make exact Obſervations of their Motions, 'tis requiſite, we uſe Tubes of 10 and 12 Feet and upwards. From the time of their Diſcovery, many curious Aſtronomers have attended their Motions round *Jupiter* with a diligent Eye; and have found, that ſometimes falling into the Shadow of *Jupiters* Body, they diſappear; and thence emerging, they again become viſible: ſometimes they are hid behind the very Globe of their great Lord; and ſometimes being juſt in his Face, his Splendor overcomes theirs, and they become inviſible, as a glimmering Lamp between the Eye and Sun.

Joh.

Borellus. *Joh. Alfons Borellus* has publiſh'd a Tract of the *Theoricks* of theſe *Medicea Sidera* (ſo named by *Galileo* in complement to his great Patron the *Duke* of *Tuſcany*) *Theoricæ Mediceorum ſiderum*, *Florent.* 1666. 4°. But none have laboured more to reduce the Obſervations of theſe little Planets to ſomething of Uſe and Advantage to the World, than the two celebrated A-ſtronomers of the preſent Age, *Caſſini* and *Flamſteed*: The former has taken great pains in Publiſhing Hypotheſes, Tables, and Precepts for calculating their Appearances and Eclipſes, (in his *Ephemerides Mediceorum ſiderum*, 1668. *Bonon.*) in order to ſetling the *Longitudes* of Places to a great Certainty. And the latter finding the Numbers of *Caſſini*'s Tables not ſo *juſt* to the preſent Time, from moſt accurate Obſervations of his own, taken by the Teleſcope and Micrometer, has fixt new Numbers, that agree about this time moſt exactly to the *Phænomena*; but with the Liberty to himſelf of altering theſe Numbers, as by future Obſervations he ſhall find Occaſion; For he is not ſo *poſitive* as to ſay, that what he ſettles (for 15 or 20 years) ſhall be *perpetual*. What he has hitherto publiſhed of this kind may be found diſperſed in the *Philoſoph. Tranſact.* wherein, he has given the World the Catalogues of theſe *Satellits Eclipſes* for ſeveral years conſecutively. In Num. 154. Pag. 404. he ſhews their Uſes, and how (by their Help) the *difference* of *Longitude* betwixt any two Places on Earth, where they ſhould be obſerved, might be determined: And *teaches* a Method of finding out, within what ſpace on our Globe any of them are obſervable. Theſe Directions he repeats in Latin, Num. 165. Pag. 760. for the uſe of Forreiners. In Num. 177 & 178. with the Catalogue of Eclipſes for the year 1686. he deſcribes a ſmall Inſtrument; and ſhews how by the help of it, of the ſaid Catalogue, and of the Tables of *Jupiters Geocentrick Places* and *Parallaxes*, the Appearances of the *Satellites* at any time in that year might be diſcovered, and delineated by Scale and

Caſſini. Flamſteed.

and Compaſs on Paper. But the Curioſities, which this excellent Aſtronomer has yet unpubliſhed, relating to this uſeful Part of Aſtronomy, are very great and ingenious; which I hope, in time, he will impart to the World; as with much Freedom and Generoſity, he now communicates them to his private Friends: In the Number of which I am very proud to reckon my ſelf.

Before I quit this Article of the *Satellites*, I cannot omit taking notice of an Obſervation, which, by mere Accident, I made ſome years ago, of a *Total Diſappearance* of all the *Satellites*. I had often attended their Motions with a good Teleſcope of 12 Foot; and often had them *all four* conſpicuous at a time, very often *three*, frequently *two*, but never leſs than *one*, and this but very rarely; till *An.* 1681. *Novemb.* 2. *Hor.* 10. *p. m. Dublinii St. Vet.* there was *not one of them viſible*; *Jupiter* there ſtood by himſelf, in all Appearance, without his Guards; and a bold *Lucian* might have pull'd him from his Throne without Reſiſtance. Some years after, I obtain'd from my learned Friend Mr. *Flamſteed* his Tables of the Motions of theſe *Satellites*; And the Poſtures of theſe *Jovial Moons*, at that time, are found by them to be as is expreſſed in *Tab.* 40. *Fig.* 3. The firſt, third and fourth were *juſt in his Face*; and were therefore drown'd by his Light, and the ſecond was *behind* his Body. The rarity of this Appearance (at leaſt to me) makes me note it ſo particularly: perhaps thoſe that are more frequently imploy'd in watching their Motions may meet ſometimes with the like Conjunctions; but I believe 'tis very ſeldom. *Hevelius*, in his conſtant Attendance on them, for more than a year and half, hit not upon ſuch an Appearance; as may be ſeen by the Hiſtory of his Obſervations at the end of his *Selenographia*; Nor *Caſſini*, as is manifeſt from his forementioned Ephemerides.

Satellites *all diſappearing.*

T. 40. *F.* 3.

Beſides theſe *four* little *Moons* about *Jupiter*, the Teleſcope diſcovers other Remarkables even in his Body. As firſt: his

Jupiter's *Belts.*

Face

Face is not all of a *Colour*; But there are in it *brighter* and *darker* Parts; and these are drawn athwart him, like broad *Zones* or *Belts* almost parallel to the Ecliptick, as is expressed

T.40. F.3. in *Fig.* 3. *Tab.* 40.

Jupiter's Spot and Rotation.

Hence the Rotation of our Earth.

The last thing that has been observed in this Planet is a *Spot*, first seen by our Ingenious Mr. *Hook May* 9. *h.* 9. *p. m.* 1665. By this *Spot* 'tis manifest, that *Jupiter* turns round his own Axis in the space of less than 10 hours, or about $9^h. 56'$. A very strong Argument to prove, that our Earth may do so likewise; since *Jupiter*, who is so considerably *bigger* than the *Earth*, has a Motion much more quick than ours in 24^h. *Kepler*, upon the Restauration of the *Pythagorean* or *Copernican Hypothesis*, did conjecture, from the Motion of the Primary Planets about the Sun as their Centre, That the Sun *moved* about its own Axis; but could not evince it, till future Observations by the *Telescope* discovered *Spots* in the Sun, and by *them* demonstrated, that the Sun revolves on its own Axis, in $25\frac{1}{4}$ days. *Jupiter* has four *secondary* Planets moving round him, and he himself in their Centre *turns* on his Axis: our *Earth* has a *secundary* Planet, the *Moon*, that moves round her *once* a Month: is it not therefore highly *probable*, that the *Earth* also *revolves diurnally* on its Axis? For a farther account of this *Spot* in *Jupiter*, I refer to the *Philosoph. Transact.* N. 1. P. 3. N. 4. P. 75. N. 8. P. 143. N. 12. P. 209. N. 15. P. 246. N. 82. P. 4039.

(12.) Reflection on the Motions of Saturn's and Jupiter's Satellits, evincing the Order of the Creation.

(12.) But before I leave *Saturn* and *Jupiter*, I cannot but take notice of one admirable Property, for the Knowledge whereof, we are beholden to the Telescope; and that is, the wonderful Agreement which is found in all the several Systems of our Vortex; as well between the General *System* of the Sun, and Primary Planets with the particular *System* of *Saturn's* or *Jupiter's* Planets, as between the particular Systems themselves, in this single Property, *That the Periodical Times of the Planets Revolutions are in a* sesquialtera Ratio *of their Distances from the*

Centre

Centre of the Planet about which they revolve. That is, As the Square of the Period of the first *Satellite* (for instance): To the Square of the Period of the second :: So the Cube of the Distance of the first from *Jupiter's* Centre: To the Cube of the Distance of the second from his Centre. This holds most exquisitely true in *Jupiter's Satellites*, as is noted by the admirably learned Mr. *Newton*, in his incomparable Treatise, *Philosophiæ Naturalis Principia Mathematica*, Lib. 3. Hypoth. 5. And the same *Law of Motion* is strictly observed by the five Primary Planets, and the Earth about the Sun. As he notes, Hypoth. 7, 8. This is also verifyed by Monf. *Cassini* in the five *Satellites* of *Saturn*; as appears by his Account of them in the *Philosoph. Transact.* Num. 92. P. 5178. N. 133. P. 831. N. 181. P. 79.

And from hence may we justly fall into the deepest Admiration, that *one* and the *same Law* of Motion should be observed in Bodies so vastly distant from each other, and which seem to have no Dependence or Correspondence with each other. This does most evidently demonstrate, that they were all at first put into Motion, by *one* and the *same unerring Hand*, even the *infinite Power* and *Wisdom* of God, who has *fixt* this *Order* amongst them *all*, and has *establish'd* a *Law*, which they cannot *transgress*. *Chance* or *dull Matter* could never produce such an *Harmonious Regularity* in the Motion of Bodies so vastly distant: This plainly shews a *Design* and *Intention* in the *first Mover*. And with Submission to the Reverend and Learned *Divines*, I am apt to think, that one Argument drawn from the *Order*, *Beauty* and *Design* of Things, is more forecible against *Atheism*, than Multitudes of Notional Proofs drawn from *Ideas*, Apparitions of *Specters*, *Witches*, &c. (not that these should lose their due Strength) For besides the *Heavens*, even the *little Globe* we inhabit affords us infinite Variety in this kind: And for my own part, I must confess, I can read

Beauty Order and Design of natural Bodies strong Proofs of a Deity.

read more Divinity in Mr. *Charlton*'s admirable *Musæum*, on a Box of beautiful *Shells*, of delicately painted *Plants*, curiously adorned *Insects*, *Serpents*, *Birds*, or *Minerals*; than in large Volumes of Notional Writers. For *Animals*, *Plants*, and *Minerals* do yield us abundant Instances, which visibly shew a *Design* or *End proposed*; which, as it cannot possibly consist with *Chance*, so neither can it be apprehended to have been so *ab æterno*: For 'tis absolutely unconceivable, that a thing *designed* for some *End* or *Purpose*, should not be so *designed* in *time*, by some *designing Being*. But I beg Pardon for this Digression, in which I am thus ingaged before I was aware. To return therefore to our Subject.

(13.) Mars

(13.) *Mars* offers himself next; who, trusting in his own Strength, is attended by no Guards; But the Prying Telescope discovers in his Face *Scars*, *Spots*, and *Ruggedness*; as we may find in the *Philosoph. Transact.* Num. 14. P. 239. &c. By these

Rotation. *Spots* the acute *Cassini* has determin'd, that he *turns* on his own *Axis* once in $24^h. 40'$. tho others assign his Revolution performed in just half that time (a Mistake easie enough) *vid. ibidem.* The fiery Face of this Planet requires a very good Telescope to view him, and a small Aperture on the Object-Glass: or else this Glaring Light makes but a confused Appearance. But however furious his Beams are, he is beholden for them (as all the rest of the Planets) to the great Fountain of Light and Heat the *Sun*. For what the great distance of *Saturn* and *Jupiter*, and their being so much *above* the *Sun* (as I may so

Increases and Decreases. speak) hinders us from seeing, *viz.* their *Increase* and *Decrease* in Light like our Moon, is very visible in the Planet *Mars*; who, in his *Quadratures* with the Sun, and in his *Perigeon*, may be seen almost *bissected*, but never *corniculated* or *falcated* as the other Inferiours, *Vid. Hevelii Selenograph.* Cap. 4. P. 66.

(14.) *The* Sun. Sun's *Spots.*

(14.) The glorious *Sun* does next present; in whose bright Face, we can hardly expect to find *dark Spots*. yet such there

are,

are, and *frequent* too. *Scheinerus* in his *Rosa Ursina* has published a large Book in *fol.* of nothing else; and *Hevelius*, at the end of his *Selenography*, has many Observations of these *Maculæ*; as also of some brighter Spots in the *Sun*, called *Faculæ*. The Way of observing them is taught at large, in the foresaid Authors; and is in short, either by admitting the Light of the Sun through a Telescope upon a white Paper, in a dark Room; or by arming the Eye with a small thin Glass *smoaked* over a Torch, Lamp, or Candle, and with it looking through a Telescope at the *Sun*'s Body. The only Discovery that has been made by these *Maculæ*, is, that the Sun revolves round his own Axis, in the Space of about $25\frac{1}{4}$ days. But for a compleat and succinct Account of the Theory of these *Maculæ* and their Motions, I refer the Reader to the excellent Mathematician *Andrew Tacquet*, *Astronom.* Lib. 8. Tract. 3. Num. 7. 'Tis only to be noted, that for these several years past, the Appearance of these *Maculæ* has been much more rare, than when *Galileo* (who certainly first discovered them) *Scheinerus*, *Hevelius*, attended their Observations, *viz.* about 50 or more years ago. About that time, one should seldom see the *Sun*'s Face (no more than now our brighter Beauties here below) free from one or more *black Patches*; but now (as if they were grown out of Fashion) he seldom wears any: One in 5 or 7 years hardly appearing: As if *now* he put them on, more of necessity, to cover an odd Pimple, that may otherwise disfigure his Countenance, than to adorn his Face. How far the *Fair Sex* should follow his Example, I dare not venture to determine. But from them we naturally fall to *Venus*.

Sun's *Rotation.*

Sun's *Spots rare of late.*

(15.) *Venus*, the brightest Planet in the Heavens. She fears not sometimes even at Noon-day to display her Beauty; and in this Armour reposing an entire Confidence, performs her Course *alone*, and free from all other Attendants.

(15.) Venus *and* Mercury.

Mercury's Wit and Quickneſs ſecures him, therefore he has no Train, but generally ſhelters himſelf under the Beams of his potent Lord the Sun.

Increaſe and Decreaſe.

But both theſe Inferiour Planets are found by the Teleſcope to *increaſe* and *decreaſe*, as our *Moon*. For ſometimes they appear *corniculated*, ſometimes *falcated*, ſometimes *gibbous*, and ſometimes *full* even on, or nigh to their Conjunctions with the Sun. By which laſt Phænomenon, 'tis manifeſt they move about the Sun, ſometimes *farther* from us, ſometimes *nigher* to us than he: and conſequently the *Ptolemaick Hypotheſis* is abſolutely *falſe*. (whatever *Hyotheſis* be true.) And for the Demonſtrable Detection of this *Error*, we are beholden to the *Teleſcope*: And I doubt not but Poſterity may, by the ſame Inſtrument, diſcover ſome *Hypotheſis* as *poſitively true*: For the *Probabilities* of the *Copernican* Syſtem are already ſo ſtrongly confirm'd thereby, that there ſeems no Room left for any farther Doubt; But Time and Labour will yet diſcover farther Proofs. How ſucceſsfully Mr. *Hook* has applyed the Teleſcope to prove the *Motion* of the *Earth*, I leave the Reader to judge upon Peruſal of his *Attempt*.

Falſity of the Ptolemaick Hypotheſis.

(16.) Diſcoveries in the Moon.

(16.) And thus at laſt are we arrived at home to contemplate our Neighbour the *Moon*. Her may we properly call *our own*, as making us the Centre of her Periodical Motion. For, as the *Satellites* about *Saturn* or *Jupiter* move round *them*; ſo moves the *Moon* as a Satellite about our Earth. *Galileo* with his Teleſcope firſt diſcover'd great Ruggedneſs in the *Moons* Face, after him *Langrenus* the King of *Spain*'s Coſmographer, attempted to draw her Picture. But the noble *Hevelius* in his curious and coſtly Work of *Selenography*, has perfected this Affair, perhaps beyond Amendment. There may we ſee the *Moons* Countenance diſtinguiſhed in an admirable difference of Parts, both for Shape and Colour. We may there ſee greater Parts that reſemble our Seas, Lakes, Rivers, Iſlands,

Peninſulas,

Peninſulas and Continents; other leſſer Spots (almoſt infinite in number) that reſemble our Mountains, Hills and Vallies. Of the *greater* Parts, thoſe that are ſomething *obſcure*, may we reckon *Seas* and *Lakes*; and the brighter may we account *Land*. For juſt ſo does our Earth appear, when, from a diſtant Height we look upon a Mixture of Land and Water enlightened by the Sun. Of the *ſmaller* Spots, thoſe that are *brighteſt* and *ſhine*, are Mountains and Rocks; and the *darker* Parts, which are uſually encompaſſed with theſe *brighter* Verges, may we eſteem *Vallies*.

Mountains in the Moon.

Now that ſome Parts of the Moon are much higher than others, is as manifeſt by the Teleſcope, as that ſome Parts of our Earth are higher than others. For if we look upon it about the Quarter-days, we ſhall plainly ſee the Edge, towards the dark Part, broken and cragged; and many little bright Spots, that are clearly ſeparated from the reſt of the enlightned Part. Which is an evident Proof that theſe are the high Tops of Eminencies, which receive the Suns Light, before the Parts *below* them are enlightned. Moreover the Moons Spots caſt their Shadows *oppoſite* to the Sun, that is, to the *Eaſtward*, whilſt the Moon is *Increaſing*, and to the *Weſtward*, on her *Decreaſe*. That theſe Mountains are very *high*, is manifeſt from the way of Meaſuring them, delivered in *Riccioli Almageſt*. 1. Pag. 208. And *Tacquet Geom. Prac.* Cap. 8. Prob. 2.

Names impoſed on the Spots.

For the better diſtinguiſhing theſe Spots, and making them more uſeful in the Obſervation of Lunar Eclipſes, there are names impoſed on them by Authours: *Hevelius* aſſigns to them the names of Places here on Earth: *Grimaldus* and *Ricciolus* give to them the names of famous Mathematicians and Aſtronomers.

Eclipſes accurately obſerved by them.

By means of theſe Spots, Lunar Eclipſes are now much more accurately obſerved than formerly; to the great Advancement of Geography and Navigation, in ſetling the Longitudes

gitudes

gitudes of Places. For now the Immersions and Emersions of these Spots from the Shadow of the Earth are most nicely determined.

Moreover, by these Spots the Moon is discovered to have *various librating Motions*, from *East* to *West*, and from *West* to *East*, also from *North* to *South*, and from *South* to *North*. But hereof we cannot now enlarge, *vid. Bullialdi Astrom. Philolaic.* Lib. 3. Cap. 13. *Hevelii Selenograph*: *Riccioli Almagest.* 1. Lib. 4. Cap. 9. Neither is it needful to insist on the Moons *Transits over*, and *Appulses to*, fixt Stars and Planets; which can never be accurately observed but by the Telescope. I cannot tell whether it be worth our while to take notice in this Place, that Monf. *Isadc Vossius* has published a fantastical Conceit of his own, for explicating the Appearance of the Moons Spots; in his *Liber variarum Observationum.*

(17.) *Planets whether inhabited.*

(17.) And now perhaps we may be allowed to sit down, and think awhile, whether all these Celestial Bodies, that thus dance round our Sun, may not be inhabited. But this Disquisition has been already so ingeniously managed by several, (particularly by the Reverend Dr. *Wilkins* Bishop of *Chester*, in his *World in the Moon*; and by Monf. *Fontenel* his *Plurality of Worlds*) that there is little left to be said on the Subject. I shall only add, that there is nothing in *Nature*, *Morality* or *Religion*, that contradicts the *Affirmative* of this Opinion. And 'tis through a narrowness of Thought that some men *deny* it. They will not think on any other sort of Creatures, than what we see here on Earth; and presently begin to ask, how should *Men* possibly live in *Saturn*'s *cold* Climates, or in the scorching *Heat* that affects *Mercury*. But shall we thus confine the *Great Creator* to our poor Conceptions? Cannot he that has made a Man, a Whale, an Elephant, a Fly, be able to create indefinite Varieties of Creatures, and all endowed with different Faculties, and various

Ways

Ways of Perception? Some adapted to one Planet, others to others? And all theſe may be ingaged in different Ways of Life and Thought; but ſhould all be ingaged in Praiſing and Serving him that gave them all their *Beings*.

(18.) But here I quit theſe remote Thoughts; and from viewing the admirable Extent, Beauty, Order and Variety of the Creation abroad, betake my ſelf *home* to our own Globe. And here we ſhall not lay aſide our *Teleſcope* as *uſeleſs*: We may imploy it on various Occaſions and divers Concerns of Human Life. The Merchant may with this diſcern afar his rich-laden Veſſels, whoſe Sides and Sails are ſwell'd, and look big with imported Wealth. The Seaman may diſcern his Friend or Foe. The wealthy Countryman may ſurvey his diſtant Herds, Plantations, and Labourers; and Generals may obſerve their wide-ſpread Troops. But endleſs would it be to touch on all; the contriving Head will find it *uſeful*, and adapt it to his own particular Concerns on many Occaſions, that cannot now be thought of. *(18.) Teleſcopes Uſe on Earth.*

(19.) And thus much ſhall ſuffice in ſhort, concerning the *Diſcoveries* made with the *Teleſcope*. And now I hope it will not be asked, *Cui Bono*? To what *End* are all theſe Diſcoveries? What Advantage is there in them? For, if the *Advancement* of *Aſtronomy* have any good in it; if the furniſhing us with a Contemplation from whence we may evince the *Power* and *Wiſdom* of an *Almighty Creator* be any Good; If affording an Opportunity of admiring the vaſt Extent, Order and Beauty of the Creation be any Good; I am ſure, hereby we reap all theſe *Goods* in an ample manner. But ſuppoſing that nothing of all theſe Advantages were at hand juſt at preſent, let not the inquiſitive Philoſopher therefore deſpond in his Enquiries. The conſideration of the *Magnet* (as I have noted before) teaches us what Secret Virtues may lurk in the ſimpleſt Things: And what admirable Uſes Poſterity may raiſe out *(19.) Uſes of the Teleſcopes Celeſtial Diſcoveries.*

out of them. The *Torricellian Experiment* was long apply'd *only* to Disquisitions concerning a *Vacuum*, before our incomparable Philosopher the Honorable Mr. *Boyle*, one of the chief Glories of the *English* Nation, discovered its Usefulness in predicting the Weather, by which 'tis become one of the most pleasant Instruments in the World. But this I say, not so much to encourage men in the Prosecution of *useless* Enquiries (for doubtless he is *best* imployed that can propose the *best* Advantages to Mankind from his Studies) But to discourage some men from exclaiming against all Labours as absolutely *useless*, whose *immediate* Use they do not apprehend.

(20.) *Of the Microscope.* (20.) And now that we are arrived at home, let us change our Instrument, and take into our hands a *Microscope*. And indeed with this, our Contemplations may be *endless*; all things affording such admirable Appearances, such curious Contexture of Parts, and such delicate vivid Colours; that the Contrivance of the *Almighty Creator* is as visible in the *meanest Insect* or *Plant*, as in the greatest *Leviathan* or strongest *Oak*. To touch upon all the Wonders this Instrument shews us, would be infinite; I shall therefore only refer to those, who have prosecuted Enquiries therewith to great Exactness.

Fontana. *Franciscus Fontana* in his *Observationes Cœlestium Terrestriumque Rerum* (wherein he challenges to himself the Invention of the *double Microscope*, *An.* 1618.) is the first (that I can learn) who published *Microscopical* Observations of some few Bodies. After Borellus. him *Borellus* at the end of his Tract, *De vero Telescopii Inventore*, Power. *An.* 1650. does the same. Next a learned *Englishman* Dr. *Power*, *An.* 1664. did the like. But all these went no farther than verbal Description; they want those curious and lively Schemes, Hook. which the learned and ingenious Mr. *Hook* presents to the World in his *Micrographia*, *An.* 1665. a Book full of admirable Discoveries by the *Microscope*, and other curious Enqui-Grew. ries. The learned Dr. *Grew*, and the excellent *Bononian* Philosopher

losopher *Marcel. Malpighius*, have laboured most successfully in the *Anatomy* of *Plants* by the Microscope; and the latter has used it much in his Books, *de Ovo incubato, de Bombyce, de Viscerum structura, &c.* The *Heer Lewenhoeck* of *Delft* in *Holland*, has lately apply'd himself with great Diligence to the use of Microscopes: of which Instrument he thinks he has a better kind than was ever yet known. When I visited this Gentleman at *Delft*, he shew'd me several that indeed were very curious; but nothing more than what I had ordinarily seen before; being composed only of one single, very minute Glass-Sphere or Hemisphere, placed between two very thin pierced *Laminæ*, or Plates of Brass, and the Object was brought to its due distance before the Glass by a fine Screw: But for his *best* sort, he beg'd our Excuse in concealing them. The Observations he has made with his Glasses are Printed in several Letters of his in *Dutch*; but for the most part, they are to be found dispers'd in the *Philosophical Transactions*. The last Author that has professedly treated of *Microscopick* Observations, is *Johan. Fran. Griendelius* in his *Micrographia nova Norimberg*. 1687. wherein he has taken a great deal of pains in giving the genuine Representations of his Objects as magnify'd.

Malpighius. Lewenhoeck. Griendelius.

I have been often delighted with the curious Appearance of many Objects seen through the Microscope. But none ever surprised me more, than the visible *Circulation* of the Blood in Water-Newts (*Lacerta aquatica*) to be seen as plainly as Water running in a River, and proportionably much more rapid. Of this I have formerly given the Account at large to the *Royal Society*. And 'tis publish'd in the *Philosophical Transact*. Num. 177. P. 1236.

Circulation of the Blood in Water-Newts.

(21.) I shall conclude all, with two remaining Uses of the *Telescope*. The first is, *To view nigh Objects therewith*. The most apposite Telescope for our Purpose, is that consisting of a *Convex Object-Glass* and *Convex Eye-Glass*. But what is deli-

(21.) *Viewing nigh Objects with the* Telescope.

 vered

vered hereof, is easily accommodated to the Telescope with a Concave Eye-Glass; but this latter, taking in but a narrow space of the Object, is not so proper for our Purpose.

Wherefore, suppose we have a Telescope, whose Object-Glass has its *Focal* Length for very *distant Objects* just 3 Foot, (how to find this exactly, I have shewn Chap. IV. Sect. 3.) This we'll call the *Solar Focus*. And suppose the Focus of the Eye-Glass be just 2 Inches; and that we were to view with this Telescope an Object 20 Foot distant from the Object-Glass. 'Tis required to find the *distinct Base* of this Object-Glass exposed now to this *nigh* Object 20 Foot distant, which in the First Part is called the *Respective Focus*.

And this is done by Prop. V. of the First Part: By this Analogy,

As the difference between the Distance of the Object and Glasses Focus:
To the Glasses Focus::
So the Distance of the Object from the Glass:
To the Distance of the respective *Focus, or distinct Base from the Glass.*

In the Numbers of our Example thus,

204 . 36 :: 240 . 42,35+ = equal to the distinct Base of this Object-Glass exposed now to this nigh Object. So that the Distance of the Object-Glass from the Eye-Glass, which, for viewing *distant* Objects, was 36+2=38 Inches; must now be 42,35+2=44,35 Inches; and so much is the Telescope to be lengthened to view this Object distant only 20 Foot.

Viewing Pictures in Miniature. The use of this Theorem is very pretty; and they who draw Pictures in *Miniature*, may practise it to good Purpose. For the small Picture (being *inverted* that it may appear *erect*, and placed in a strong Light, and thus looked at with a Telescope)

may

may be made to appear full as *bigg* as the natural Face, or *bigger* if we pleaſe. And by this means, the leaſt Errors of the Painter may be eaſily diſcovered, which cannot be done ſo well by a ſingle Convex-Glaſs; as will be manifeſt to thoſe who ſhall try and compare both Ways. And this naturally leads me to the laſt Uſe of the Teleſcope I ſhall here mention; and 'tis as it were the *Converſe* of this I have juſt ſpoke of. 'Tis,

(22.) *To Meaſure the Diſtance of an Object at one Station by a Teleſcope.* (22.) *Meaſuring Diſtances by a Teleſcope.*

This is the great *Deſideratum* in *Practical Geometry*; and always reputed *impoſſible.* Whatever Attempts have been made towards it have been always found to reſult at laſt to a *double Station,* tho ſome way or other diſguiſed by the Contrivance. This is evidently ſhewn by *Tacquet* (*Geom. Pract.* Lib. 1. *ad Finem* Cap. 5.) concerning the Method propoſed by *Clavius.* And may be ſhewn of any other, that I have ever yet heard of.

Towards the latter end of the year 1665. (as we may find Num. 7. P. 123. *Philoſoph.Tranſact.*) Monſ. *Auzout* did propoſe to Mr. *Hook*, to exchange with him a Secret he had in Opticks, for another which Mr. *Hook* had. Monſ. *Auzout*'s was, *Locorum Diſtantias ex unicâ ſtatione, abſque ullo Inſtrumento Mathematico metiri.* And therein he declares the Inſtrument, he uſes for this Purpoſe, to be a *great Teleſcope*, with ſome neceſſary Tables. Adding withal, "That tho the Practice do not al-" together anſwer the *Theory* of his *Invention*, becauſe that the " Length of the Teleſcope admits of ſome Latitude; yet one " comes *near enough*, and perhaps as *juſt*, as by moſt of the " Ways ordinarily uſed with Inſtruments. But tho Mr. *Hook* diſcovered the Secret which Monſ. *Auzout* deſired from him, (which is that I mention at the End of Chap. IV.) yet Monſ. *Auzout* never (that I know) publiſhed any thing farther of this *Hiſtory of this Affair.*

Invention of his own. And indeed I cannot ſee, how juſtly Monſ. *Azout* could ſay, that his Performance was, *abſque ullo Inſtrumento Mathematico*; and at the ſame time tell us, that it was done by a *great Teleſcope*; which ſurely is a *Mathematick Inſtrument* in the higheſt ſenſe: Perhaps he means, 'tis none of thoſe *Mathematick Inſtruments* commonly uſed in this Practice.

Mr. *Oldenburg*, who then publiſh'd the *Philoſoph. Tranſact.* adds Pag. 125. " That the Secret of meaſuring the Diſtance " of Places by a Teleſcope fitted for that Purpoſe, and for one " Station, is a Thing already known (if I am not miſsin- " formed) to ſome Members of the *Royal Society*; who have " been a good while ſince conſidering of it, and have contri- " ved Ways for the doing of it. Whether the ſame with " thoſe of Monſ. *Azout*, I know not; nor have I (at the Di- " ſtance that I am now from them) Opportunity of particular " Information.

This is in ſhort the Hiſtory of this Affair. Wherein (for ought as ever I could learn) nothing more has been done hitherto. So that what I ſhall propoſe therein is my own Thought and Contrivance. Whereof I ſhall firſt declare the Method; and afterwards ſhall not conceal the Difficulties thereof: not doubting but more ingenious Heads may have Contrivances of the ſame kind far exceeding mine; tho nothing herein has ever yet been publiſh'd.

The Author's Propoſal.

Wherefore let the Diſtance of the Object from the Object-Glaſs be called d, the *Solar Focus* of the Object-Glaſs, or its *Focus* at very diſtant Objects f, the *reſpective Focus* or diſtinct Baſe at a nigh Object r.

By Prop. V. and the Rule immediately mentioned:

It ſhall be — — — $d - f . f :: d . r$

And Compounding — $d . f :: d + r . r$

Permute — — — — $r . f :: r + d . d$

Divide — — — — $r - f . f :: r . d$

Which

Which laſt Analogy is the Rule I give for finding the Diſtance of an Object by a Teleſcope, thus expreſſed in Words,

As the Difference between the Reſpective Focus *and* Solar Focus:
To the Solar Focus::
So the Reſpective Focus:
To the Diſtance of the Object from the Object-Glaſs.

It now remains to ſhew how to obtain exactly the three firſt Terms of this Analogy; That thereby we may find the *fourth* required. And firſt for the *Solar Focal Length* of an Object-Glaſs, 'tis ſhewn before in Chap. IV. Sec. 3. how to procure it exactly. Secondly for the *Reſpective Focus*, or diſtinct Baſe of any *nigh* Object we look at, I propoſe the Way to obtain it thus:

As the *Micrometer* is contrived to *open* and *cloſe* in the Focus of the Object-Glaſs, and the Indices give the exact Meaſure of this *Opening*: So may we adapt an Inſtrument, which may *advance* or *withdraw* the curious Point of a ſlender Needle *to* or *from* the Object-Glaſs. And an *Index* (after the manner of the *Micrometer*) may ſhew how much the ſlender Point is withdrawn from the Object-Glaſs. Then looking through the Teleſcope at the Object, whoſe Diſtance we meaſure, let us withdraw the Needle, till by moving the Eye before the Eye-Glaſs, we perceive the Needle, as it were, *fixt* upon the Object: (as is taught Chap. V. Sect. 4.) Then is the Needle in the *Reſpective Focus*. And by obſerving the *Index* aforementioned, we have the Meaſure of this *Reſpective Focus* from the Object-Glaſs: And conſequently the Difference between it and the *Solar Focus*. With which we are to work according to the Rule, and we obtain what was required, *viz.* the *Diſtance* of the Object. Thus ſuppoſe, in the foregoing Example, the Glaſſes Focus be = 36 Inches, and the Re-

ſpective

ſpective Focus be meaſured 42,45 : The Analogy will then ſtand thus,

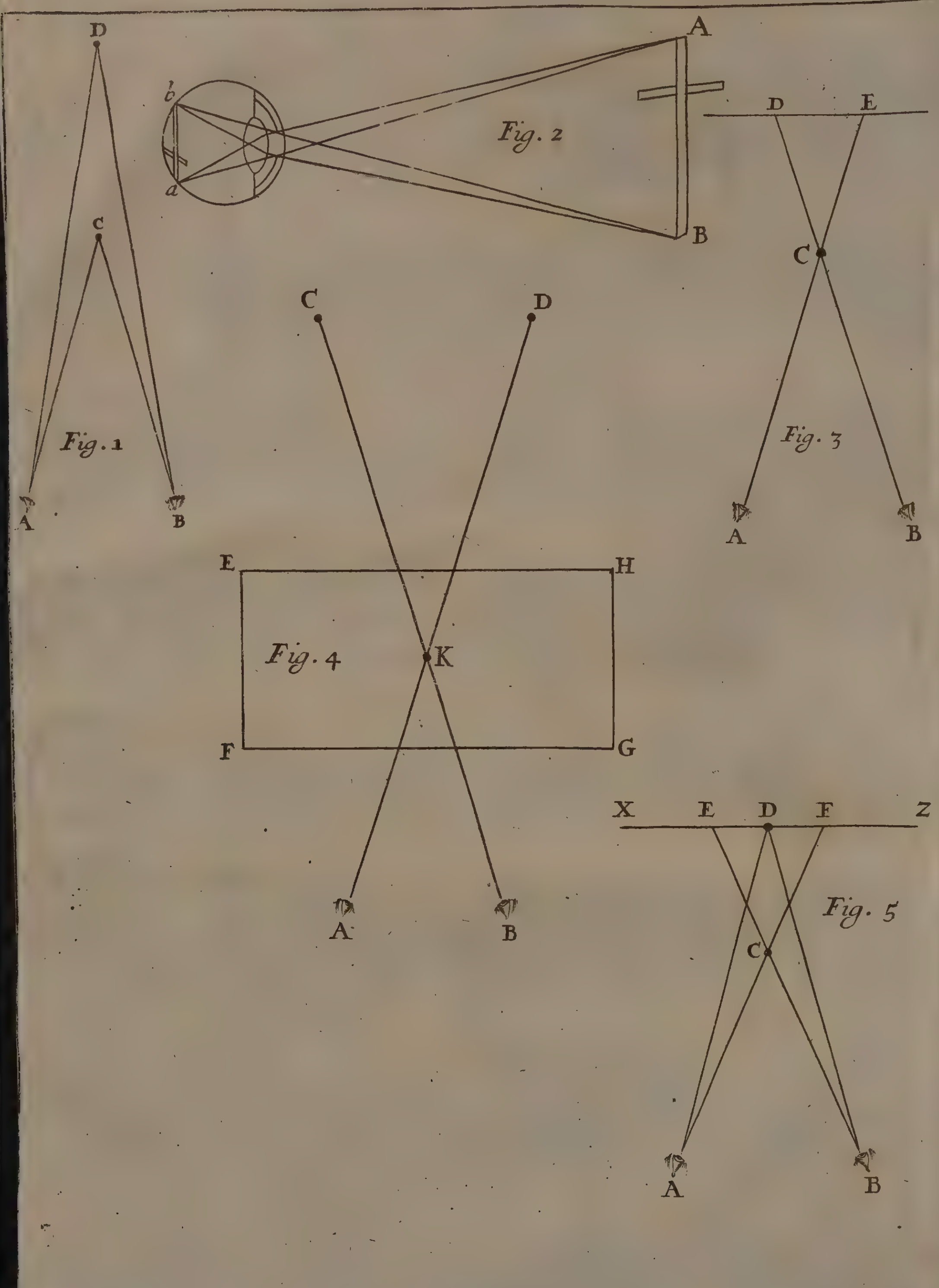
42,35 − 36 = 6,35 . 36 :: 42,35 . 240 = to the Diſtance of the Object from the Object-Glaſs. And thus may we make a *Table* to any *Teleſcope*, that upon the firſt Sight of the *Reſpective Focus* ſhall give the Diſtance of the Object.

Difficulties and Uncertainty thereof.

But now I ſhall not conceal the Difficulties, that attend this Method of obſerving Diſtances. And firſt, The Eye can never be *juſtly* certain, when the Needle's Point is *exactly* in the Reſpective Focus; for tho the Way I propoſe of examining it (*viz.* by moving the Eye before the Eye-Glaſs, and obſerving whether or not the Needle ſeems to move on the Object) be the beſt Expedient, and moſt certain that I can think of at preſent: Yet we ſhall find, that it admits of ſome *Latitude*; and that the Needle may be moved a little more *forward* or *backward*, without ſhewing any viſible Motion thereof on the Object; and this too moſt eſpecially, the longer the Teleſcopes are that we uſe, and the wider the Aperture. But for the Remedy hereof, 'tis beſt to obſerve where the Needle (being *too nigh* the Object-Glaſs) *begins firſt* to move on the Object; as alſo to obſerve, where the Needle (being *too far* from the Object-Glaſs) *begins firſt* to appear to move on the Object; and to take the *middle* between theſe two Stations of the Needle for the *true Reſpective Focus.*

Secondly, where the Diſtance of the Object is *very great*, and vaſtly diſproportional to the Solar Focal Length of the Object-Glaſs; the Obſervation will not be very accurate. And therefore it is that Monſ. *Auzout* in the forementioned Paſſage out of the *Philoſoph. Tranſact.* confines his Practice to *great Teleſcopes*. For unleſs they be *ſo*, a conſiderable Alteration in the Diſtance of the Object makes no ſenſible Alteration in the *Reſpective Focus* of the *Object-Glaſs.*

But



Tab. 41. pag. 287

But if we believe the ſaid Monſ. *Auzout*, by this Way, (if this Way which I propoſe be his, as I know not whether it be or not) *one comes near enough to Exactneſs, and perhaps as juſt as by moſt of the Methods ordinarily uſed.* However, tho the Practice, through ſome accidental Difficulties, do not ſo exactly anſwer the Theory : Yet it cannot be deny'd, that the Theory is true. And perhaps farther Improvements may bring it to Perfection.

Chap. VII.

An Optick Problem of Double Viſion.

IN *Tab.* 41. *Fig.* 1. AB are the two Eyes, C the nigher Object, D the farther Object; If both Eyes open are fixt upon C, the Object D ſhall ſeem double, and then ſhutting the left Eye A, the left Image of D diſappears; and ſhutting the Right Eye B, the right Image of D diſappears. *Tab.* 41. *Fig.* 1.

But if both Eyes open are fixed on D, then C ſhall ſeem double, and if the left Eye A be ſhut, the right Image of C vaniſhes, if the Right Eye B be ſhut, the left Image of C vaniſhes.

This is the Declaration of the Phænomenon, which any one may experiment by placing two Candles, D 3 Foot, and C 1 Foot diſtant from the Eyes, and then ſtanding ſo that the Noſe and two Objects may lie in or near a right Line, he fixing his Eyes on either Object, alternately open and ſhut them. I chooſe two Candles, and at that Diſtance and Poſture, becauſe the Experiment thereby will be more ſenſibly evident; tho it hold in any two Objects at whatever Diſtance, till the Diſtance be ſo great, that the Interval between the two

Eyes,

Eyes bears no ſenſible proportion thereto, that is, till the Angle ACB (and much more ADB) is ſo very ſmall, that the Lines AC, BC, may be taken to run as it were Parallel.

This being declared, I explicate the Reaſon of this Appearance, as follows.

In the firſt Caſe, both Eyes being fixed on C, if the Right Eye B be ſhut, the Object D will appear to the left hand of C; then ſhutting the Left Eye A, and opening the Right Eye B, and looking at C, D will appear to the Right Hand of C; therefore opening both A and B, and looking ſtedfaſtly on C, the Object D will appear on both ſides C, that is, both to the Right and Left Hand of C, and therefore double. And 'tis manifeſt, the Right Eye B receives the Right Image of D, and the Left Eye A the Left Image of D; therefore in this Caſe, to the Right Eye being ſhut, the Right Image of D diſappears, and to the Left Eye being ſhut, the Left Image of D diſappears.

But in the ſecond Caſe, fixing both Eyes on D, C ſeems double, for the Left Eye A being ſhut, C appears to B on the Left Hand of D; then the Right Eye being ſhut, the Left Eye A ſees C on the Right Hand of D; ſo that in this Caſe, the Right Eye receives the Left Image of C, and the Left Eye the Right Image of C, and conſequently the opening or ſhutting of either, does in this Caſe make the contrary Image of C diſappear. And why C ſhould ſeem double, is plain in this Caſe; for the Right Eye ſees it on the Left Hand of D, and the Left Eye on the Right Hand of D; ſo that both Eyes ſee it on both Hands of D, and therefore double. But then in the firſt Caſe, the Object D, and not the Object C, ſeems double. And in the ſecond Caſe, the Object C, and not the Object D, ſeems double. For in Viſion, there is a Difference between *looking* and *ſeeing*; what-

ever

ever Object I *look* at with both Eyes appears *ſingle*, and all others more remote or nigher, that I *ſee*, appear *double*; for upon the Object, I *look* at, the Optick Axes do concur, but not ſo on thoſe I only *ſee*. And this is the Reaſon that at any time I can make all the Objects about me ſeem confuſed, only by turning the Optick Axes ſo, that they may concur in the free Air; as if there were a certain viſible Object there before them, tho really there be none.

And that the Concurrence of the *Axes Optici* in a ſingle Point or Object is ſufficient to make that Object ſeem but one beſides the Proof of the foregoing Experiments, I ſhall endeavour to evince, or at leaſt to explicate, by an other known Affection of Viſion; the Explanation whereof is allowed by all men as ſatisfactory, 'tis this, in *Tab.* 41. *Fig.* 2. T. 41. F. 4.
the Image *a b* of the Object AB is painted on the *Retina* inverted, and yet the Eye (or rather the Soul by means of the Eye) ſees the Object erect and in its natural Poſture; Becauſe the Mind takes no notice of what happens to the Rays in the Eye by Refraction or Decuſſation, but in its direction towards the Object; it follows ſtreight alongſt the Rays as they by their Impulſe and in their plain Courſe lead it, and conſequently following the Rays *a A*, it is directed ſtrait to the upper part of the Object; and alſo following the Rays *b B*, it is directed to the lower part of the Object, and ſo of the reſt: for ſuppoſe the Ball of the Eye taken out of its Socket, or Cavity in the Skull, and a Man receives in the Socket an impulſe by a Stick coming in the poſture of *B b*, and hitting him on the upper part of the Cavity, ſurely he would never look for the Original of this Blow at *A*, but would be certainly directed to hunt back as it were alongſt the Stick *b B* towards the Place from whence the Stroak comes. So the mind does hunt back by means of each Pencil of Rays (which are as it were a Stick giving the *Retina* a certain

 Impulſe)

Impulſe) to the Point from whence it comes, and is thereby directed ſtrait thereto. To apply this to what I intend, I ſay then, that the Mind or viſive Faculty (if I may have leave to uſe that Word for a thing we all underſtand and cannot better expreſs) takes no notice that there are two *Axes Optici*, or two Pictures made by thoſe *Axes Optici* on each *Retina*, but following back, and hunting *counter* alongſt theſe Axes, it is directed to, and determined in *one ſingle* Point, and therefore it ſees it as *one*.

And ſeeing this Speculation does naturally lead me to the Conſideration of an Opinion, firſt as I think ſtarted by the celebrated *Gaſſendus*, and ſince embraced by many, *viz.* that we ſee but with one Eye at once one and the ſame Point of an Object, *otiante alio*, (as they term it) whilſt the other is idle and does nothing. I ſhall not think it improper to ſubjoyn to my former Diſcourſe ſomething relating to that Opinion, confining my ſelf only to what may ſomething illuſtrate my former Explanations, for to enumerate all the Experiments that prove we ſee with two Eyes, would ſwell this far beyond the limits I deſign it. Therefore to the Matter. v. p113

Againſt our ſeeing with two Eyes at once one and the ſame Point of an Object, it is commonly objected, that if it were ſo, every Object would ſeem in two places at once, *vide* Gaſſendi *Epiſt.* 4. *de Magnit. Solis humilis & ſublimis, &* Taqueti *Opt. lib.* 1. *Prop.* 2. Thus in Tab. 41. Fig. 3. If the Eyes A, B, look at the Object C, and both ſee it at a time, A would ſee it on the oppoſite Wall ſuppoſe at E, and B at D.

T. 41. F. 3.

I am ſo far from thinking this an Objection, that I aſſert firſt, that we do ſee all Objects in two places, and that this is not taken notice of by us upon theſe Accounts. Firſt, the common Objects of our ſight are large, and the Axes of our Eyes

directed

directed to but one Physical Point thereof at a time, and then we cannot be expected to see such in two places at once; for when I read on a flat broad Book, where is the opposite Wall for a single small Letter to be seen on at two places, the Letters themselves are fixt to the Surface that determines the Sight. What I Instance in Letters on a Book may be accommodated to most other Objects. Secondly, There are few Objects disposed fitly for shewing us that we see them in two places, for either their Bulk hinders the Experiment from appearing so plainly, or their distance from each other or from the Eyes. Thirdly, The Wall D E and the Parts thereof are seen so confusedly by the Eyes A, B, when fixed on C, that we cannot so clearly observe this Experiment; and this happens to all Objects more remote or nigher to us than C, when the Eyes are fixed on C, still supposing that it self be but in a moderate Distance from us, that is, in such a Distance from us, as may bear some considerable Proportion to the Distance of the two Eyes from each other. Fourthly, the chief Reason why we do not perceive so plainly the Object C to cover both D and E is, because, though D be cover'd by C from the Eye B, yet it is not covered from the Eye A, and therefore we think it not covered at all. Also though E be covered by C from the Eye A, yet it is not covered from the Eye B, and consequently because we see E, tho but with one Eye B, we imagine it not covered at all, so that both the Points D and E being open to both the Eyes A and B, *viz.* the Point D to the Eye A, and the Point E to the Eye B, we think neither of them covered, whereas really they are both obscur'd, each to its proper Eye, as will be evident by winking. And from hence is manifest the falsity of *Taquet*'s Assertion in the forecited Place, *viz.* that opening both Eyes A and B (*Fig.* 3.) and looking at C, it shall appear only to cover E (he imagining that most Men see generally with their left Eye) whereas I

ſay it ſhall appear to obſcure neither E nor D, tho to each Eye reſpectively it cover each. Fifthly, whereas the Bulk of Objects (as I have noted) does hinder the Experiment from being ſo ſenſibly evident; if by any Experiment I can ſhew how a large Object may be doubled, I think 'twill be ſufficiently plain. Therefore in *Tab*. 41. *Fig*. 4. let A, B, be the two Eyes , before which at a convenient Diſtance place the two Candles C, D, then take a large piece of Paper EFGHK, in whoſe middle at K there is a ſmall Hole. This Paper ſo place between the Eyes and Candles, that the Eye A may through the Hole K ſee the Candle D; and at the ſame time the Eye B may ſee through the Hole K the Candle C; the due Diſtance of the Paper that is requiſite for this will be found by Tryals, and winking alternately with the Eyes. When all things are in this Poſture, open both Eyes A and B, and direct them to look at either of the Candles, I ſay the Hole K ſhall ſeem double; and what is proved of the Point K is true of all other Points in the Paper, ſo the whole Paper appears double; but no Points thereof are ſo evidently doubled as the Point K, becauſe they want the Advantage of Lights behind them to render the Experiment more ſenſible. And upon this Occaſion I cannot but hint to all Perſons that are deſirous to make Experiments about Viſion, that they always imploy the moſt luminous and vigorous Objects they can poſſibly, for many Experiments will be evident by them, that will not ſenſibly ſucceed with others.

T.41.F.2.

But I aſſert ſecondly, that unleſs we ſaw with both Eyes, the two firſt mentioned Experiments would not ſucceed, and they may be reckoned amongſt the greateſt Arguments that can be produced for it. For ſuppoſe in *Fig*. 1. A and B to look ſtedfaſtly on C, I ſay they ſee C in two Places at once, for they ſee it both on the Right hand and Left hand

of

of D. For tis the ſame thing to ſee C between two D's, as to ſee it at two Places at once. And this I'll declare more fully *Tab.* 41. *Fig.* 5. ſuppoſing the two Eyes A and B fixed on the Object C, I ſay C appears on the oppoſite Wall X Z in two Places, *viz.* at E and F (tho neither E nor F are obſcur'd thereby for the foregoing Reaſon) for let us ſuppoſe the Object D of the firſt Figure to be placed duly on this Wall; ſhutting A, the Object C ſhall appear to B on the left hand of D by the Angle D B E: then ſhutting B, the Object C ſhall appear to A on the right hand of D, by the Angle D A F; wherefore to both open at a time and looking at it, it ſhall appear on both ſides of D, to A on the right ſide, to B on the left ſide, and this happens by doubling of D; ſo that from hence this Paradoxical Corollary ariſes, *That an Object may be ſeen in two Places, and yet not ſeen double.* T.41. F.5.

They that aſſert that we ſee but with one Eye, whilſt the other is idle and does nothing, aſſert likewiſe that all viſual Spirits recede from the idle Eye, and only ſupply the Eye that ſees; but I ſee no Reaſon why the Eyes A, B, looking at C, either of them ſhould be ſaid deſtitute of Spirits in reſpect of C, which they ſee ſingle, and yet both repleniſhed with Spirits in reſpect of D, which they ſee Double. The Images of C and D both are painted on the *Retina* of A as well as of B, and therefore A and B both being repleniſhed with Spirits to ſee D, (even by the Conceſſion of our Adverſaries) why not to ſee C alſo, at which they ſtedfaſtly look?

To conclude, I propoſe it to the learned and ingenious Dr. *Briggs*, or to any other of the Philoſophical Spirits of this Age to explain the foregoing Phænomena by the Doctor's Theory of Viſion. And as a Concluſion to the whole ſhall only add one Experiment that Demonſtrates we ſee with

both

both Eyes at once; and 'tis, that which is commonly known and practiſed in all Tennis-Courts, that the beſt Player in the World Hoodwinking one Eye ſhall be beaten by the greateſt Bungler that ever handled a Racket; unleſs he be uſed to the Trick, and then by Cuſtom he gets an Habit of uſing one Eye only.

APPENDIX.

Fig. 1.

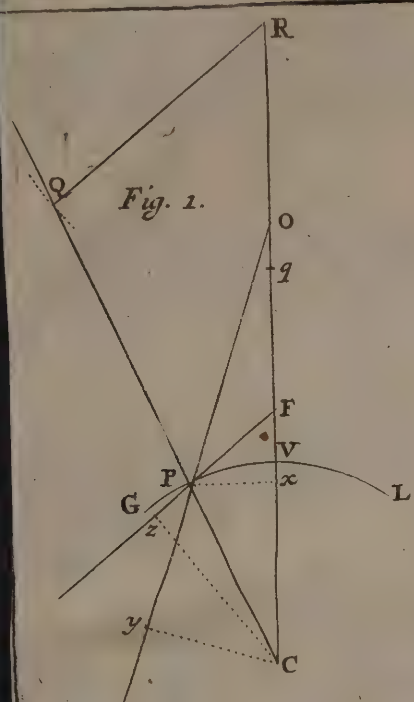


Fig. 2.

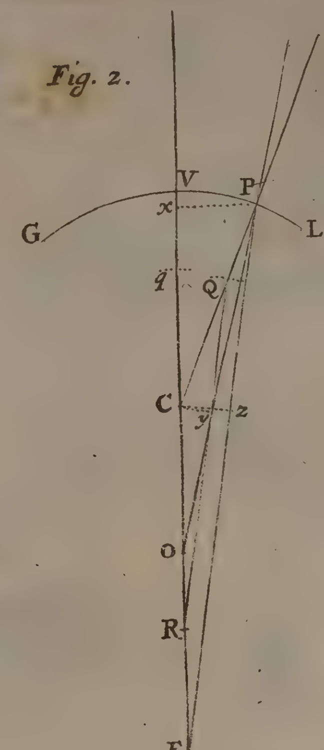


Fig. 3.

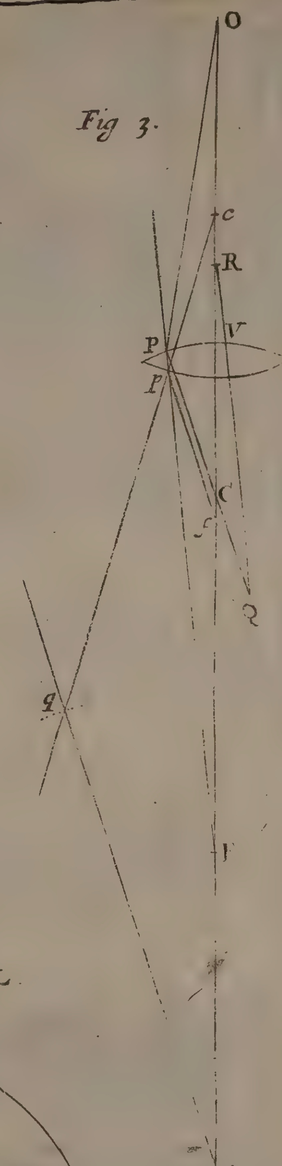


Fig. 4.

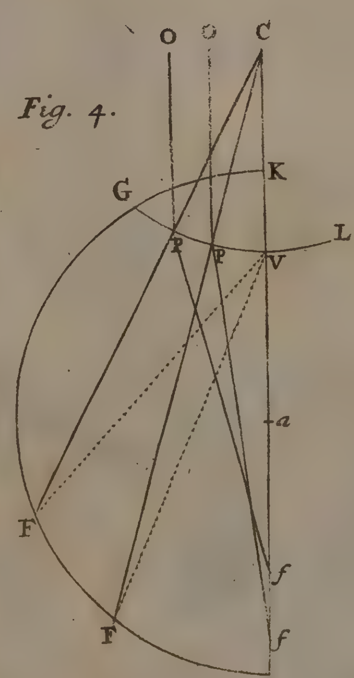
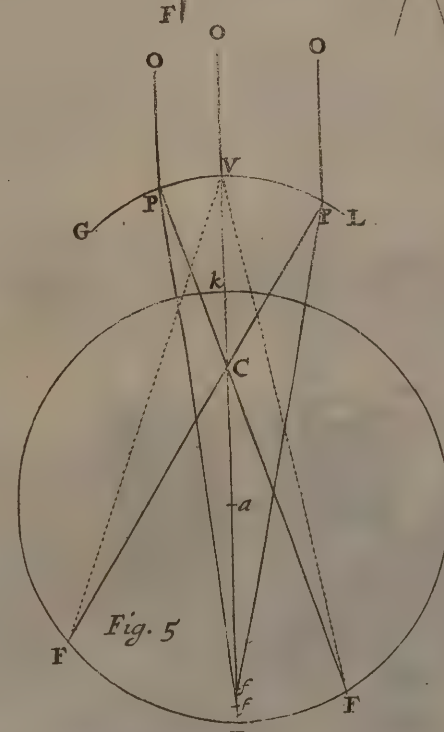
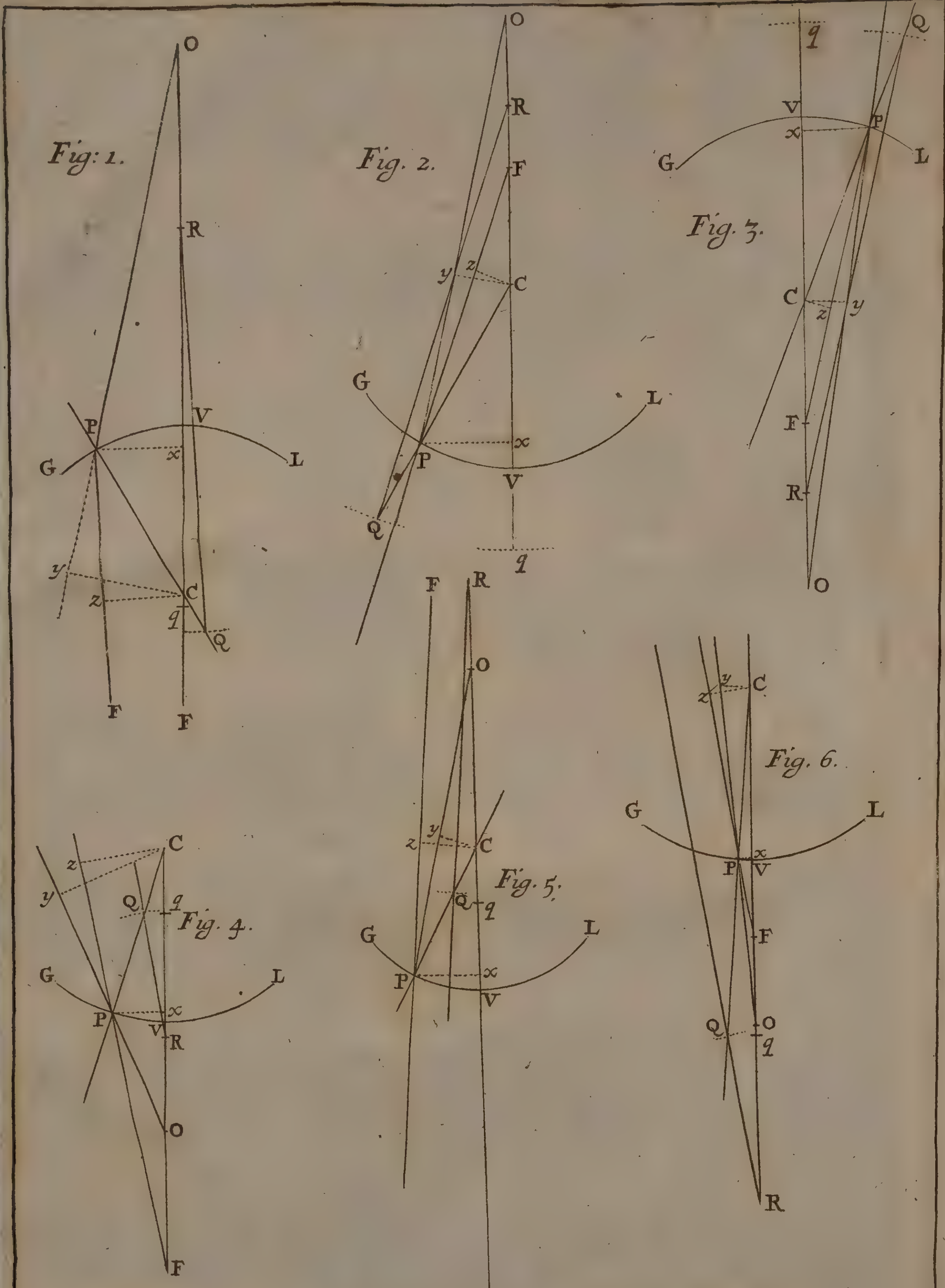


Fig. 5





Tab. 42. Pag. Ult

APPENDIX.

Whilst this Book was in the Press my Affairs were such, that I could not attend the Perusal and Correction thereof, but therein have made use of my Friend Mr. E. Halley, *who was willing to do me that Service: He, after the first Part hereof was finished, sent me a Proposition of his own, which I took to be of that Consequence in Dioptricks, that I importuned him to permit it to be subjoyned by way of* Appendix *to my Treatise, it being of that Extent as to comprehend the whole Doctrine of the* Foci *of Spherical Glasses of all Sorts, exposed either to Diverging, Converging, or Parallel Rays. It is as follows.*

PROPOSITION.

TO find the *Focus* of any Parcel of Rays *Diverging from*, or *Converging to* a given Point in the *Axis* of a *Spherical Lens*, and inclined thereto under the same Angle; the *ratio* of the Sines in Refraction being known.

Let G L *Tab.* 42. be the *Lens*, P any Point in its Surface, V the Pole thereof, C the Center of the Sphere whereof it is a Segment, O the Object or Point in the Axis, to or from which the Rays do proceed, O P a given Ray: and let the *ratio* of Refraction be as *r* to *s*; make C R to C O as *s* to *r* for the immersion of a Ray, or as *r* to *s* for the Emersion, (that is, as the Sines of the Angles in the Medium which the Ray enters, to their corresponding Sines in the Medium out of which it comes) and laying C R from C towards O, the Point R shall be the same for all the Rays of the Point O. Then draw the Radius P C if need be continued, and with the Centre R and Distance O P sweep a touch of an Arch intersecting P C in Q; the Line Q R *Tab.* 42.

being

being drawn ſhall be parallel to the refracted Ray, and PF being made parallel thereto ſhall interſect the Axis in the Point F, which is the Focus ſought Or make it as CQ : CP :: CR : CF and CF ſhall be the Diſtance of the Focus from the Centre of the Sphere.

Demonſtration.

Let fall the Perpendiculars *Px* on the Axis, *Cy* on the given Ray and *Cz* on the refracted Ray. By the Conſtruction PF and QR are parallel, whence the △s QRC and PFC are ſimular, and CR to QR as CF to PF, that is, CR to OP as CF to PF. Now CF : PF :: *Cz* : *Px ob ſimilia Triang.* whence CR : OP :: *Cz* : *Px*, and CR : *Cz* :: OP : *Px*. Again CR is to CO as the Sines of Refraction, by conſtruction, that is as *s* to *r* or *r* to *s*; and as CR to *Cz*, ſo CO $= \frac{r}{s}$ or $\frac{s}{r}$ CR to $\frac{r}{s}$ or $\frac{s}{r}$ *Cz*, and ſo PO to *Px* : But as PO to *Px*, ſo CO to *Cy*. *Ergo Cy* $= \frac{r}{s}$ or $\frac{s}{r}$ *Cz*, that is, *Cy* to *Cz* is as the Sines of Refraction, but *Cy* is the Sine of the Angle of Incidence, and *Cz* of the refracted Angle. *Ergo conſtat Propoſitio.*

The ſeveral Caſes of Rays Diverging or Converging as they enter the curve Surface of a *Convex* or *Concave Lens*, are for the Readers Eaſe delineated in the firſt four Figures of *Tab.* 42. as are in *Fig.* 5 and 6 thereof, and in *Fig.* 1, 2. of *Tab.* 43. the like Caſes of emerging Rays. All which are drawn with the ſame Letters to their reſpective Points, only in ſome the Point F falling far diſtant, is to be underſtood in the Interſection of the Line PF with the Axis.

This thus demonſtrated in the moſt difficult Caſes, will give all the Rules for the Foci of Rays parallel to the Axis as likewiſe for the principle Focus, where the Rays neareſt the Axis do unite, all which Rules I ſhall collect in theſe following Corollaries.

Cor. 1. If OP be equal to CR, then the Points Q and C are coincident, and the Rays OP after Refraction run on parallel to the Axis.

Cor. 2.

Cor. 2. If the Point Q fall on the ſame ſide the Axis as is the Point P, then the Beams after Refraction do tend on, either diverging or converging as before: But if Q fall on the other ſide the Axis, as in *Fig.* 1. the diverging Rays are made to converge by a Convex, or the converging to diverge by a Concave Glaſs.

Cor. 3. If O P do exceed C R, the Focus is in all Caſes on the ſame ſide of the Glaſs as is the Centre of the Sphere C. But contrarywiſe if O P be leſs than C R, the Focus falls on the other ſide of the Glaſs beyond the Vertex V.

Cor. 4. An Object may be ſo placed, that the Rays next the Axis of a Convex-Glaſs ſhall have an imaginary Focus, tranſmitting diverging Rays, when the more remote Parts thereof ſhall make them converge to a real Focus.

Cor. 5. If O V the Diſtance of the Object from the Pole or Vertex of the Glaſs, be taken inſtead of O P, then will C Q be the difference of O V and C R, and as that difference to C R, ſo the Radius C V to C F the Diſtance of the principal Focus from the Centre of the Sphere whereof the Glaſs is a Segment: or elſe as C Q to O P or R Q:: ſo P C to V F the focal diſtance from the Pole of the Glaſs. Whence follows a general Rule for the Foci of all Glaſſes, only according to *Corol.* 3. if O V do exceed C R, the Focus is on the ſame ſide of the Glaſs as is the Centre of the Sphere. But if C R be the greater, then the Focus is on the oppoſite ſide of the Glaſs, whence it will be determined whether the Focus be real or imaginary.

Cor. 6. What has hitherto been ſaid of one Surface of a *Lens* is eaſily applicable to the other; taking F the *Focus* of the firſt Surface as an Object, and uſing it as O in the Figures for emerging Rays, whereby the *Focus* of both Surfaces will be determined, as in *Tab.* 43. *Fig.* 3. where I have given an Example.

Cor. 7. Hitherto we have conſidered only *oblique Rays* either *Diverging* or *Converging*; it now remains to add ſomething con-

cerning *Rays parallel* to the *Axis*: In this Caſe the Point O muſt be conſidered as infinitely diſtant, and conſequently OP, OC, and CR are all infinite; and OP and OC are in this Caſe to be accounted as always equal, (ſince they differ but by a Part of the Radius of the Sphere GPVL, which is no part of either of them,) wherefore the *ratio* of CR to OP will be always the ſame, *viz.* as *s* to *r* for immerging Rays, and as *r* to *s* for thoſe that emerge. And by this Propoſition CF is to PF in the ſame *ratio*. It remains therefore to ſhew on the Baſe CP, to find all the Triangles CPF wherein CF is to PF in the *ratio* given by the degree of Refraction. This Problem has been very fully conſidered by the celebrated Dr. *Wallis* in his late Treatiſe of Algebra, pag. 258. to which I refer; but I muſt here repeat the Conſtruction thereof, *Tab.* 43. *Fig.* 4, 5.

Let GPVL be a *Lens*, VC or PC the Radius of its Sphere, and let it be required to find all the Points *f*, *f*, ſuch as *Cf*, may be to *Pf* in the given *ratio* of *s* to *r* for immerging Rays, or as *r* to *s* for the emerging. Divide CV in K, and continue CV to F, that CK may be to VK, and CF to VF in the propoſed *ratio*: Then divide KF equally in the Point *a*, and with that Center ſweep the Circle FKF; this Circle being drawn gives readily all the Foci of the Parallel Rays OP, OP. For having continued CP till it interſect the Circle in F, PF ſhall be always equal to *Vf* the Diſtance of the Focus of each reſpective Parcel of Rays OP, from the Vertex or Pole of the *Lens*.

To demonſtrate this, draw the prickt line VF, and by what is delivered by Dr. *Wallis* in the aforecited Place, VF and CF, will be alwayes in the ſame propoſed *ratio*. Again *Vf* being made equal to PF, CF and *Cf* will be likewiſe equal, as are CP, VC; and the Angles P*Cf*, VCF being *ad verticem* are alſo equal: Wherefore *Pf* will be equal to VF, and conſequently *Cf* to *Pf* in the ſame *ratio* as CF to VF, whence

and

and by what foregoes, the Points f, f are the ſeveral reſpective Foci of the ſeveral Parcels of Rays, OP, OP. Q. E. D.

That CF is to PF in the *ratio* of the Refraction, in the caſe of parallel Rays, will be yet more evident, if it be conſider'd, that the Angle at C is equal to the Angle of Incidence, and the Angle at P to the refracted Angle; wherefore PF the ſide oppoſite to the Angle at C, is as the Sine of the Angle of Incidence, and CF oppoſite to the Angle at P, is as the Sine of the reſpective refracted Angle; whence in all Caſes of parallel Rays, CF is to PF in the ſame conſtant *ratio* of Refraction.

If it ſhall be deſired to effect in Numbers what we have here done by Lines, it will be moſt eaſie to adapt a Calculus to the foregoing Geometrical Conſtruction. For if in the Triangle POC there be given the Radius CP equal to *Unity*, CO the diſtance of the Object from the Centre of the Sphere, and the Perpendicular $\mathcal{P}x$ equal to the Sine of the Angle PCO, the ſide PO = QR will be equal to $\sqrt{CO^{q.} + CP^{q.} \pm 2CO \text{ in } \sqrt{CP^{q.} - \mathcal{P}x^{q.}}}$ Then as QR or PO to $\mathcal{P}x$ :: ſo CR to the Sine of the Angle CQR, and the Complement to 180$^{gr.}$ of the ſum of the Angles CPO and CQR is the Angle CRQ = CFP; and as $\mathcal{P}x$ to PO ſo the Sine of the Angle CRQ to CQ; and as CQ to CP ſo CR to CF, which is the diſtance of the reſpective Focus of all the Rays PO from the Centre of the Sphere C.

But the Foci of Rays parallel to the *Axis* may be more readily computed, following the Footſteps of the Conſtruction delivered in *Coroll.* 7. (*Tab.* 43. *Fig.* 4, 5.) for thereby it will appear, that the Radius of the Circle KF, *viz.* aF, is equal to $\frac{rs}{rr-ss}$ CP, and $Ca = \frac{rr}{rr-ss}$ CP, for emerging Rays, as in *Fig.* 4; but for immerging Rays, as in *Fig.* 5, Ca will be found to be $\frac{ss}{rr-ss}$ CP: and ſuppoſing the diſtance of the Ray from the

Axis $= \mathcal{P}x$, in the Caſe of parallel Rays Emerging, the diſtance of the Focus will be found, $PF = \frac{rr}{rr-ss}\sqrt{CP^q - \mathcal{P}x^q} + \sqrt{\frac{rs}{rr-ss}CP^q - \frac{rr}{rr-ss}Px^q} - CP$: that is, r to s being as 3 to 2, $PF = \frac{9}{5}\sqrt{CP^q - \mathcal{P}x^q}. + \sqrt{\frac{6}{5}CP^q - \frac{9}{5}Px^q} - CP$. And for immerging Rays, the Focal diſtance is found by a like Rule, $PF = \frac{ss}{rr-ss}\sqrt{CP^q - Px^q} + \sqrt{\frac{rs}{rr-ss}CP^q - \frac{ss}{rr-ss}Px^q} + CP$: that is r and s being as 3 to 2 as before, PF is equal to $\frac{4}{5}\sqrt{CP^q - Px^q} + \sqrt{\frac{6}{5}CP^q - \frac{4}{5}Px^q} + CP$. Theſe Canons are ſo eaſily deduced from the Conſtructions, that I ſhall not need to trouble the Reader with their Demonſtration; only I ſhall add two Tables which I computed from them, with little more work than a continual Addition; which may by way of Example, ſerve to inſtruct and exerciſe the young Student in this part of Mathematicks.

Suppoſe C P the Radius of the Sphere of Glaſs 2 Inches, and the *ratio* of Refraction as 3 to 2; at each tenth of an Inch diſtance from the Axis, the *Foci* are as follows.

For Emerging Rays.

Px	PF
0	√ 12,9600 + √ 5,7600 — 2
1	√ 12,9276 + √ 5,7276 — 2
2	√ 12,8304 + √ 5,6304 — 2
3	√ 12,6684 + √ 5,4684 — 2
4	√ 12,4416 + √ 5,2416 — 2
5	√ 12,1500 + √ 4,9500 — 2
6	√ 11,7936 + √ 4.5936 — 2
7	√ 11,3724 + √ 4,1724 — 2
8	√ 10,8864 + √ 3,6864 — 2
9	√ 10,3356 + √ 3,1356 — 2
10	√ 9 ,7200 + √ 2,5200 — 2

For Immerging Rays.

Px	PF
0	√ 2,5600 + √ 5,7600 + 2
1	√ 2,5536 + √ 5,7536 + 2
2	√ 2,5344 + √ 5,7344 + 2
3	√ 2,5024 + √ 5,7024 + 2
4	√ 2,4576 + √ 5,6576 + 2
5	√ 2,4000 + √ 5,6000 + 2
6	√ 2,3296 + √ 5.5296 + 2
7	√ 2,2464 + √ 5.4464 + 2
8	√ 2,1504 + √ 5.3504 + 2
9	√ 2.0416 + √ 5.2416 + 2
10	√ 1.9200 + √ 5.1200 + 2

But

But it is to be Noted, that these *Foci* for Immerging Rays, must not be taken for the Foci of a Plano-Convex, when the Convex Side is towards the Object, for the plane Side by its Refraction, does contract the Focal length by about a Semidiameter of the Sphere; These suppose the Body of Glass continued, as in the First Proposition of this Treatise.

FINIS.

ERRATA.

Page	Line	For	Read
3	26	V	IV
4	12	Propositions	Proportions
50	23	6th.	16th.
57	17	Convexity	Concavity
58	5	Plano-Convex	Plano-Concave
67	8	4th.	5th.
141	24	Breadth of the Object	½ Breadth of the Object
143	6	*Dele* XV	
149	12	We cannot add	We are to add
160	4	Lemma	Lemma. I.
193	4	CF or FZ + FD	CF (or FZ) + FD.
219	13	*l h* more than *l i*	*l h* is greater than *l i*
219	14	*h g* less	*h g* is less
220	1	shall be	as shall be
264	22	his	this

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where the object is most confus'd p. 130
'tis requisite